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Preferences with Frames: A New Utility Specification that Allows for the Framing of Risks

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Abstract

Experimental work on decision-making shows that, when people evaluate risk, they often engage in "narrow framing": that is, in contrast to the prediction of traditional utility functions defined over wealth or consumption, they often evaluate risks in isolation, separately from other risks they are already facing. While narrow framing has many potential applications to understanding attitudes to real-world risks, there does not currently exist a tractable preference specification that incorporates it into the standard framework used by economists. In this paper, we propose such a specification and demonstrate its tractability in both consumption/portfolio choice and equilibrium settings.

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1 Introduction

When economists model the behavior of an individual agent, they typically use utility functions defined over total wealth or consumption. Such utility functions make a precise prediction as to how the agent evaluates a new gamble he is offered: he merges the new gamble with other risks he is already facing to determine its effect on the distribution of his future wealth or consumption, and then checks if the new distribution is an improvement or not.

The experimental literature on decision-making under risk has uncovered many instances in which people do not appear to evaluate gambles this way: instead of merging a new gamble with other risks they are already facing and checking if the combination is attractive, they often evaluate the new gamble in isolation, separately from their other risks. Such behavior is known as "narrow framing" (Kahneman and Lovallo, 1993; Kahneman, 2003). "Framing" is a general term used to describe the way people think about and evaluate gambles, while "narrow" emphasizes that people sometimes evaluate a new gamble by thinking about the distribution of the gamble, taken alone, and not just about the distribution of their overall wealth once the gamble is added to their other risks. More formally, narrow framing means that the agent derives utility *directly* from the outcome of a specific gamble he is offered, and not just indirectly via its contribution to his total wealth. Equivalently, he derives utility from the gamble's outcome *over and above* what would be justified by a concern for his overall wealth risk.

While narrow framing has been documented most clearly in experimental settings, it also has the potential to address attitudes to risk in the field. In particular, there are numerous real-world situations in which people appear to neglect simple opportunities for diversification. Stock market non-participation – the refusal, over many decades, of many households to allocate even a small amount of their wealth to the stock market, even though equity is relatively uncorrelated with other major household risks – is one example (Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995). Another is home bias – the refusal of many households to diversify what holdings of domestic stock they do have with even a small position in foreign stock (French and Poterba, 1991). Yet another example is the relatively large fraction of wealth that some households allocate to just a few individual stocks (Curcuru et al., 2004).

At least in the absence of frictions, it is hard to explain such behavior with traditional utility functions defined only over wealth or consumption. Investors who pay attention to the riskiness of their overall wealth are invariably keen to take advantage of opportunities for diversification. An investor who evaluates risks in isolation, however, misses these opportunities, thereby making it easier to understand why he might fail to exploit them. For example, an investor who evaluates an individual stock in isolation will fail to notice its diversification benefits; he will therefore be reluctant to buy it, leaving him with a portfolio made up of too few stocks. Similarly, an investor who thinks about foreign stock market risk in isolation will fail to notice the diversification benefits, and is therefore more likely to exhibit home bias.

In spite of these potential applications, it has not been easy for economists to explore the implications of narrow framing for real-world phenomena, for the simple reason that there are no tractable preference specifications that allow for narrow framing. The development of such preferences faces many hurdles. The specification should allow the agent to derive utility directly from specific gambles he is taking, but also, as in traditional models, to derive utility from total wealth or consumption. In other words, it must allow for both narrow and traditional "broad" framing at the same time. Moreover, it must be implementable even in the dynamic settings favored by economists.

In this paper, we present an intertemporal preference specification that meets these requirements. In our framework, the agent gets utility from a gamble's outcome both indirectly, via its contribution to his total wealth, but also directly. The specification is very tractable, both in partial equilibrium, allowing for an investigation of portfolio and consumption choice, but also in equilibrium, allowing for an exploration of the effect of narrow framing on prices. We present examples of both kinds of analysis. We also show that, in our framework, the agent's indirect value function takes a simple form, making it easy for the researcher to calibrate our preferences by checking the agent's attitude to timeless monetary gambles.

In constructing our preference specification, we do not take an axiomatic approach. The reason is that, in extreme cases, narrow framing can lead the agent to choose a dominated alternative, a prediction that is hard to reconcile with a set of normative axioms. We therefore take another approach, which is simply to posit a specification that allows for narrow framing, and to show that it has attractive properties as well as a number of potential applications, both in partial equilibrium and in full equilibrium settings.

In order to explore applications of narrow framing, a researcher needs two things: a tractable preference specification that allows for narrow framing; and a theory of which risks agents will frame narrowly. In this paper, we focus on the first component, in other words, on developing a tractable preference specification. The second component is equally important, but is not the focus of our study. When necessary, we will rely on the most prominent currently-available theory of framing, namely Kahneman's (2003) "accessibility" theory. We describe this theory in Section 2.

The one previous attempt to incorporate narrow framing into standard preferences is that of Barberis, Huang, and Santos (2001). They investigate the implications, for the pricing of the aggregate stock market, when investors are loss averse, in other words, more sensitive to losses – even small losses – than to gains of the same magnitude. For part of their analysis, Barberis, Huang, and Santos (2001) take the gains and losses to be gains and losses in stock market wealth, rather than in total wealth; in this case, then, they are implicitly assuming that investors frame narrowly, and their preference specification reflects this.

While Barberis, Huang, and Santos' (2001) specification is tractable in equilibrium settings, it also has some serious limitations. First, it is intractable in partial equilibrium and so cannot be used to understand observed portfolio choice. Second, the agent's indirect value function cannot be explicitly computed, making it hard to calibrate the utility function by checking attitudes to monetary gambles. Finally, to ensure stationarity in equilibrium, the narrow framing term in their specification requires an ad-hoc scaling by aggregate consumption.

In this paper, we show that our new preference specification improves on that of Barberis, Huang, and Santos (2001) in several significant ways. Our preferences *are* tractable in partial equilibrium; they *do* admit an explicit value function; and they do *not* require any ad-hoc scaling. Even in equilibrium settings, where Barberis, Huang, and Santos' (2001) specification is tractable, our formulation offers an important advantage: since it leads to an explicit value function, it allows the researcher to check whether the parameter values used in any particular application are *reasonable*, in terms of predicting sensible attitudes to timeless monetary gambles.

Barberis, Huang, and Thaler (2006) apply the narrow framing preferences that we develop here in an analysis of the stock market participation puzzle, while Barberis and Huang (2007) apply them in an study of the equity premium. The distinct contribution of this paper is the formal derivation, in full generality, of the equations that govern portfolio choice, asset pricing, and attitudes to timeless monetary gambles. For example, it is in this paper that we derive the first-order conditions for optimal consumption and portfolio choice; and it is in this paper that we show how narrow framing can be incorporated into a full equilibrium setting.

In Section 2, we review some experimental evidence on narrow framing. In Section 3, we present a preference specification that allows for narrow framing. Section 4 specifies a general portfolio problem, derives the first-order conditions of optimality and presents a simple example. Section 5 explains how an agent with our preferences evaluates timeless monetary gambles, while Section 6 explores the equilibrium implications of narrow framing. Section 7 concludes.

2 Narrow Framing

Before presenting any formal analysis, we first review some of the experimental evidence on narrow framing. The classic demonstration is due to Tversky and Kahneman (1981), who ask 150 subjects the following question:[1]

Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer:

Choice (I) Choose between:

A. a sure gain of \$240

B. 25% chance to gain \$1,000 and 75% chance to gain nothing

Choice (II) Choose between:

C. a sure loss of \$750

D. 75% chance to lose \$1,000 and 25% chance to lose nothing.

Tversky and Kahneman (1981) report that 84% of subjects chose A, with only 16% choosing B, and that 87% chose D, with only 13% choosing C. In particular, 73% of subjects chose the combination A&D, namely

$$\text{25\% chance to win \$240,} \quad \text{75\% chance to lose \$760,} \tag{1}$$

which is surprising, given that this choice is dominated by the combination B&C, namely

$$\text{25\% chance to win \$250,} \quad \text{75\% chance to lose \$750.} \tag{2}$$

It appears that, instead of focusing on the *combined* outcome of decisions I and II – in other words, on the outcome that determines their final wealth – subjects are focusing on the outcome of each decision separately. Indeed, subjects who are asked *only* about decision I do overwhelmingly choose A; and subjects asked *only* about decision II do overwhelmingly choose D.

In more formal terms, we cannot model the typical subject as maximizing a utility function defined only over total wealth. Rather, his utility function appears to depend *directly* on the outcome of each of decisions I and II, rather than just indirectly, via the contribution of each decision to overall wealth. As such, this is an example of narrow framing.

More recently, Barberis, Huang, and Thaler (2006) argue that the rejection of the gamble

$$(110, \frac{1}{2}; -100, \frac{1}{2}),$$

[1] For more evidence and discussion of narrow framing, see Kahneman and Tversky (1983), Tversky and Kahneman (1986), Redelmeier and Tversky (1992), Read, Loewenstein, and Rabin (1999), and Rabin and Thaler (2001).

to be read as "win \$110 with probability $\frac{1}{2}$, lose \$100 with probability $\frac{1}{2}$, independent of other risks," observed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) in a majority of subjects, may also be evidence of narrow framing. They reason that the subjects who are offered this gamble are typically already facing *other* kinds of risk – labor income risk, housing risk, or financial market risk, say. In the absence of narrow framing, they must therefore evaluate the 110/100 gamble by mixing it with these other risks and then checking if the combination is attractive. It turns out that the combination *is*, quite generally, attractive: since the 110/100 gamble is independent of other risks, it offers useful diversification benefits, which, even if "first-order" risk averse in the sense of Segal and Spivak (1990), people can enjoy. The rejection of the 110/100 gamble therefore suggests that people are *not* fully merging the gamble with their other risks, but that, to some extent, they are evaluating it in isolation; in other words, that they are framing it narrowly.

As noted in the Introduction, our goal is not to provide a new theory of when narrow framing occurs and when it does not, but rather to provide tools for exploring the implications of any specific framing hypothesis. Nevertheless, it may be helpful, as we enter the formal analysis, to keep at least one possible theory of framing in mind.

One candidate theory is proposed by Kahneman (2003). He argues that narrow framing occurs when decisions are made intuitively, rather than through effortful reasoning. Since intuitive thoughts are by nature spontaneous, they are heavily shaped by the features of the situation at hand that come to mind most easily; to use the technical term, by the features that are most "accessible." When an agent is offered a new gamble, the distribution of the gamble, considered separately, is often more accessible than the distribution of his overall wealth once the new gamble has been merged with his other risks. As a result, if the agent thinks about the gamble intuitively, the distribution of the gamble, taken alone, may play a more important role in decision-making than would be predicted by traditional utility functions defined only over wealth or consumption.

In Tversky and Kahneman's (1981) example, the outcome of each one of choices A, B, C, or D is highly accessible. Much less accessible, though, is the *overall* outcome once two choices – A&D, say, or B&C – are combined: the distributions in (1) and (2) are less "obvious" than the distributions of A, B, C, and D given in the original question. As a result, if subjects use their intuition when responding, the outcome of each of decisions I and II may play a bigger role than predicted by traditional utility functions. Similar reasoning applies in the case of the 110/100 gamble.[2]

[2] Another possible source of narrow framing is non-consumption utility, such as regret. Regret is the pain we feel when we realize that we would be better off today if we had taken a different action in the past. Even if a gamble that an agent accepts is just one of many risks that he faces, it is still linked to a specific decision, namely the decision to accept the gamble. As a result, it exposes the agent to possible future regret: if the gamble turns out badly, he may regret the decision to accept it. Consideration of non-consumption utility therefore leads quite naturally to preferences that depend *directly* on the outcomes of specific gambles that

While Tversky and Kahneman's (1981) experiment provides strong evidence of narrow framing, it is also somewhat extreme, in that, in this example, narrow framing leads subjects to choose a dominated alternative. In general, narrow framing does not necessarily lead to violations of dominance. All the same, Tversky and Kahneman's (1981) example does raise the concern that, when applied to asset pricing, narrow framing might sometimes give rise to arbitrage opportunities. To ensure that this does not happen, it is important to focus on applications to *absolute* pricing – in other words, to the pricing of assets, like the aggregate stock market, which lack perfect substitutes: when substitutes are imperfect, there are no riskless arbitrage opportunities. We would not expect narrow framing to have much useful application to *relative* pricing: in this case, any impact that narrow framing had on prices would create an arbitrage opportunity that could be quickly exploited.

While narrow framing has primarily been documented in experimental settings, it is intuitively clear, as noted in the Introduction, that it may also be able to address well-known observations about attitudes to real-world risks. This points to the usefulness of developing a formal preference specification that allows for narrow framing. We now present such a specification.

3 Preferences that Allow for Framing

We work in discrete time throughout. At time t, the agent, whose wealth is denoted W_t, chooses a consumption level C_t and allocates his post-consumption wealth, $W_t - C_t$, across n assets. His wealth therefore evolves according to

$$\widetilde{W}_{t+1} = (W_t - C_t)(\sum_{i=1}^{n} \theta_{i,t} \widetilde{R}_{i,t+1}) \equiv (W_t - C_t)\widetilde{R}_{W,t+1}, \tag{3}$$

where $\theta_{i,t}$ is the fraction of post-consumption wealth allocated to asset i, $\widetilde{R}_{i,t+1}$ is the gross return on asset i between time t and $t+1$, and $\widetilde{R}_{W,t+1}$ is the gross return on wealth over the same interval.

We can think of each of the n assets available to the agent as a "gamble." His gamble in asset i, for example, consists of putting down capital of $\theta_{i,t}(W_t - C_t)$ at time t, and receiving an uncertain payoff of $\theta_{i,t}(W_t - C_t)\widetilde{R}_{i,t+1}$ at time $t+1$. We want to allow for the possibility that the agent frames one or more of these n gambles narrowly; in other words, that he gets utility from their outcomes directly, and not just indirectly via their contribution to total wealth risk. How can this be modeled?

A useful starting point for developing preferences that allow for narrow framing is recur-

the agent faces.

sive utility, in which the agent's time t utility, V_t, is given by

$$V_t = W(C_t, \mu(\widetilde{V}_{t+1}|I_t)), \tag{4}$$

where $\mu(\widetilde{V}_{t+1}|I_t)$ is the certainty equivalent of the distribution of future utility, $\widetilde{V}_{t+1}$, conditional on time t information, I_t, and $W(\cdot,\cdot)$ is an aggregator function that aggregates current consumption C_t with the certainty equivalent of future utility to give current utility (see Epstein and Zin, 1989, for a detailed discussion). Most implementations of recursive utility assign $W(\cdot,\cdot)$ the CES form

$$W(C, x) = ((1-\beta)C^\rho + \beta x^\rho)^{\frac{1}{\rho}},\ 0 < \beta < 1,\ 0 \neq \rho < 1, \tag{5}$$

and assume homogeneity of $\mu(\cdot)$. If a certainty equivalent functional is homogeneous, it is necessarily homogeneous of degree one, so that

$$\mu(k\widetilde{x}) = k\mu(\widetilde{x}),\ k > 0. \tag{6}$$

In its current form, the specification in (4) does not allow for narrow framing: an investor with these preferences cares about the outcome of a gamble he is offered only to the extent that the outcome affects his overall wealth risk. These preferences can, however, be naturally extended to allow for narrow framing. Suppose that the agent frames $n-m$ of the n assets narrowly – specifically, assets $m+1$ through n. In terms of Kahneman's (2003) accessibility theory of framing, asset n, say, might be framed narrowly because information about the distribution of its future returns is very accessible; in particular, more accessible than information about the distribution of the agent's overall wealth once a position in asset n has been added to his holdings of other assets. Under this view, the fact that the distribution of asset n's returns is so accessible means that it plays a larger role in the agent's decision-making than traditional utility functions would suggest.

We propose that narrow framing of this kind can be captured by the following preference specification:

$$V_t = W\left(C_t, \mu(\widetilde{V}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right), \tag{7}$$

where

$$\begin{aligned}
W(C, x) &= ((1-\beta)C^\rho + \beta x^\rho)^{\frac{1}{\rho}},\ 0 < \beta < 1,\ 0 \neq \rho < 1 && (8)\\
\mu(k\widetilde{x}) &= k\mu(\widetilde{x}),\ k > 0 && (9)\\
\widetilde{G}_{i,t+1} &= \theta_{i,t}(W_t - C_t)(\widetilde{R}_{i,t+1} - R_{i,z}),\ i = m+1, \ldots, n && (10)\\
\overline{v}(x) &= \left\{ \begin{array}{lcl} x & \text{for} & x \geq 0 \\ \lambda x & & x < 0 \end{array} \right.,\ \lambda > 1. && (11)
\end{aligned}$$

Relative to the usual recursive specification in (4)-(6), we maintain the standard assumptions for $W(\cdot,\cdot)$ and $\mu(\cdot)$. The difference is that we now add $n-m$ new terms to the second argument of $W(\cdot,\cdot)$, one for each of the $n-m$ assets that the investor is framing narrowly. For example, $\widetilde{G}_{n,t+1}$ is the specific gamble the investor is taking by investing in asset n. By adding in the new term $b_0 E_t(\overline{v}(\widetilde{G}_{n,t+1}))$, we allow the investor to get utility directly from the outcome of this gamble, rather than just indirectly via its contribution to next period's wealth. Equivalently, the investor now gets utility from the outcome of this specific gamble over and above what would be justified by its implications for his total wealth risk. In less formal terms, he now evaluates his investment in asset n in isolation, to some extent.

The simplest way to express the gamble the investor is taking by investing in asset n, say, is

$$\widetilde{G}_{n,t+1} = \theta_{n,t}(W_t - C_t)(\widetilde{R}_{n,t+1} - 1), \tag{12}$$

the amount invested in the asset, $\theta_{n,t}(W_t - C_t)$, multiplied by its net return, $\widetilde{R}_{n,t+1} - 1$. This corresponds to equation (10) with $R_{i,z} = 1$. In this case, so long as $\theta_{n,t} > 0$, a positive net return is considered a gain and, from (11), is assigned positive utility; a negative net return is considered a loss and is assigned negative utility.

By using the more general specification in equation (10), we allow for some flexibility as to what counts as a gain – in other words, as to what kind of gamble outcome is assigned positive utility. Treating any positive net return as a gain – in other words, setting $R_{i,z} = 1$ – is one possibility, but another that we consider later in the paper sets $R_{i,z} = R_{f,t}$. In this case, an asset's return is only considered a gain, and hence is only assigned positive utility, if it exceeds the risk-free rate.[3]

The next issue is to specify the utility $\overline{v}(\cdot)$ the agent receives from narrowly framed gains and losses. We propose the piecewise linear specification in (11). There are at least two ways of motivating this. The first is tractability. One way to increase tractability is to impose homotheticity. Since $\mu(\cdot)$ is homogeneous of degree one, homotheticity obtains so long as $\overline{v}(\cdot)$ is *also* homogeneous of degree one. At the same time, to ensure that the first-order conditions associated with the maximization problem are both necessary and sufficient for optimality, we need $\overline{v}(\cdot)$ to be concave. The only function that is both homogeneous of degree one and concave is precisely the piecewise linear function in (11).

Another possible line of argument is that $\overline{v}(\cdot)$ should be modeled as closely as possible after Kahneman and Tversky's (1979) prospect theory – a descriptive theory, based on extensive experimental evidence, of decision-making under risk. The reason is that, in those experimental settings where people have been shown to be evaluating a gamble in isolation, they often also appear to be following the rules of prospect theory in deciding whether to accept the gamble. For example, the experiment of Tversky and Kahneman (1981) discussed in Section 2 points not only to narrow framing, but also, through the preference for A

[3]Since $R_{i,z}$ determines whether a particular outcome is a gain or a loss, it is an example of what the literature on decision-making calls a "reference point." An ongoing research effort tries to understand what determines the reference points that people use in practice (Koszegi and Rabin, 2007).

over B and for D over C, to risk aversion over gains and risk-seeking over losses, mirroring the prospect theory value function's concavity (convexity) in the region of gains (losses). Similarly, the rejection of the 110/100 gamble suggests not only narrow framing, but also a greater sensitivity to losses than to gains, in line with the kink in the prospect theory value function.[4]

Equations (10) and (11) show that we have adopted two of the main features of prospect theory in our specification of $\overline{v}(\cdot)$: outcomes are described in terms of gains and losses relative to a reference return $R_{i,z}$, and the agent is more sensitive to losses than to gains. The two other elements of prospect theory – the concavity (convexity) of the value function in the region of gains (losses), and the probability weighting function – are more difficult to incorporate, because they induce risk-seeking on the part of the agent, which means that the first-order conditions for the maximization problem are no longer sufficient for optimality.

The parameter b_0 controls the degree of narrow framing. A b_0 of 0 means no narrow framing at all, while a large b_0 means that the investor is evaluating each of assets $m+1$ through n almost entirely in isolation. For simplicity, we take the degree of narrow framing to be the same for all $n-m$ assets, but our analysis extends very easily to the more general case where

$$V_t = W\left(C_t, \mu(\widetilde{V}_{t+1}|I_t) + \sum_{i=m+1}^{n} b_{i,0} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right). \tag{13}$$

Finally, we note that the preferences in (7)-(11) are dynamically consistent. Today, the agent knows how he will frame future gains and losses and what function $\overline{v}(\cdot)$ he will apply to those narrowly framed gains and losses. Moreover, he takes all of this into account when making today's decisions. Standard dynamic programming techniques can therefore be applied, and dynamic consistency follows.[5]

4 The Consumption-Portfolio Problem

In this section, we derive the first-order conditions for optimal consumption and portfolio choice, and illustrate the tractability of our framework by solving a simple portfolio problem.

[4]Kahneman (2003) suggests an explanation for why prospect theory and narrow framing might appear in combination like this. He argues that prospect theory captures the way people act when they make decisions intuitively, rather than through effortful reasoning. Since narrow framing is also thought to derive, at least in part, from intuitive decision-making, it is natural that prospect theory would be used in parallel with narrow framing.

[5]Of course, if the agent does not correctly forecast the way he will frame future gains and losses, dynamic inconsistency can arise – but this is not the case here.

The Bellman equation that corresponds to (7) is

$$\begin{aligned} V_t &= J(W_t, I_t) = \max_{C_t,\theta_t} W\left(C_t, \mu(J(\widetilde{W}_{t+1}, I_{t+1})|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right) \\ &= \max_{C_t,\theta_t}\left[(1-\beta)C_t^{\rho} + \beta\left[\mu(J(\widetilde{W}_{t+1}, I_{t+1})|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right]^{\rho}\right]^{\frac{1}{\rho}}. \end{aligned} \tag{14}$$

Given the form of $\widetilde{G}_{i,t+1}$ in equation (10), we can show that

$$J(W_t, I_t) = A(I_t)W_t \equiv A_t W_t, \tag{15}$$

so that

$$A_t W_t = \max_{C_t,\theta_t}\left[(1-\beta)C_t^{\rho} + \beta(W_t - C_t)^{\rho}\left[\mu(A_{t+1}\theta_t'\widetilde{R}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{i,z})))\right]^{\rho}\right]^{\frac{1}{\rho}}, \tag{16}$$

where

$$\theta_t = (\theta_{1,t}, \ldots, \theta_{n,t})',\ \ R_t = (R_{1,t}, \ldots, R_{n,t})'. \tag{17}$$

Equation (16) shows that the consumption and portfolio decisions are separable. In particular, the portfolio problem is

$$B_t^* = \max_{\theta_t}\left[\mu(A_{t+1}\theta_t'\widetilde{R}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{i,z})))\right], \tag{18}$$

and after defining

$$\alpha_t \equiv \frac{C_t}{W_t}, \tag{19}$$

the consumption problem becomes

$$A_t = \max_{\alpha_t}[(1-\beta)\alpha_t^{\rho} + \beta(1-\alpha_t)^{\rho}(B_t^*)^{\rho}]^{\frac{1}{\rho}}. \tag{20}$$

The first-order condition for optimal consumption choice α_t^* is

$$(1-\beta)(\alpha_t^*)^{\rho-1} = \beta(1-\alpha_t^*)^{\rho-1}(B_t^*)^{\rho}, \tag{21}$$

and the second-order condition confirms that equation (21) is not only necessary but also sufficient for a maximum. Combining equations (20) and (21) gives

$$A_t = (1-\beta)^{\frac{1}{\rho}}(\alpha_t^*)^{1-\frac{1}{\rho}}, \tag{22}$$

and similarly,

$$A_{t+1} = (1-\beta)^{\frac{1}{\rho}}(\alpha_{t+1}^*)^{1-\frac{1}{\rho}}, \tag{23}$$

which, when substituted into (18), allows us to rewrite the portfolio problem as

$$B_t^* = \max_{\theta_t} \left[\mu((1-\beta)^{\frac{1}{\rho}} {\alpha_{t+1}}^{1-\frac{1}{\rho}} \theta_t' \widetilde{R}_{t+1} | I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{i,z}))) \right]. \tag{24}$$

In Section 4.1, we present a simple numerical example of a portfolio problem in the presence of narrow framing. For that example, and for many others a researcher might be interested in, the portfolio problem can easily be solved using only equations (21) and (24). For some applications, though, it can be useful to lay out in full the necessary and sufficient first-order conditions for consumption and portfolio choice, and we do this in Proposition 1 below.

Since our emphasis is on the effects of narrow framing, Proposition 1 restricts the form of the certainty equivalent functional $\mu(\cdot)$ to the simple case of

$$\mu(\widetilde{x}) = (E(\widetilde{x}^{\zeta}))^{\frac{1}{\zeta}}, \; 0 \neq \zeta < 1. \tag{25}$$

However, the same method of proof used for Proposition 1 can also be applied to other explicitly defined forms of $\mu(\cdot)$, whether expected utility or not, that satisfy the homogeneity property (9). For example, it is straightforward to derive the first-order conditions that hold when $\mu(\cdot)$ takes the non-expected utility form proposed by Chew (1983), namely "weighted utility":

$$\mu(\widetilde{x}) = \left(\frac{E(\widetilde{x}^{1-\gamma+\delta})}{E(\widetilde{x}^{\delta})} \right)^{\frac{1}{1-\gamma}}, \; \gamma \neq 1. \tag{26}$$

Proposition 1: *The necessary and sufficient first-order conditions for the decision problem that maximizes (7), subject to (3), (8), (10), (11), and (25), are, for each t, that*

$$\left(\frac{1-\alpha_t}{\alpha_t}\right)^{1-\frac{1}{\rho}} \left[\beta^{\frac{1}{\rho}} \left[E_t(\alpha_{t+1}^{\zeta(1-\frac{1}{\rho})} (\theta_t' \widetilde{R}_{t+1})^{\zeta}) \right]^{\frac{1}{\zeta}} + b_0 \left(\frac{\beta}{1-\beta}\right)^{\frac{1}{\rho}} \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{i,z}))) \right] = 1 \tag{27}$$

and that there exists ψ_t such that, for $i = 1, \ldots n$,

$$\psi_t - (1-\beta)^{\frac{1}{\rho}} [E_t(\alpha_{t+1}^{\zeta(1-\frac{1}{\rho})} (\theta_t' \widetilde{R}_{t+1})^{\zeta})]^{\frac{1}{\zeta}-1} E_t[\alpha_{t+1}^{\zeta(1-\frac{1}{\rho})} (\theta_t' \widetilde{R}_{t+1})^{\zeta-1} \widetilde{R}_{i,t+1}] \tag{28}$$
$$\begin{cases} = b_0 1_{\{i>m\}} \mathrm{sgn}(\theta_{i,t}) E_t[\overline{v}(\mathrm{sgn}(\theta_{i,t})(\widetilde{R}_{i,t+1} - R_{i,z}))] & \text{for } \theta_{i,t} \neq 0 \\ \in [b_0 1_{\{i>m\}} E_t(\overline{v}(\widetilde{R}_{i,t+1} - R_{i,z})), -b_0 1_{\{i>m\}} E_t(\overline{v}(R_{i,z} - \widetilde{R}_{i,t+1}))] & \text{for } \theta_{i,t} = 0 \end{cases}.$$

Proof of Proposition 1: See the Appendix.

Equation (27) is simply a rearrangement of the first-order condition for consumption choice in equation (21). Equation (28) is the first-order condition for the portfolio problem in (24); in the Appendix, we show it to be both necessary and sufficient. The right-hand side of equation (28) is non-zero only if asset i is framed narrowly, in other words, only if $i > m$. Since $\overline{v}(\cdot)$ is not smooth at zero, the first-order condition takes the form of an inequality when $\theta_{i,t} = 0$.

Equations (27) and (28) lend themselves very naturally to the backward induction method of dynamic programming. Given $(\alpha_{t+1}, \theta_{t+1})$, equation (28) can be solved for θ_t, and with θ_t in hand, equation (27) can be solved for α_t.

Applications of recursive utility often consider the special case of (25) in which $\zeta = \rho$, in other words, the case where the exponent in the certainty equivalent functional is the same as the exponent in the aggregator function in (8). The corollary below presents the simplified first-order conditions that apply in this case. We use $1-\gamma$ to denote the common value of ζ and ρ.

Corollary: *When $\zeta = \rho = 1-\gamma$, the necessary and sufficient first-order conditions for the decision problem that maximizes (7), subject to (3), (8), (10), (11), and (25), are, for each t, that*

$$(\frac{1-\alpha_t}{\alpha_t})^{\frac{-\gamma}{1-\gamma}} \left[\beta^{\frac{1}{1-\gamma}} \left[E_t(\alpha_{t+1}^{-\gamma}(\theta_t' \widetilde{R}_{t+1})^{1-\gamma})\right]^{\frac{1}{1-\gamma}} + b_0(\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}} \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{i,z})))\right] = 1 \tag{29}$$

and that there exists ψ_t such that, for $i = 1, \ldots, n$,

$$\psi_t - (1-\beta)^{\frac{1}{1-\gamma}} [E_t(\alpha_{t+1}^{-\gamma}(\theta_t' \widetilde{R}_{t+1})^{1-\gamma})]^{\frac{\gamma}{1-\gamma}} E_t[\alpha_{t+1}^{-\gamma}(\theta_t' \widetilde{R}_{t+1})^{-\gamma} \widetilde{R}_{i,t+1}] \tag{30}$$
$$\begin{cases} = b_0 1_{\{i>m\}} \mathrm{sgn}(\theta_{i,t}) E_t[\overline{v}(\mathrm{sgn}(\theta_{i,t})(\widetilde{R}_{i,t+1} - R_{i,z}))] & \text{for } \theta_{i,t} \neq 0 \\ \in [b_0 1_{\{i>m\}} E_t(\overline{v}(\widetilde{R}_{i,t+1} - R_{i,z})), -b_0 1_{\{i>m\}} E_t(\overline{v}(R_{i,z} - \widetilde{R}_{i,t+1}))] & \phantom{\text{for }} \theta_{i,t} = 0 \end{cases}.$$

Conditions (28) and (30) simplify slightly when $\theta_{i,t} > 0$. In this case,

$$\mathrm{sgn}(\theta_{i,t}) E_t[\overline{v}(\mathrm{sgn}(\theta_{i,t})(\widetilde{R}_{i,t+1} - R_{i,z}))] = E_t(\overline{v}(\widetilde{R}_{i,t+1} - R_{i,z})). \tag{31}$$

4.1 An example

To illustrate the effects of narrow framing, we now use the preceding analysis to solve a simple portfolio problem in which the investor allocates his wealth across three assets. Asset 1 is riskless and earns a constant gross risk-free rate of R_f in each period. Assets 2 and 3 are risky, with gross returns $\widetilde{R}_{2,t+1}$ and $\widetilde{R}_{3,t+1}$ given by

$$\log \widetilde{R}_{i,t+1} = g_i + \sigma_i \widetilde{\varepsilon}_{i,t+1}, \; i = 2, 3, \tag{32}$$

where

$$\begin{pmatrix} \tilde{\varepsilon}_{2,t} \\ \tilde{\varepsilon}_{3,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix}\right), \text{ i.i.d. over time.} \tag{33}$$

The investor's wealth evolves according to

$$\widetilde{W}_{t+1} = (W_t - C_t)((1 - \theta_{2,t} - \theta_{3,t})R_f + \theta_{2,t}\widetilde{R}_{2,t+1} + \theta_{3,t}\widetilde{R}_{3,t+1}), \tag{34}$$

where $\theta_{2,t}$ ($\theta_{3,t}$) is the fraction of post-consumption wealth allocated to asset 2 (3).

We can simplify the portfolio problem further, and still demonstrate the effects of narrow framing, by making one additional assumption: that the fraction of the investor's wealth in asset 2 is fixed at $\theta_{2,t} = \overline{\theta}_2$, so that the investor simply has to split the remainder of his wealth between the riskless asset and risky asset 3. We can think of asset 2 as a non-financial asset, such as housing wealth or human capital, and asset 3 as the domestic stock market. In this case, given a fixed position in the non-financial asset, the investor is thinking about how to allocate the rest of his wealth between the risk-free asset and a risky stock market. Alternatively, asset 2 could be domestic stock and asset 3, foreign stock.

We now investigate what happens if, in making this decision, the investor frames asset 3 narrowly, so that his preferences are given by

$$\begin{aligned} V_t &= W(C_t, \mu(\widetilde{V}_{t+1}) + b_0 E_t(\overline{v}(\widetilde{G}_{3,t+1}))) & (35) \\ W(C, x) &= ((1-\beta)C^{1-\gamma} + \beta x^{1-\gamma})^{\frac{1}{1-\gamma}}, \; 0 < \beta < 1, \, 0 < \gamma \neq 1 & (36) \\ \mu(\tilde{x}) &= (E(\tilde{x}^{1-\gamma}))^{\frac{1}{1-\gamma}}, \, 0 < \gamma \neq 1 & (37) \\ \widetilde{G}_{3,t+1} &= \theta_{3,t}(W_t - C_t)(\widetilde{R}_{3,t+1} - R_f) & (38) \\ \overline{v}(x) &= \begin{cases} x & \text{for } x \geq 0 \\ \lambda x & \phantom{\text{for }} x < 0 \end{cases}, \; \lambda > 1. & (39) \end{aligned}$$

Comparing this to the general specification in equations (7)-(11), we have set $n = 3$ and $m = 2$ – in words, there are three assets and just one of them is framed narrowly – and we have given $\mu(\cdot)$ the simplest possible form: power utility with an exponent equal to the exponent in the aggregator function $W(\cdot, \cdot)$. Finally, we have set the reference return $R_{i,z}$ equal to the risk-free rate R_f, so that a return is only considered a gain – in other words, is only assigned positive utility – if it exceeds the risk-free rate.

In terms of Kahneman's (2003) accessibility theory of framing, we can interpret the narrow framing of asset 3 as indicating that information about the distribution of that asset is very accessible to the investor – more so than information about the distribution of his total wealth once a position in asset 3 is merged with his fixed position in asset 2.[6]

[6] One could argue that the investor should also frame the outcome of asset 2 narrowly, on the grounds that the distribution of that asset's returns may also be more accessible than the distribution of overall wealth once the two risky assets are combined. While it is clear, from equation (7), that this can easily be accomodated, it adds little to the intuition of this section. For simplicity, then, we assume that only asset 3 is framed narrowly.

We use the return process parameters shown in Table 1. For simplicity, we set the mean returns on the two risky assets, and also their volatilities, to the same value. Their correlation is $\omega = 0.1$. A fixed fraction $\overline{\theta}_2 = 50\%$ of wealth is allocated to asset 2, the non-financial asset, and the gross risk-free rate is 1.02%. Finally, we set the preference parameter β, which has little direct influence on attitudes to risk, to 0.98, and consider a range of values for the remaining preference parameters: γ, λ, and b_0.

Before presenting the results, we outline a simple way of solving this problem. Given the i.i.d investment opportunity set, we conjecture that

$$(\theta_{3,t}, \alpha_t, A_t) = (\theta_3, \alpha, A), \ \forall\ t. \tag{40}$$

The portfolio problem in (24) then becomes

$$B^* = \max_{\theta_3} \left[(1-\beta)^{\frac{1}{1-\gamma}} \alpha^{\frac{-\gamma}{1-\gamma}} [E((\theta' \widetilde{R}_{t+1})^{1-\gamma})]^{\frac{1}{1-\gamma}} + b_0 E(\overline{v}(\theta_3(\widetilde{R}_{3,t+1} - R_f))) \right]. \tag{41}$$

The only difficulty here is that the portfolio problem depends on the consumption policy constant α. This can be addressed in the following way. Given a candidate optimal consumption policy α, solve (41) for that α. Substitute the resulting B^* into equation (21) to generate a new candidate α, and continue this iteration until convergence occurs.

Table 2 shows the portfolios chosen by an investor who maximizes the utility function in (35). Recall that a fixed 50% of the investor's wealth is held in risky asset 2. For four pairs of values of (γ, λ), and for a wide range of values of b_0, the table lists the percentage of the investor's *remaining* wealth that is allocated to risky asset 3, as opposed to the risk-free asset. For example, a figure of 100% in the table means that all remaining wealth, or 50% of total wealth, is invested in asset 3.

Note first that, when $b_0 = 0$, in other words, when there is no narrow framing at all, the investor either puts all of his remaining wealth, or the vast majority of it, into asset 3. The intuition is simple. Asset 3 not only earns a premium over the risk-free rate, but is also almost uncorrelated with asset 2, thereby offering the investor substantial diversification. Since the investor does not frame narrowly, he pays attention to overall wealth risk, and therefore finds the diversification very attractive.

At the other end of the table, when $b_0 = 0.5$, the investors puts *none* of his remaining wealth into risky asset 3. At this level of narrow framing, the investor evaluates asset 3 so much in isolation that he misses its diversification benefits. Instead, he focuses narrowly on the asset's potential gains and losses, and since, through the parameter λ, he is much more sensitive to losses than to gains, he rejects the asset completely.

The portfolio choice results in Table 2 – specifically, the fact that, for a wide range of values of b_0, the investor allocates nothing to asset 3, thereby completely ignoring its diversification benefits – immediately suggest a number of possible applications for narrow framing.

To see this, note that there are many contexts in which investors do indeed appear to reject obvious diversification opportunities. In what has come to be known as the stock market participation puzzle, many U.S. households have for decades refused to add even a small amount of stock market risk to their portfolios, even though the stock market is relatively uncorrelated with other major household risks (Heaton and Lucas, 2000). Very similar is the home bias puzzle: the fact that many investors have historically refused to diversify what holdings of domestic stock they do have with even a small position in foreign stock, in spite of the low correlation between the two asset classes. Yet another example is the fact that some households invest a relatively large fraction of their wealth in just a few individual stocks.

In the absence of frictions, it is hard to explain such behavior with traditional utility functions defined only over wealth or consumption. Investors who pay attention to the riskiness of their overall wealth are invariably keen to take advantage of opportunities for diversification. This is true even for utility functions that exhibit "first-order" risk aversion (Barberis, Huang, and Thaler, 2006). Table 2 suggests that narrow framing, on the other hand, *can* potentially explain the widespread under-diversification in household portfolios: an investor who evaluates risks in isolation misses diversification opportunities, thereby making it easier to understand why he might fail to exploit them.

5 Attitudes to Timeless Gambles

Economists are often interested in attitudes to timeless gambles – gambles whose uncertainty is resolved immediately. Attitudes to such gambles can be used to decide if a particular parameterization of a utility function corresponds to "reasonable" risk aversion or not. In this section, we show how an agent who engages in narrow framing would evaluate a timeless gamble.

The earlier literature has already discussed how an agent with the recursive utility specification in (4) would evaluate a timeless gamble (Epstein and Zin, 1989). The reason we need to do more analysis is that, if the agent frames some risks narrowly, as the agent with the preferences in (7)-(11) does, then he may also frame timeless gambles narrowly.

The narrow framing of a timeless gamble can be motivated, as before, by Kahneman's (2003) notion of accessibility. Suppose that, at time τ, the agent is offered a timeless gamble $\widetilde{g}$, a 50:50 bet to win \$$x$ or lose \$$y$, independent of other risks, and whose outcome provides no information about future investment opportunities. The gamble payoffs, x and y, are highly accessible and, in particular, may be more accessible than the distribution of overall wealth once $\widetilde{g}$ is mixed with the agent's other risks. As a result, the distribution of the gamble, taken alone, may play a more important part in the agent's decision-making than would be predicted by traditional utility functions.

Even if the timeless gamble $\widetilde{g}$ is framed narrowly, there is still some flexibility in how it is evaluated. One possible approach, proposed in the earlier literature on recursive utility, is that, when evaluating a timeless gamble, the investor inserts an infinitesimal time step $\Delta\tau$ around the moment where the gamble's uncertainty is resolved, and then applies the recursive calculation over this time step (Epstein and Zin, 1989). In this case, then, the investor waits for the outcome of the timeless gamble to be revealed and then decides what fraction of his wealth to consume between τ and $\tau + 1$.

Under this approach, the investor's utility after taking the gamble is[7]

$$V_\tau = W(0, \mu(\widetilde{V}_{\tau+\Delta\tau}|I_\tau) + b_0 E(\overline{v}(\widetilde{g}))). \tag{42}$$

Since

$$\mu(\widetilde{V}_{\tau+\Delta\tau}|I_\tau) = \mu(A_{\tau+\Delta\tau}(I_{\tau+\Delta\tau})\widetilde{W}_{\tau+\Delta\tau}|I_\tau) = A_\tau\mu(W_\tau + \widetilde{g}|I_\tau) = A_\tau\mu(W_\tau + \widetilde{g}), \tag{43}$$

where the second equality comes from the fact that $\widetilde{g}$ provides no information about future investment opportunities, the third from the fact that $\widetilde{g}$ is independent of time τ information, and A_τ is defined in equation (15), equation (42) becomes

$$V_\tau = W\left(0, A_\tau\mu(W_\tau + \widetilde{g}) + b_0(\frac{x - \lambda y}{2})\right). \tag{44}$$

If the investor chooses not to take the gamble, this reduces to

$$V_\tau = W(0, A_\tau W_\tau). \tag{45}$$

The gamble is therefore accepted iff

$$A_\tau\mu(W_\tau + \widetilde{g}) + b_0(\frac{x - \lambda y}{2}) > A_\tau W_\tau. \tag{46}$$

If $\widetilde{g}$ is small relative to the investor's wealth, and if μ is "smooth" – in the sense of exhibiting "second-order" risk aversion, say, as in Segal and Spivak (1990) – then $\mu(W_\tau + \widetilde{g}) \approx W_\tau + E(\widetilde{g})$ and condition (46) becomes

$$\frac{x}{y} > \frac{A_\tau + b_0\lambda}{A_\tau + b_0}. \tag{47}$$

[7] To keep equation (42) simple, we suppose that the agent frames only the timeless gamble narrowly. It is straightforward to extend the calculations to the case where the agent also frames some of his other risks narrowly.

Note that, if b_0 is large relative to A_τ, the gamble is accepted iff the ratio $x{:}y$ exceeds λ. Intuitively, when b_0 is large, the investor evaluates the gamble largely in isolation and therefore accepts it only if its ratio of gain to loss exceeds his sensitivity to losses, λ.

A second possibility is that the investor evaluates the timeless gamble $\widetilde{g}$ over the same time interval he uses to evaluate his other risks, which, from (7), is the time interval between τ and $\tau+1$. In this case, then, the investor makes the time τ consumption decision before seeing the outcome of the timeless gamble.

Under this approach, if the investor does not take the gamble, his utility, from (15), is

$$V_\tau = A_\tau W_\tau. \tag{48}$$

If he does take the gamble, his utility is

$$V_\tau = \widehat{A}_\tau W_\tau = W(\widehat{C}_\tau, \mu(\widetilde{V}_{\tau+1}|I_\tau) + b_0 E\overline{v}(\widetilde{g})), \tag{49}$$

where the hats over $\widehat{A}_\tau$ and $\widehat{C}_\tau$ are a reminder that, if the gamble is accepted, optimal consumption and portfolio policies are affected. Since

$$\mu(\widetilde{V}_{\tau+1}|I_\tau) = \mu(A_{\tau+1}\widetilde{W}_{\tau+1}|I_\tau) = \mu(A_{\tau+1}((W_\tau - \widehat{C}_\tau)\widetilde{R}_{W,\tau+1} + \widetilde{g})|I_\tau), \tag{50}$$

equation (49) becomes

$$V_\tau = W\left(\widehat{C}_\tau, (W_\tau - \widehat{C}_\tau)\mu\left(A_{\tau+1}(\widetilde{R}_{W,\tau+1} + \frac{\widetilde{g}}{W_\tau - \widehat{C}_\tau})|I_\tau\right) + b_0(\frac{x-\lambda y}{2})\right). \tag{51}$$

The investor therefore takes the gamble iff

$$W\left(\widehat{C}_\tau, (W_\tau - \widehat{C}_\tau)\mu\left(A_{\tau+1}(\widetilde{R}_{W,\tau+1} + \frac{\widetilde{g}}{W_\tau - \widehat{C}_\tau})|I_\tau\right) + b_0(\frac{x-\lambda y}{2})\right) > A_\tau W_\tau. \tag{52}$$

We now present an illustrative example. Consider an investor who, at time τ, has wealth of $\$500,000$ invested in a risky asset with gross return $\widetilde{R}_{t+1}$, given by

$$\log \widetilde{R}_{t+1} \sim N(0.04, 0.03), \text{ i.i.d over time.} \tag{53}$$

The investor is offered a timeless gamble $\widetilde{g}$, a 50:50 bet to win \$200 or lose \$100, independent of other risks. Suppose that the investor engages in narrow framing, so that he evaluates the timeless gamble according to either (46) or (52). We set

$$\begin{aligned} W(C,x) &= ((1-\beta)C^{1-\gamma} + \beta x^{1-\gamma})^{\frac{1}{1-\gamma}},\ 0<\beta<1,\ 0<\gamma\neq 1 && (54)\\ \mu(\widetilde{x}) &= (E(\widetilde{x}^{1-\gamma}))^{\frac{1}{1-\gamma}},\ 0<\gamma\neq 1, && (55)\end{aligned}$$

so that, as in Section 4.1, $\mu(\cdot)$ has a power utility form with exponent equal to the exponent in the aggregator function.

The top panel in Figure 1 shows, for $\beta = 0.98$ and $\gamma = 1.5$, the range of values of b_0 and λ for which the agent rejects the 200/100 gamble when the gamble is evaluated according to the first method laid out above, corresponding to equation (46).[8] The figure shows that, for high values of b_0, the agent rejects the gamble when $\lambda > 2$ and accepts it otherwise. The intuition is simple. For high values of b_0, the agent effectively evaluates the gamble in isolation: whether he takes it or not is therefore determined by his sensitivity to narrowly framed losses, λ. If he is more than twice as sensitive to losses as to gains, the 200/100 gamble, with its 2:1 ratio of gain to loss, becomes unattractive.

As b_0 falls, it takes higher values of λ to reject the gamble. To understand this, consider the extreme case where $b_0 = 0$. In this case, the preferences in (7), coupled with (54) and (55), collapse to standard power utility. For such preferences, the investor is almost risk-neutral to small gambles and is therefore delighted to accept a small, independent, actuarially favorable gamble like 200/100. As b_0 falls towards 0 then, the investor becomes more and more interested in the 200/100 gamble and progressively higher values of λ are required to scare him away from it.

The bottom panel in Figure 1 shows, again for $\beta = 0.98$ and $\gamma = 1.5$, the range of values of b_0 and λ for which the agent rejects 200/100 when the gamble is evaluated using the second method laid out above, corresponding to equation (52).[9] The figure shows that, for this gamble, the alternative procedure produces identical results. The reason is that, since the 200/100 gamble is small relative to the investor's wealth, it makes little difference whether the time τ consumption decision is made before or after observing the gamble's outcome.

Sometimes, the researcher is interested not in whether the agent accepts or rejects a gamble x/y, but in what premium π the agent would pay to avoid a symmetric gamble $\tilde{g}$: a 50:50 bet to win or lose a fixed amount x, say. For an agent who engages in narrow framing, the premium can easily be computed using the analysis above.

For example, following the first evaluation method in equation (44), the utility from taking the gamble is

$$V_\tau = W(0, A_\tau \mu(W_\tau + \tilde{g}) + b_0 \frac{x}{2}(1 - \lambda)), \tag{56}$$

while the utility after paying the premium is

$$V_\tau = W(0, A_\tau \mu(W_\tau - \pi) - b_0 \lambda \pi). \tag{57}$$

The premium paid is therefore given by

$$A_\tau \mu(W_\tau + \tilde{g}) + b_0 \frac{x}{2}(1 - \lambda) = A_\tau (W_\tau - \pi) - b_0 \lambda \pi, \tag{58}$$

[8] See the Appendix for computational details.

[9] See the Appendix for computational details.

so that

$$\pi = \frac{A_\tau(W_\tau - \mu(W_\tau + \tilde{g})) + b_0\frac{x}{2}(\lambda - 1)}{A_\tau + b_0\lambda}. \tag{59}$$

Alternatively, the gamble can be evaluated according to the method assumed in equation (51), leading, in many cases, to very similar results.

6 Equilibrium Analysis

We now show that our preference specification is tractable not only in partial equilibrium, but also in a full equilibrium setting, thereby allowing us to study the impact of narrow framing on asset prices.

The simplest way of implementing narrow framing in an equilibrium context is to assign our preferences to a representative agent. In this case, the first-order conditions (29) and (30) give the relationship that must hold between aggregate consumption and asset returns. The following lemma rewrites those first-order conditions in a way that brings the role of consumption out more clearly and that is therefore easier to apply in equilibrium settings.

Lemma: *Suppose that asset* 1 *is the risk-free asset and that the reference return is set to* $R_{i,z} = R_{f,t}$, $\forall i$. *If, moreover,* $\theta_{i,t} > 0$, $\forall i > 1$, *then the first-order conditions for consumption and portfolio choice in equations (29)-(30) reduce to*

$$\left[\beta R_{f,t} E_t((\frac{C_{t+1}}{C_t})^{-\gamma})\right]\left[\beta E_t((\frac{C_{t+1}}{C_t})^{-\gamma} R_{W,t+1})\right]^{\frac{\gamma}{1-\gamma}} = 1 \tag{60}$$

$$\frac{E_t((\frac{C_{t+1}}{C_t})^{-\gamma}(R_{i,t+1} - R_{f,t}))}{E_t((\frac{C_{t+1}}{C_t})^{-\gamma})} + b_0 1_{\{i>m\}} R_{f,t}(\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}}(\frac{1-\alpha_t}{\alpha_t})^{\frac{-\gamma}{1-\gamma}} E_t(\overline{v}(R_{i,t+1} - R_{f,t})) = 0, \tag{61}$$

$$i = 2, \ldots, n.$$

Proof of Lemma: See the Appendix.

One last equation that will prove useful is the weighted sum of the equations in (61), where the i'th equation is weighted by $\theta_{i,t}$:

$$\frac{E_t((\frac{C_{t+1}}{C_t})^{-\gamma}(R_{W,t+1} - R_{f,t}))}{E_t((\frac{C_{t+1}}{C_t})^{-\gamma})} + b_0 R_{f,t}(\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}}(\frac{1-\alpha_t}{\alpha_t})^{\frac{-\gamma}{1-\gamma}} \sum_{i=m+1}^{n} E_t(\theta_{i,t}\overline{v}(R_{i,t+1} - R_{f,t})) = 0. \tag{62}$$

We now show how equations (60)-(62) can be implemented in a simple equilibrium setting. To illustrate the tractability of our framework, we use it to analyze the effect of narrow framing on the magnitude of the equity premium.

6.1 An example

Consider an economy with just three assets. Asset 1, the risk-free asset, is in zero net supply and earns a gross return $R_{f,t}$. Assets 2 and 3 are risky and are in positive net supply. Asset 2 can be thought of as a non-financial asset, such as housing wealth or human capital, and earns a gross return of $\widetilde{R}_{N,t+1}$. Asset 3 is the stock market and earns a gross return of $\widetilde{R}_{S,t+1}$.

We investigate the implications for the risk-free rate and equity premium when the representative agent frames stock market risk narrowly, in other words, when he has the preferences

$$\begin{aligned}
V_t &= W(C_t, \mu(\widetilde{V}_{t+1}) + b_0 E_t(\overline{v}(\widetilde{G}_{S,t+1}))) && (63)\\
W(C, x) &= ((1-\beta)C^{1-\gamma} + \beta x^{1-\gamma})^{\frac{1}{1-\gamma}},\ 0 < \beta < 1,\ 0 < \gamma \neq 1 && (64)\\
\mu(\tilde{x}) &= (E(\tilde{x}^{1-\gamma}))^{\frac{1}{1-\gamma}},\ 0 < \gamma \neq 1 && (65)\\
\widetilde{G}_{S,t+1} &= \theta_{S,t}(W_t - C_t)(\widetilde{R}_{S,t+1} - R_{f,t}) && (66)\\
\overline{v}(x) &= \begin{cases} x & \text{for} \quad x \geq 0 \\ \lambda x & \phantom{\text{for}} \quad x < 0 \end{cases},\ \lambda > 1, && (67)
\end{aligned}$$

where $\theta_{S,t}$ is the fraction of wealth allocated to the stock market and where, relative to the general specification in (7)-(11), we have set $n = 3$ and $m = 2$, given $\mu(\cdot)$ a power utility form with exponent equal to the exponent in the aggregator function $W(\cdot,\cdot)$, and set the reference return $R_{i,z}$ equal to the risk-free rate $R_{f,t}$.

In terms of Kahneman's (2003) accessibility theory of framing, we can interpret the narrow framing of the stock market as indicating that information about the distribution of stock returns is very accessible to the investor, perhaps because of regular exposure to such information in books, newspapers, and other media; and, in particular, that this information is more accessible than information about the distribution of overall wealth once the stock market is merged with the investor's holdings of the non-financial asset.[10]

We consider an equilibrium in which: (i) the risk-free rate is a constant R_f; (ii) consumption growth and stock returns are distributed as

$$\begin{aligned}
\log \frac{C_{t+1}}{C_t} &= g_C + \sigma_C \varepsilon_{C,t+1} && (68)\\
\log R_{S,t+1} &= g_S + \sigma_S \varepsilon_{S,t+1}, && (69)
\end{aligned}$$

[10]One could argue that the investor should also frame the outcome of the non-financial asset narrowly, on the grounds that the distribution of that asset's returns may also be more accessible than the distribution of overall wealth once the two risky assets are combined. While it is clear, from (7), that this can easily be accomodated, doing so leaves the equity premium largely unaffected. For simplicity, then, we assume that only stock market risk is framed narrowly.

where

$$\begin{pmatrix} \varepsilon_{C,t} \\ \varepsilon_{S,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_{CS} \\ \rho_{CS} & 1 \end{pmatrix}\right), \text{ i.i.d. over time;} \tag{70}$$

(iii) the consumption-wealth ratio α_t is a constant α, which, using

$$R_{W,t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{1}{1-\alpha}\frac{C_{t+1}}{C_t}, \tag{71}$$

implies that

$$\log R_{W,t+1} = g_W + \sigma_W \varepsilon_{W,t+1}, \tag{72}$$

where

$$\begin{aligned} g_W &= g_C + \log \frac{1}{1-\alpha} && (73)\\ \sigma_W &= \sigma_C && (74)\\ \varepsilon_{W,t+1} &= \varepsilon_{C,t+1}; && (75) \end{aligned}$$

and (iv) the fraction of total wealth made up by the stock market, $\theta_{S,t}$, is a constant over time, θ_S, so that

$$\theta_{S,t} = \frac{S_t}{S_t + N_t} = \theta_S, \ \forall t, \tag{76}$$

where S_t and N_t are the total market value of the stock and of the non-financial asset, respectively.

One question that arises immediately is whether the structure we propose in conditions (i)-(iv) can be embedded in a general equilibrium framework. Condition (iv), the condition that $\theta_{S,t}$ be constant over time, make this a non-trivial challenge. For example, this condition cannot emerge from the simplest model of the production sector, the Lucas tree. In the Appendix, we show that a slightly richer model of the production sector *can* be consistent with conditions (i)-(iv). While we place this analysis in the Appendix, we emphasize that it represents one of our paper's more important contributions: it is this analysis that clears the way for a numerical investigation of the equilibrium implications of narrow framing.

Under conditions (i)-(iv), equations (60)-(62) simplify to

$$\beta^{\frac{1}{1-\gamma}}(1-\alpha)^{\frac{-\gamma}{1-\gamma}} R_f E((\frac{C_{t+1}}{C_t})^{-\gamma})(E((\frac{C_{t+1}}{C_t})^{1-\gamma}))^{\frac{\gamma}{1-\gamma}} = 1 \tag{77}$$

$$\frac{E((\frac{C_{t+1}}{C_t})^{-\gamma}(R_{S,t+1} - R_f))}{E((\frac{C_{t+1}}{C_t})^{-\gamma})} + b_0 R_f (\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}}(\frac{1-\alpha}{\alpha})^{\frac{-\gamma}{1-\gamma}} E(\overline{v}(R_{S,t+1} - R_f)) = 0 \tag{78}$$

$$\frac{E((\frac{C_{t+1}}{C_t})^{-\gamma}(R_{W,t+1} - R_f))}{E((\frac{C_{t+1}}{C_t})^{-\gamma})} + b_0 R_f (\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}}(\frac{1-\alpha}{\alpha})^{\frac{-\gamma}{1-\gamma}} \theta_S E(\overline{v}(R_{S,t+1} - R_f)) = 0. \tag{79}$$

We now use these three equations to compute the equilibrium equity premium. First, we set the return and consumption process parameters to the values in Table 3. These values are estimated from annual data spanning the 20th century and are standard in the literature. Then, for given preference parameters β, γ, b_0, and λ, and a given stock market fraction of total wealth θ_S, equations (77)-(79) can be solved for the consumption-wealth ratio α, the risk-free rate R_f, and the mean log stock return g_S, thereby giving us the equity premium. Luckily, analytical expressions for all the expectation terms in equations (77)-(79) are available, making the calculations straightforward. These expressions are given in the Appendix.[11]

Table 4 presents the results. We take $\beta = 0.98$, $\theta_S = 0.3$, and consider various values of γ, λ, and b_0; the parameter β has little effect on attitudes to risk, and the results are quite similar over a range of values of θ_S. The table shows that narrow framing of stocks can generate a substantial equity premium at the same time as a low risk-free rate. The parameter triple $(\gamma, \lambda, b_0) = (1.5, 3, 0.02)$, for example, produces an equity premium of 5.45% and a risk-free rate of 2.3%. The intuition is simple: if the agent gets utility directly from changes in the value of the stock market and, via the the parameter λ, is more sensitive to losses than to gains, he finds the stock market risky and will only hold the available supply if compensated by a high average return.[12]

The results in Table 4 confirm the findings of Barberis, Huang, and Santos (2001), whose paper is, to our knowledge, the only other attempt to incorporate narrow framing into standard preferences. They study the implications for the equity premium when investors are loss averse, in other words, more sensitive to losses – even small losses – than to gains of the same magnitude. For part of their analysis, Barberis, Huang, and Santos (2001) take the gains and losses to be gains and losses in stock market wealth, rather than in total wealth; in this case, then, they are implicitly assuming that investors frame narrowly, and their preference specification reflects this. In their analysis, they find, as in Table 4, that, in combination with loss aversion, narrow framing of the stock market can generate large equity premia.

In Sections 4 and 5, we have already seen two important ways in which our preference specification improves on that of Barberis, Huang, and Santos (2001). Our specification is tractable in partial equilibrium, while theirs is not. And our preference specification allows for an explicit value function, while their does not, making it difficult to calibrate their utility function by computing attitudes to timeless monetary gambles.

[11] The goal of this section is to provide a framework for equilibrium analysis of narrow framing, and, in particular, to derive the first-order conditions (77)-(79). In what follows, we present some brief numerical computations based on these equations. This is not the focus of our paper, however. For more detailed numerical analysis, see Barberis and Huang (2007), who take equations (77)-(79) as their starting point.

[12] Of course, in assigning our preferences to a representative agent, we are assuming that properties of individual preferences survive aggregation. We do not prove any results about aggregation here, but we can gain some insight from Chapman and Polkovnichenko (2006), who study the effect of heterogeneity on the link between "disappointment aversion" preferences and the equity premium. They find that the equity premium in a heterogeneous agent model, while lower than in a representative agent model, is still sizeable.

We now show that, even in an equilibrium setting, where Barberis, Huang, and Santos' (2001) model is also tractable, our specification offers an important advantage. To see this, note that another question a researcher may be interested in is whether the parameters used by narrow framing models to generate high equity premia are *reasonable*, in terms of making sensible predictions about attitudes to timeless monetary gambles. Since Barberis, Huang, and Santos' (2001) preferences do not admit an explicit value function, they cannot address this question. Our preferences, on the other hand, can easily do so.

Consider, for example, a simple thought experiment proposed by Epstein and Zin (1990) and Kandel and Stambaugh (1991). The experiment posits an agent with current wealth of $\$75,000$ and asks what premium the agent would pay to avoid a 50:50 bet to win or lose $\$25,000$; and also, what premium he would pay to avoid a 50:50 bet to win or lose \$250. By comparing the premia predicted by a particular parameterization of a utility function to our intuition as to what the answers should be, we can judge how reasonable that parameterization is.

The columns labelled π_L and π_S in Table 4 present, for each preference parameterization we consider, the premia that would be charged by the representative agent in our economy, given his equilibrium holdings of risky assets and current wealth of $\$75,000$. The quantities π_L and π_S correspond to the large and small gambles, respectively. We compute π_L and π_S using equation (59). The parameter A_τ in that equation can be computed from equation (22) using the consumption-wealth ratio α obtained from equations (77)-(79).

The table shows that, while all the parameterizations produce reasonable values of π_L, the predicted values of π_S are more reasonable for $b_0 \leq 0.03$ in the case of $(\gamma, \lambda) = (1.5, 2)$, and for $b_0 \leq 0.01$ in the case of $(\gamma, \lambda) = (1.5, 3)$. Our model therefore provides an additional insight not available using Barberis, Huang, and Santos' (2001) specification: that narrow framing of stocks can easily produce large equity premia while also matching reasonable attitudes to large-scale gambles; but that if the researcher is also interested in matching attitudes to *small*-scale gambles, there are limits to the size of the equity premium that narrow framing can generate.

7 Conclusion

Experimental work on decision-making shows that, when people evaluate risk, they often engage in "narrow framing": that is, in contrast to the prediction of traditional utility functions defined over wealth or consumption, they often evaluate risks in isolation, separately from other risks they are already facing. While narrow framing has many potential applications to understanding attitudes to real-world risks, there does not currently exist a tractable preference specification that incorporates it into the standard framework used by economists. In this paper, we propose such a specification and demonstrate its tractability in both portfolio choice and equilibrium settings.

8 Appendix

Proof of Proposition 1: Substituting the expression for B_t^* in equation (24) into the following rearrangement of equation (21),

$$(\frac{\beta}{1-\beta})^{\frac{1}{\rho}}(\frac{1-\alpha_t}{\alpha_t})^{1-\frac{1}{\rho}}B_t^* = 1, \tag{80}$$

gives

$$(\frac{\beta}{1-\beta})^{\frac{1}{\rho}}(\frac{1-\alpha_t}{\alpha_t})^{1-\frac{1}{\rho}}\left[\mu((1-\beta)^{\frac{1}{\rho}}\alpha_{t+1}^{1-\frac{1}{\rho}}\theta_t'\widetilde{R}_{t+1}|I_t) + b_0\sum_{i=m+1}^{n}E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1}-R_{i,z})))\right] = 1, \tag{81}$$

which, after applying the form of $\mu(\cdot)$ in equation (25), gives equation (27).

To derive equations (28), let $K(\theta_t)$ equal the argument being maximized in equation (24), and recall that the certainty equivalent functional is given by (25), so that

$$K(\theta_t) \equiv H(\theta_t) + G(\theta_t), \tag{82}$$

where

$$\begin{aligned} H(\theta_t) &\equiv (1-\beta)^{\frac{1}{\rho}}\mu({\alpha_{t+1}}^{1-\frac{1}{\rho}}\theta_t'\widetilde{R}_{t+1}|I_t) \\ &= (1-\beta)^{\frac{1}{\rho}}[E_t(\alpha_{t+1}^{\zeta(1-\frac{1}{\rho})}(\theta_t'R_{t+1})^{\zeta})]^{\frac{1}{\zeta}} \qquad (83) \\ G(\theta_t) &\equiv b_0\sum_{i=m+1}^{n}E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1}-R_{i,z}))). \qquad (84) \end{aligned}$$

The optimal portfolio weights θ_t can be computed by solving

$$\max_{\theta_t}\left[H(\theta_t) + G(\theta_t) + \psi_t\left(1-\sum_{i=1}^{n}\theta_{i,t}\right)\right], \tag{85}$$

where the Lagrange multiplier ψ_t satisfies

$$\sum_{i=1}^{n}\theta_{i,t} = 1. \tag{86}$$

Since $\mu(\cdot)$ is strictly concave, and α_{t+1} and $\{\widetilde{R}_{i,t+1}\}_{i=1}^{n}$ are all non-zero random variables, $H(\theta_t)$ is also strictly concave in θ_t. Moreover, since $\overline{v}(\cdot)$ is concave in θ_t, so is $G(\theta_t)$. The argument to be maximized in (85) is therefore strictly concave in θ_t and any local maximum is also a global maximum.

Since $H(\theta_t)+G(\theta_t)$ has well-defined first derivatives everywhere except at $\theta_{i,t}=0$ for $i > m$, the necessary and sufficient conditions for optimality, other than the standard constraint (86), are

$$\frac{\partial H(\theta_t)}{\partial\theta_{i,t}} + \frac{\partial G(\theta_t)}{\partial\theta_{i,t}} - \psi_t = 0, \quad \text{for } \theta_{i,t} \neq 0, \tag{87}$$

and

$$\begin{cases} \frac{\partial H(\theta_t)}{\partial \theta_{i,t}} + \frac{\partial G(\theta_t)}{\partial \theta^+_{i,t}} - \psi_t \leq 0 \\ \frac{\partial H(\theta_t)}{\partial \theta_{i,t}} + \frac{\partial G(\theta_t)}{\partial (-\theta^-_{i,t})} - \psi_t \geq 0 \end{cases}, \quad \text{for } \theta_{i,t} = 0. \tag{88}$$

Writing out the partial derivatives in full gives conditions (28).

Computing Attitudes to Timeless Gambles

Condition (46) can be easily implemented as soon as A_τ is computed. Given that investment opportunities are i.i.d., it is straightforward to show that $A_t = A$, $\forall t$, and that $\alpha_t = \alpha$, $\forall t$. The quantity A can then be computed from (16), where, given our simplifying assumption that the investor does not frame any of his pre-existing risks narrowly, b_0 can be set to 0. Equation (18) then becomes

$$B^* = A(E(\widetilde{R}^{1-\gamma}_{t+1}))^{\frac{1}{1-\gamma}}, \tag{89}$$

which, when substituted into equation (21), gives

$$\alpha = 1 - \beta^{\frac{1}{\gamma}}(E(\widetilde{R}^{1-\gamma}_{t+1}))^{\frac{1}{\gamma}}. \tag{90}$$

A can then be computed from equation (22).

To implement condition (52), note that the left-hand side can be written

$$\max_{\alpha} \; \{(1-\beta)\alpha^{1-\gamma}+\beta(1-\alpha)^{1-\gamma}\left[A(E(\widetilde{R}_{\tau+1} + \frac{\widetilde{g}}{W_\tau(1-\alpha)})^{1-\gamma})^{\frac{1}{1-\gamma}} + b_0 \frac{x-\lambda y}{2W_\tau(1-\alpha)}\right]^{1-\gamma}\}^{\frac{1}{1-\gamma}} W_\tau. \tag{91}$$

This maximization can be performed numerically.

Proof of Lemma: Note that

$$\alpha_{t+1}\theta'_t R_{t+1} = \alpha_{t+1}\frac{W_{t+1}}{W_t - C_t} = \frac{\alpha_t C_{t+1}}{(1-\alpha_t)C_t}. \tag{92}$$

Substituting this into equation (29) gives,

$$\beta^{\frac{1}{1-\gamma}}\left[E_t((\frac{C_{t+1}}{C_t})^{-\gamma}\theta'_t R_{t+1})\right]^{\frac{1}{1-\gamma}} + b_0(\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}}(\frac{1-\alpha_t}{\alpha_t})^{\frac{-\gamma}{1-\gamma}} \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(R_{i,t+1} - R_{f,t}))) = 1. \tag{93}$$

Substituting equation (92) into conditions (30), recalling that $\theta_{i,t} > 0$ for $i > 1$, and then taking the difference between equation (30) for security $i > 1$ and equation (30) for security 1, gives, for $i > 1$,

$$\begin{aligned} &\beta^{\frac{1}{1-\gamma}}[E_t((\frac{C_{t+1}}{C_t})^{-\gamma}\theta'_t R_{t+1})]^{\frac{\gamma}{1-\gamma}} E_t[(\frac{C_{t+1}}{C_t})^{-\gamma}(R_{i,t+1} - R_{f,t})] + \\ &b_0 1_{\{i>m\}}(\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}}(\frac{1-\alpha_t}{\alpha_t})^{\frac{-\gamma}{1-\gamma}} E_t(\overline{v}(R_{i,t+1} - R_{f,t})) = 0. \end{aligned} \tag{94}$$

Note that equation (94) also holds trivially for $i = 1$. Subtracting

$$\sum_{i=1}^{n} \theta_{i,t}(\text{eqn(94) for asset } i) \tag{95}$$

from equation (93) gives equations (60) and (61).

Simplifying the First-order Conditions (77)-(79)

Using

$$\begin{aligned} E(e^{a\varepsilon}) &= e^{a^2/2} && (96) \\ E(1_{\varepsilon<\widehat{\varepsilon}}) &= N(\widehat{\varepsilon}) && (97) \\ E(1_{\varepsilon<\widehat{\varepsilon}} e^{a\varepsilon}) &= e^{a^2/2} N(\widehat{\varepsilon} - a), && (98) \end{aligned}$$

the first-order conditions (77)-(79) become

$$\begin{aligned} \alpha &= 1 - \beta^{\frac{1}{\gamma}} R_f^{\frac{1-\gamma}{\gamma}} e^{\frac{1}{2}(1-\gamma)\sigma_C^2} && (99) \\ 0 &= b_0 R_f (\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}} (\frac{1-\alpha}{\alpha})^{\frac{-\gamma}{1-\gamma}} \left[e^{g_S + \frac{1}{2}\sigma_S^2} - R_f + (\lambda - 1) \left[e^{g_S + \frac{1}{2}\sigma_S^2} N(\widehat{\varepsilon}^S - \sigma_S) - R_f N(\widehat{\varepsilon}^S) \right] \right] + \\ & \quad e^{g_S + \frac{1}{2}\sigma_S^2 - \gamma \sigma_S \sigma_C \rho_{CS}} - R_f && (100) \\ 0 &= b_0 R_f (\frac{\beta}{1-\beta})^{\frac{1}{1-\gamma}} (\frac{1-\alpha}{\alpha})^{\frac{-\gamma}{1-\gamma}} \theta_S \left[e^{g_S + \frac{1}{2}\sigma_S^2} - R_f + (\lambda - 1) \left[e^{g_S + \frac{1}{2}\sigma_S^2} N(\widehat{\varepsilon}^S - \sigma_S) - R_f N(\widehat{\varepsilon}^S) \right] \right] + \\ & \quad \frac{1}{1-\alpha} e^{g_C + \frac{1}{2}\sigma_C^2 - \gamma \sigma_C^2} - R_f, && (101) \end{aligned}$$

where

$$\widehat{\varepsilon}^S = \frac{\log(R_f) - g_S}{\sigma_S}. \tag{102}$$

A General Equilibrium Model to support conditions (i)-(iv) of Section 6.1

We now show that the structure described in conditions (i)-(iv) of Section 6.1 can be embedded in a simple general equilibrium model. To repeat, the conditions are that: (i) the risk-free rate is a constant R_f; (ii) $R_{S,t+1}$ and $\frac{C_{t+1}}{C_t}$ follow the i.i.d processes in (68)-(70); (iii) the consumption-wealth ratio is a constant α; and (iv) the fraction of wealth made up by the stock market is a constant, θ_S. Of course, the last condition implies that the fraction of total wealth made up by the non-financial asset, $\theta_{N,t}$, is also constant over time, and equal to $\theta_N = 1 - \theta_S$.

Consider an economy with two firms. Asset 3 in Section 6.1, stock, is a claim to the payout of one of the firms, the "stock" firm, say, while asset 2 in that section, the non-financial asset, is a claim to the payout of the other firm, the "non-stock" firm. Total output in the economy, Y_t, is the sum of the output of the stock firm, $Y_{S,t}$, and of the non-stock firm, $Y_{N,t}$,

$$Y_t = Y_{S,t} + Y_{N,t}. \tag{103}$$

Each firm divides its output between a consumption good payout and capital investment,[13]

$$Y_{S,t} = \widehat{C}_{S,t} + I_{S,t}, \qquad Y_{N,t} = \widehat{C}_{N,t} + I_{N,t}, \tag{104}$$

so that

$$Y_t = \widehat{C}_t + I_t, \tag{105}$$

where $\widehat{C}_t$ and I_t are the total consumption good payout and total investment in the economy,

$$\widehat{C}_t = \widehat{C}_{S,t} + \widehat{C}_{N,t}, \qquad I_t = I_{S,t} + I_{N,t}. \tag{106}$$

The production technologies are

$$Y_{S,t+1} = f_S(I_{S,t}, I_{N,t}), \qquad Y_{N,t+1} = f_N(I_{S,t}, I_{N,t}). \tag{107}$$

For simplicity, labor input is not modeled explicitly, and capital investment made at time t is assumed 100% depreciated after $t+1$.

We write $\widehat{S}_{t-}$ and $\widehat{S}_t$ ($\widehat{N}_{t-}$ and $\widehat{N}_t$) to denote the total market value of all shares of the stock firm (non-stock firm) at time t, immediately before and after the consumption good payout, respectively, so that

$$\widehat{S}_{t-} = \widehat{C}_{S,t} + \widehat{S}_t, \qquad \widehat{N}_{t-} = \widehat{C}_{N,t} + \widehat{N}_t. \tag{108}$$

We also define the total market value of both firms, before and after the consumption good payout, as

$$\widehat{W}_{t-} = \widehat{S}_{t-} + \widehat{N}_{t-}, \qquad \widehat{W}_{t+} = \widehat{S}_t + \widehat{N}_t. \tag{109}$$

The conditions for general equilibrium are the conditions for capital market optimality

$$E_t(m\frac{\partial Y_{S,t+1}}{\partial I_{S,t}}) = 1, \qquad E_t(m\frac{\partial Y_{N,t+1}}{\partial I_{N,t}}) = 1, \qquad \forall t, \tag{110}$$

where m is the stochastic discount factor; and the market clearing conditions, both for the consumption good and for shares in the firms,

$$\widehat{C}_t = C_t, \qquad \widehat{S}_t = S_t, \qquad \widehat{N}_t = N_t, \qquad \forall t. \tag{111}$$

The last equation implies

$$W_t - C_t = \widehat{W}_{t+}, \qquad W_t = \widehat{W}_{t-}. \tag{112}$$

[13] If the consumption equilibrium in conditions (i)-(iv) shares a variable with the production economy we consider here, we distinguish the latter with a hat sign.

We seek an equilibrium with the following properties:

$$\begin{aligned}\widehat{C}_t &= \xi Y_t, \quad I_t = (1-\xi)Y_t, \quad \forall t \\ I_{S,t} &= \zeta I_t = \zeta(1-\xi)Y_t, \quad I_{N,t} = (1-\zeta)I_t = (1-\zeta)(1-\xi)Y_t, \quad \forall t, \end{aligned} \tag{113}$$

and where

$$\widehat{S}_t = A_S I_{S,t}, \quad \widehat{N}_t = A_N I_{N,t}, \quad \widehat{W}_{t-} = \Lambda Y_t. \tag{114}$$

Under these assumptions, the returns on the stock firm, on the non-stock firm, and on total wealth are

$$\widehat{R}_{S,t+1} = \frac{\widehat{S}_{(t+1)-}}{\widehat{S}_t} = \frac{(Y_{S,t+1} - I_{S,t+1}) + A_S I_{S,t+1}}{A_S I_{S,t}} = \frac{Y_{S,t+1} + (A_S - 1)\zeta(1-\xi)Y_{t+1}}{A_S I_{S,t}} \tag{115}$$

$$\widehat{R}_{N,t+1} = \frac{\widehat{N}_{(t+1)-}}{\widehat{N}_t} = \frac{(Y_{N,t+1} - I_{N,t+1}) + A_N I_{N,t+1}}{A_N I_{N,t}} = \frac{Y_{N,t+1} + (A_N - 1)\zeta(1-\xi)Y_{t+1}}{A_N I_{N,t}} \tag{116}$$

$$\widehat{R}_{W,t+1} = \frac{\widehat{W}_{(t+1)-}}{\widehat{W}_{t+}} = (1-\xi)\frac{\Lambda}{\Lambda - \xi}\frac{Y_{t+1}}{I_t}. \tag{117}$$

Note also that since

$$\widehat{W}_{t-} = \widehat{C}_t + \widehat{S}_t + \widehat{N}_t, \tag{118}$$

we have

$$\Lambda = \xi + (1-\xi)(A_S\zeta + A_N(1-\zeta)). \tag{119}$$

We can now state:

Proposition 2: *There exists a consumption-production general equilibrium in which the consumption and return processes are given by equations (68), (69), (72), and (77)-(79), and the production process is given by (113)-(114), with*

$$Y_{S,t+1} = (I_{S,t}I_{N,t})^{\frac{1}{2}}\nu_{S,t+1}, \quad Y_{N,t+1} = (I_{S,t}I_{N,t})^{\frac{1}{2}}\nu_{N,t+1}, \quad \forall t, \tag{120}$$

where

$$\nu_{S,t+1} = \frac{1+\alpha}{\alpha}\left(\frac{\theta_S}{\theta_N}\right)^{\frac{1}{2}}\left[\exp(g_S + \sigma_S\varepsilon_{S,t+1}) - \frac{1}{1+\alpha}\exp(g_C + \sigma_C\varepsilon_{C,t+1})\right] \tag{121}$$

$$v_{S,t+1} + \nu_{N,t+1} = 2(\theta_S\theta_N)^{\frac{1}{2}}\frac{1}{1-\alpha}\exp(g_C + \sigma_C\varepsilon_{C,t+1}), \tag{122}$$

and where the constant coefficients are given by

$$\xi = \frac{1+\alpha}{2}, \; \Lambda = \frac{1+\alpha}{2\alpha}, \; A_S = A_N = \frac{1+\alpha}{\alpha}, \; \zeta = \theta_S. \tag{123}$$

Proof of Proposition 2: First note that conditions (i)-(iv) of Section 6.1 hold if and only if:

$$\begin{aligned}
C_t &= \widehat{C}_t, \text{ for some } t, \text{ to set the scale,} && (124)\\
\frac{C_t}{W_t} &= \frac{\widehat{C}_t}{\widehat{W}_t} = \frac{\xi Y_t}{\Lambda Y_t}, \quad \forall t, \text{ which implies } \alpha = \frac{\xi}{\Lambda}, && (125)\\
\theta_S &= \widehat{\theta}_S \equiv \frac{\widehat{S}_t}{\widehat{S}_t + \widehat{N}_t} = \frac{A_S\zeta}{A_S\zeta + A_N(1-\zeta)}, \quad \theta_N = \widehat{\theta}_N \equiv \frac{\widehat{N}_t}{\widehat{S}_t + \widehat{N}_t} = \frac{A_N(1-\zeta)}{A_S\zeta + A_N(1-\zeta)} && (126)\\
R_{S,t} &= \widehat{R}_{S,t}, \quad R_{N,t} = \widehat{R}_{N,t}, \quad \forall t. && (127)
\end{aligned}$$

Note that the last condition implies $R_{W,t} = \widehat{R}_{W,t}$, $\forall t$.

We now prove the proposition by explicit construction. Suppose that

$$Y_{S,t+1} = (I_{S,t})^a (I_{N,t})^b \nu_{S,t+1}, \qquad Y_{N,t+1} = (I_{S,t})^{a'} (I_{N,t})^{b'} \nu_{N,t+1}. \tag{128}$$

Then[14]

$$\begin{aligned}
\frac{\partial Y_{S,t+1}}{\partial I_{S,t}} &= a\frac{Y_{S,t+1}}{I_{S,t}} = aA_S\widehat{R}_{S,t+1} - a(A_S-1)\zeta(1-\xi)\frac{Y_{t+1}}{I_{S,t}}\\
&= aA_S\widehat{R}_{S,t+1} - a(A_S-1)\frac{\Lambda-\xi}{\Lambda}\widehat{R}_{W,t+1}, && (129)
\end{aligned}$$

and similarly

$$\frac{\partial Y_{N,t+1}}{\partial I_{N,t}} = b'A_N\widehat{R}_{N,t+1} - b'(A_N-1)\frac{\Lambda-\xi}{\Lambda}\widehat{R}_{W,t+1}. \tag{130}$$

The conditions for capital market optimality in (110) become

$$aA_S - a(A_S-1)\frac{\Lambda-\xi}{\Lambda} = 1, \qquad b'A_N - b'(A_N-1)\frac{\Lambda-\xi}{\Lambda} = 1. \tag{131}$$

Our independent equations are therefore (119), (125), (126), (127) and (131), and the unknowns are ξ, Λ, A_S, A_N, ζ, a, b, a', b', $\nu_{S,t+1}$, and $\nu_{N,t+1}$. Since we have some extra degrees of freedom, we can simplify by setting $A_S = A_N$. Then, from (131), $a = b'$. We also have

$$a + b = a' + b' = 1. \tag{132}$$

Further assuming that $a = b$, we have

$$a = b = a' = b' = \frac{1}{2}. \tag{133}$$

[14] Here, we are effectively assuming that the "stock" firm is one of infinitely many identical "stock" firms, and likewise for the "non-stock" firm. This simplifies the analysis by allowing us to ignore strategic behavior. Even in an economy where firms do behave strategically, however, an equilibrium satisfying conditions (i)-(iv) of Section 6.1 can still be constructed.

Finally, we obtain

$$\zeta = \theta_S, \qquad A \equiv A_S = A_N = \frac{\Lambda - \xi}{1 - \xi}, \qquad \frac{A}{2} - \frac{(A-1)}{2}\frac{\Lambda - \xi}{\Lambda} = 1. \tag{134}$$

Combining these, we obtain

$$\xi = \frac{1+\alpha}{2}, \qquad \Lambda = \frac{1+\alpha}{2\alpha}, \qquad A_S = A_N = \frac{1+\alpha}{\alpha}, \tag{135}$$

as in the proposition. Putting these solutions into equations (115)-(117), we obtain

$$\widehat{R}_{W,t+1} = \frac{1}{2}(\theta_S\theta_N)^{\frac{1}{2}}(\nu_{S,t+1} + \nu_{N,t+1}) \tag{136}$$

$$\widehat{R}_{S,t+1} = \frac{\alpha}{1+\alpha}(\frac{\theta_N}{\theta_S})^{\frac{1}{2}}\nu_{S,t+1} + \frac{1-\alpha}{2(1+\alpha)}(\theta_S\theta_N)^{\frac{1}{2}}(\nu_{S,t+1} + \nu_{N,t+1}) \tag{137}$$

$$\widehat{R}_{N,t+1} = \frac{\alpha}{1+\alpha}(\frac{\theta_N}{\theta_S})^{\frac{1}{2}}\nu_{N,t+1} + \frac{1-\alpha}{2(1+\alpha)}(\theta_S\theta_N)^{\frac{1}{2}}(\nu_{S,t+1} + \nu_{N,t+1}). \tag{138}$$

Setting equation (136) equal to equation (72), we obtain equation (122). Setting equation (137) equal to equation (69), we obtain equation (121). That $R_{N,t+1} = \widehat{R}_{N,t+1}$ follows from the portfolio identity.

9 References

Barberis, N. and M. Huang (2007), "The Loss Aversion / Narrow Framing Approach to the Equity Premium Puzzle," forthcoming in R. Mehra, ed., *Handbook of Investments: Equity Premium*, Amsterdam: North Holland.

Barberis, N., Huang, M., and T. Santos (2001), "Prospect Theory and Asset Prices," *Quarterly Journal of Economics* 116, 1-53.

Barberis, N., Huang, M., and R. Thaler (2006), "Individual Preferences, Monetary Gambles and the Equity Premium," *American Economic Review* 96, 1069-1090.

Chapman, D., and V. Polkovnichenko (2006), "Heterogeneity in Preferences and Asset Market Outcomes," Working paper, University of Texas, Austin.

Chew S. (1983), "A Generalization of the Quasilinear Mean with Applications to the Measurement of Income Inequality and Decision Theory Resolving the Allais Paradox," *Econometrica* 51, 1065-1092.

Curcuru, S., Heaton, J., Lucas, D., and D. Moore (2004), "Heterogeneity and Portfolio Choice: Theory and Evidence," forthcoming in Y. Ait-Sahalia and L. Hansen, eds., *Handbook of Financial Econometrics*, Amsterdam: North Holland.

Epstein, L. and S. Zin (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Perspective," *Econometrica* 57, 937-969.

French, K. and J. Poterba (1991), "Investor Diversification and International Equity Markets," *American Economic Review* 81, 222-226.

Gabaix, X. and D. Laibson (2006), "Bounded Rationality and Directed Cognition," Working paper, Harvard University.

Haliassos, M. and C. Bertaut (1995), "Why Do So Few Hold Stocks?" *Economic Journal* 105, 1110-1129.

Heaton, J. and D. Lucas (2000), "Portfolio Choice in the Presence of Background Risk," *Economic Journal* 110, 1-26.

Kahneman, D. (2003), "Maps of Bounded Rationality: Psychology for Behavioral Economics," *American Economic Review* 93, 1449-75.

Kahneman D. and D. Lovallo (1993), "Timid Choices and Bold Forecasts: A Cognitive

Perspective on Risk Taking," *Management Science* 39, 17-31.

Kahneman, D. and A. Tversky (1979), "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47, 263-291.

Kahneman, D. and A. Tversky (1983), "Choices, Values, and Frames," *American Psychologist* 39, 341-350.

Kandel, S. and R. Stambaugh (1991), "Asset Returns and Intertemporal Preferences," *Journal of Monetary Economics* 27, 39-71.

Koszegi, B., and M. Rabin (2007), "Reference-dependent Risk Attitudes," *American Economic Review*, forthcoming.

Mankiw, N.G. and S. Zeldes (1991), "The Consumption of Stockholders and Non-stockholders," *Journal of Financial Economics* 29, 97-112.

Rabin, M. and R. Thaler (2001), "Anomalies: Risk Aversion," *Journal of Economic Perspectives* 15, 219-232.

Read, D., Loewenstein, G., and M. Rabin (1999), "Choice Bracketing," *Journal of Risk and Uncertainty* 19, 171-197.

Redelmeier, D. and A. Tversky (1992), "On the Framing of Multiple Prospects," *Psychological Science* 3, 191-193.

Segal, U. and A. Spivak (1990), "First Order versus Second Order Risk Aversion," *Journal of Economic Theory* 51, 111-125.

Tversky A. and D. Kahneman (1981), "The Framing of Decisions and the Psychology of Choice," *Science* 211, 453-458.

Tversky, A. and D. Kahneman (1986), "Rational Choice and the Framing of Decisions," *Journal of Business* 59, 5251-78.

Tversky, A. and D. Kahneman (1992), "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297-323.

Table 1: Parameter values for the return processes in a portfolio choice problem with three assets. Asset 1 is riskless and earns the gross risk-free rate R_f. Assets 2 and 3 are risky: g_2 and σ_2 (g_3 and σ_3) are the mean and standard deviation of log gross returns on asset 2 (asset 3); ω is the correlation of log returns on assets 2 and 3. Finally, $\overline{\theta}_2$ is the fixed fraction of wealth held in asset 2.

Parameter	
R_f	1.02%
g_2	0.04
σ_2	0.10
g_3	0.04
σ_3	0.10
ω	0.01
$\overline{\theta}_2$	0.50

Table 2: The table shows, for given aversion to consumption risk γ, sensitivity to narrowly framed losses λ, and degree of narrow framing b_0, the percentage of his remaining wealth that an investor with 50% of his wealth already invested in one risky asset would invest in another similar, weakly correlated risky asset.

	$\gamma = 1.5$ $\lambda = 2$	$\gamma = 1.5$ $\lambda = 3$	$\gamma = 5$ $\lambda = 2$	$\gamma = 5$ $\lambda = 3$
$b_0 = 0$	100	100	90	90
$b_0 = 0.005$	100	100	88	68
$b_0 = 0.01$	100	100	86	44
$b_0 = 0.02$	100	70	82	0
$b_0 = 0.03$	100	0	76	0
$b_0 = 0.04$	100	0	72	0
$b_0 = 0.05$	100	0	68	0
$b_0 = 0.1$	100	0	40	0
$b_0 = 0.2$	52	0	0	0
$b_0 = 0.3$	0	0	0	0
$b_0 = 0.4$	0	0	0	0
$b_0 = 0.5$	0	0	0	0

Table 3: Parameter values for a representative agent equilibrium model: g_C and σ_C are the mean and standard deviation of log consumption growth, respectively, σ_S is the standard deviation of log stock returns, and ρ_{CS} is the correlation of log consumption growth and log stock returns.

Parameter	
g_C	1.84%
σ_C	3.79%
σ_S	20.0%
ρ_{CS}	0.10

Table 4: The table shows, for given aversion to consumption risk γ, sensitivity to narrowly framed losses λ, and degree of narrow framing b_0, the risk-free rate R_f and equity premium EP generated by narrow framing in a simple representative agent economy. π_L (π_S) is the premium the representative agent would pay, given his equilibrium holdings of risky assets and current wealth of $75,000, to avoid a 50:50 bet to win or lose $25,000 ($250).

γ	λ	b_0	R_f	EP	π_L	π_S
1.5	2	0	4.7%	0.12%	$6,371	$0.63
1.5	2	0.01	4.2%	1.39%	$6,336	$18
1.5	2	0.02	3.7%	2.41%	$6,312	$31
1.5	2	0.03	3.4%	3.15%	$6,296	$39
1.5	2	0.04	3.1%	3.66%	$6,286	$44
1.5	3	0	4.7%	0.12%	$6,371	$0.63
1.5	3	0.005	4.1%	1.54%	$6,836	$20
1.5	3	0.010	3.4%	2.99%	$7,237	$37
1.5	3	0.015	2.8%	4.35%	$7,552	$50
1.5	3	0.020	2.3%	5.45%	$7,773	$60

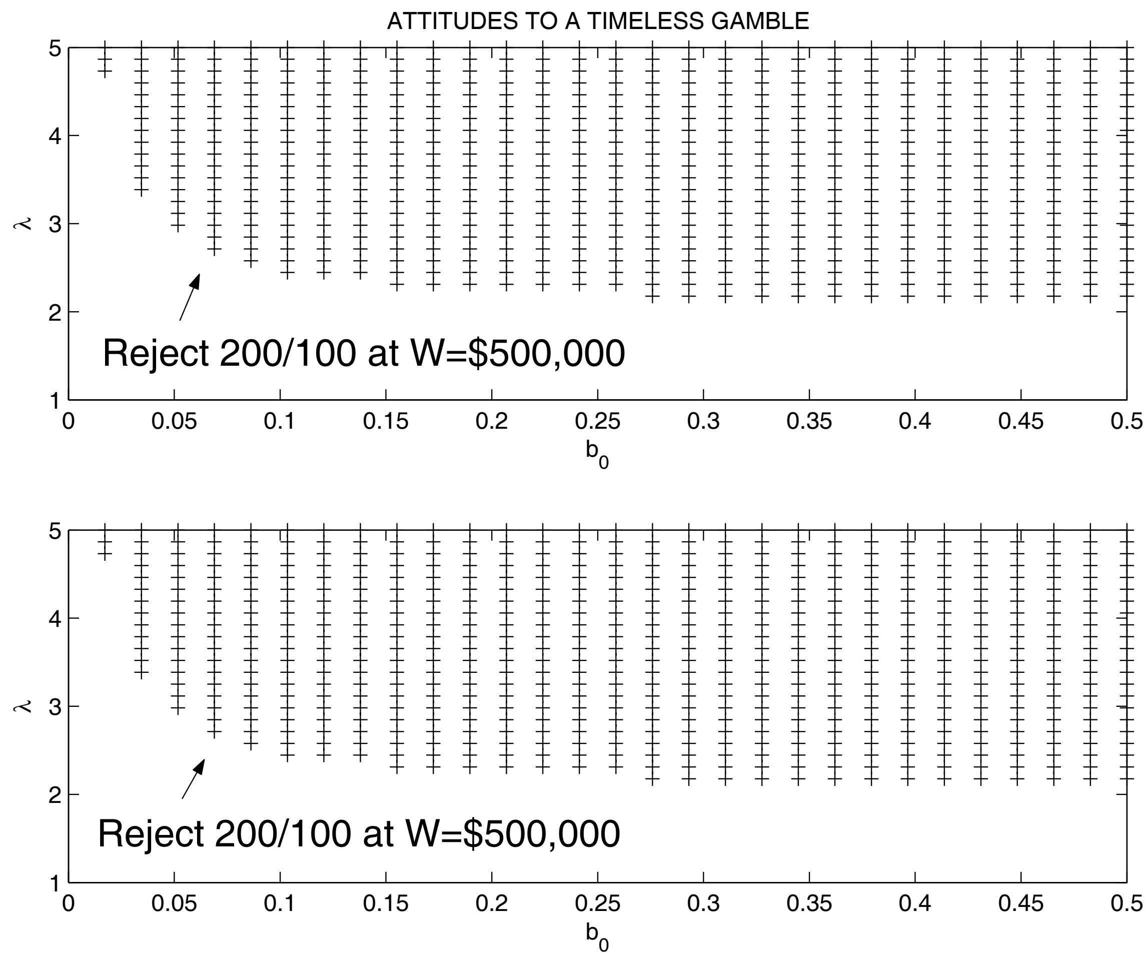

Figure 1. The figure shows how an agent who frames narrowly would react, at a current wealth level of $500,000, to a timeless gamble offering a 50:50 chance to win $200 or lose $100. The two plots correspond to different assumptions as to how the agent evaluates the gamble.