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The Supply and Demand for Safe Assets

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Abstract

There is a demand for safe assets, either government bonds or private substitutes, for use as collateral. Government bonds are safe assets, given the government's power to tax, but their supply is driven by fiscal considerations, and does not necessarily meet the private demand for safe assets. Unlike the government, the private sector cannot produce riskless collateral. When the private sector reaches its limit (the quality of private collateral), government bonds are net wealth, up to the governments own limits (taxation capacity). The economy is fragile to the extent that privately-produced safe assets are relied upon. In a crisis, government bonds can replace private assets that do not sustain borrowing anymore, raising welfare.

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1 Introduction

In 2001 the Bank for International Settlements presciently noted that the use of collateral in financial markets had become so widespread that there was a looming problem: "With growth of collateral use so rapid, concern has been expressed that it could outstrip the growth of the effective supply of these preferred assets . . . The increase in collateralized transactions has occurred while the supply of collateral with inherently low credit and liquidity risks has not kept pace. Securities markets continue to grow, but many major government bond markets are expanding only slowly or even contracting. The latter phenomenon was particularly evident in the United States in the second half of the 1990s." (p.2). Indeed, as we learned during the financial crisis of 2007-2008, with a shortage of government bonds, private agents relied heavily on privately-produced "safe assets." Privately-produced near-riskless assets, e.g., AAA/Aaa asset-backed securities, were created in response to this shortage.

Safe assets are important, as explained recently by the International Monetary Fund in their Global Financial Stability Report:

Safe assets are used as a reliable store of value and aid capital preservation in portfolio construction. They are a key source of liquid, stable collateral in private and central bank repurchase (repo) agreements and in derivatives markets, acting as the lubricant or substitute of trust in financial transactions. As key components of prudential regulation, safe assets provide banks with a mechanism for enhancing their capital and liquidity buffers. As benchmarks, safe asset support the pricing of other riskier assets. Finally, safe assets have been a critical component of monetary policy operations (IMF (2012, p. 82)).

Where do safe assets come from? Empirical evidence suggests that the private sector creates more near-riskless assets when the supply of government debt is low and reduces privately-created near-riskless assets when the supply of government debt is high. Krishnamurthy and Vissing-Jorgensen (2012b) show that the net supply of government debt is strongly negatively correlated with the net supply of private near-riskless debt. The substitution between public and private safe debt is also shown by Krishnamurthy and Vissing-Jorgensen (2012a) who document that changes in the supply of outstanding U.S. Treasuries have large effects on the yields of privatecreated assets. Gorton, Lewellen, and Metrick (2010) also find this yield spreads relationship between government debt and privately-produced substitutes. They document that the share of safe assets in the U.S. economy, including both U.S. Treasury debt and privately-created near-riskless debt has remained constant as a percentage of all U.S. assets since 1952. Xie (2012) shows that the issuance of asset-backed securities tends to occur when the outstanding government debt is low and Sunderam (2012) documents the same phenomenon with respect to asset-backed commercial paper.

Krishnamurthy and Vissing-Jorgensen (2012b) also empirically show that financial crises are more likely when the quantity of outstanding U.S. Treasuries is low, and so the privately-produced debt is high. In fact, during the recent financial crisis, the Term Securities Lending Facility (TSLF) allowed banks to borrow U.S. Treasuries while posting privately-created near-riskless bonds that had become impaired during the crisis as collateral. Hrung and Seligman (2011) argue that the TSLF was "uniquely effective relative to other policies" in dealing with the recent crisis. This is consistent with Gourinchas and Jeanne (2012) who argue that "macroeconomic shortages of safe assets can create financial instability. Crises, when they occur, further exacerbate the shortage that gave rise to it."

By "safe assets" we mean government debt and privately-created high quality debt, in particular, asset-backed securities. Such safe assets are used to collateralize repo, derivative positions, and are needed as collateral in clearing and settlement. See IMF (2012). Further, because they are "information-insensitive" (in the nomenclature of Dang, Gorton, and Holmström (2012)) they are highly liquid and hence can store value without fear of capital losses in times of stress, a form of private money.

The evidence that the private sector fills in privately-created debt when the outstanding amount of government bonds is low suggests that Ricardian Equivalence (i.e., Barro (1974)) does not always hold. There is a demand for government bonds to use as collateral. In this paper we provide a rationale for why this is the case based on the details of the role of collateral in the economy. Using a model of information acquisition about collateral values, based on Dang, Gorton, and Holmström (2012) and Gorton and Ordonez (2012), privately-produced bonds can be used as collateral, potentially relaxing borrowing restrictions for firms. Collateral is needed to enable borrowing by firms, borrowing such that the lenders do not produce information about the collateral, but simply lend. Government bonds are preferred to privatelyproduced collateral if that collateral is not of sufficiently high quality to enable the optimal borrowing. In this case government bonds are not neutral because they can be used as collateral, increasing production towards the optimal borrowing.

In normal times, lenders do not have incentives to acquire information about the value of privately-produced assets that firms use as collateral for borrowing. Hence, a large volume of assets can be used to sustain borrowing in the economy. As in Gorton and Ordonez (2012), a "crisis" occurs when there is a public arrival of bad news such that lenders have incentives to acquire information about the value of the privately-produced assets, only lending to firms with assets of high value, reducing the volume of assets that can be successfully used as collateral. This is also the definition of crises adopted by Gourinchas and Jeanne (2012).

We show here that government bonds have a (non-Ricardian) benefit during crises. In normal times, it may not matter whether the government finances its expenses with taxes or bonds, if government bonds do not relax borrowing constraints. In contrast, during crises the private assets that can be successfully used as collateral to back borrowing declines. Then government bonds can replace private assets that do not sustain borrowing anymore, constituting positive wealth and breaking Ricardian Equivalence since financing with bonds become superior to taxes, consistent with the evidence of Hrung and Seligman (2011). Even if government bonds are not net wealth normally, Ricardian Equivalence breaks down exactly during times in which bonds may be needed, during a crisis.

However, there are limits to the use of government bonds as collateral. On the one hand, taxes reduce the incentives to work and invest in the economy. On the other hand, when bonds are used as collateral and some lenders are foreign, some bonds end up outside the (domestic) economy, increasing the tax pressure domestically since those bonds are not used to cover taxes.

In our model, household lenders make loans directly to firms, and the loans must be collateralized. We abstract from financial intermediaries for the sake of simplicity, but we have in mind financial contracts like sale and repurchase agreements (repo) which involve a lender making a loan against collateral that can be either a government bond or a private asset. Also, more generally loans are made against collateral, as senior secured bank loans for example.

Our results clearly differ from the well-known Ricardian Equivalence result, which states that under certain conditions households internalize the government's budget constraint; hence it does not matter whether a government finances its spending with debt or with taxes. It does not matter for consumption plans of households whether the government finances public spending using current taxes or future taxes. The reason is that households would reduce current consumption either to pay taxes or to save to pay future taxes. One of the main conditions for Ricardian Equivalence to hold is that capital markets are perfect. In essence, under liquidity constraints, government bonds can additionally provide liquidity services, increasing households wealth because they relax liquidity constraints. In his paper, Barro (1974) explores this possibility in a very reduced way. We provide the details of how liquidity is created with safe assets, how the value of such liquidity is determined, and provide conditions under which Ricardian Equivalence does not hold. There is a very large literature on Ricardian Equivalence, with mixed empirical evidence.¹

Closest to our work is Saint-Paul (2000) who also shows that government debt can relax borrowing constraints because it can be used as collateral. In his setting, financial contracts involve costly state verification (i.e., Townsend (1979)). When the borrowers wealth includes government debt, the need for monitoring is reduced, although government debt "crowds out" private investment. Costly state verification is necessary in an environment where the production of information by a lender can be necessary because borrowers may not report the truth about unobservable project outcomes.

There are significant differences between Saint-Paul's setting and ours. The raison detre for debt is different in the two models. We adopt the concept of debt from Dang, Gorton, and Holmström (2012) who show that information production is not desirable; information-insensitive debt, that is debt where it is not desirable to produce private information ex ante, can optimally support more borrowing and hence higher output and consumption. In Townsend, it may be optimal to produce information ex post. The difference in the concept of debt is important because another difference concerns the possibility of a financial crisis. There are no crises in Saint-Paul's model whereas in our setting it can happen that information is produced about the collateral

¹Bernheim (1987), Seater (1993) and Elmendorf and Mankiw (1999) review this extensive literature. For the empirical evidence (or lack of it) see Feldstein (1982), Kormendi (1983), Barro (1987), Evans (1987), Plosser (1987), Bohn (1992 and 1998)), Ricciuti (2003), Laubach (2007), and Rohn (2010) among others. Notably, this empirical literature is not conclusive about the effects of government debt on interest rates, while Krishnamurthy and Vissing-Jorgensen (2012a) analyze spreads, with clearer results.

(as in Gorton and Ordonez (2012)), leading to a financial crisis with decreased output and consumption.

The paper proceeds as follows. In Section 2 we introduce the model and display a benchmark equilibrium without the presence of the government. We show how a financial crisis can occur. Then we introduce the government in a simple way. We analyze the conditions under which government debt is neutral in normal times and positive net wealth during a crisis. We focus on what happens during a financial crisis in order to demonstrate the main point about the use of government bonds as collateral. In Section 3 we introduce a more realistic setting in which the government sells bonds to finance investments in infrastructure. This allows us to think of Ricardian Equivalence in terms of a "convenience yield" and so we can show the link between the model results and the empirical work of Krishnamurthy and Vissing-Jorgensen (2012b and 2012a). We also show that the likelihood of a financial crisis depends on the amount of government debt outstanding. The economy is fragile to the extent that privately produced collateral is needed. The last section concludes.

2 Model

Assume a country with an overlapping generations structure in which each generation has mass 1 of risk neutral individuals who live for three periods; initial, intermediate and final. We call individuals living in their initial period *newborns*, individuals living in their intermediate period *young* and individuals living in their final period *old*. Their utility is linear in the consumption of a perishable numeraire good.

At the end of the initial period, each newborn receives a unit of an asset that we call land, which can be taken to the next period and, as we discuss later, can be potentially used as collateral. Land can be either good or bad. Good land generates C units of numeraire at the end of the intermediate period, while bad land does not generate any numeraire.

There are two possible aggregate states in the economy that govern the average quality of land. The "normal" state (*H*) is one in which the fraction of land that is of good quality is p_H . In a "low" state (*L*) only a fraction $p_L < p_H$ of land is of good quality. The low state may correspond to a crisis, as will be seen. We assume the state of the economy is known at the beginning of the intermediate period, but the individual quality of each unit of land is not known. It is possible, however, to privately observe the quality of a unit of land by spending γ units of numeraire.

At the beginning of the intermediate period, each young individual has a unit of land carried over from the initial period, observes the aggregate state and receives a stock of managerial skills $E = K^*$ (that does not generate any disutility if used). These managerial skills can be combined with numeraire in a production technology that generates more numeraire at the end of the intermediate period. The problem is that young individuals have a technology for the production of numeraire, but they have no numeraire to use as inputs. The production function is Leontief, generating $Y_1 = A \min\{E, K\}$ units of numeraire with probability q and 0 otherwise. We assume that production is efficient (i.e., qA > 1). So, the optimal scale of production is given by $K^* = E$, such that the optimal ex-ante expected production is $Y_1^* = qAK^*$.

Finally, at the beginning of the final period, old individuals have a unit of labor endowment, which generates linear disutility if used to work. They each have a Cobb-Douglas production technology that just depends on labor decisions and generates $Y_2 = L^{\alpha}$ of numeraire at the end of the final period. This implies that optimally the old generation optimal labor supply is $L^* = \alpha^{\frac{1}{1-\alpha}}$.

It is clear that, even though it is optimal for young individuals to borrow numeraire at the beginning of the intermediate period to produce during the intermediate period, they cannot use the land (which transforms into numeraire at the end of the period) and the old generation is not able to lend numeraire because their production accrues at the end of the period. To allow for the existence of potential lenders we assume there is a foreign country, the "rest-of-the-world" with perfectly competitive potential lenders that have an endowment of numeraire \overline{K} each period.²

To justify a role for land, we assume that the output of the intermediate period production function is non-verifiable, which implies that young individuals cannot borrow numeraire by issuing claims against their expected production. However, the young can use land as collateral against a loan from foreign lenders. We further assume $C > K^*$ (land known to be good can sustain the optimal loan size) and $\overline{K} > K^*$

²As will become clearer, we assume foreign individuals as lenders to isolate the effects of bond redistribution on taxation pressures, and to study the role of lending sources on the likelihood of crises. The notion of the rest-of-the-world as the lenders, however, is realistic in the build-up to the recent financial crisis. See, for example, Bertaut et al. (2011) and Bernanke et al. (2011). As noted by the IMF (2012, p. 81): "Prior to the crisis, global current account imbalances encouraged safe asset purchases by official reserve managers and sovereign wealth funds."

(there is enough numeraire in the rest of the world to sustain optimal production at the intermediate period).

The timing is summarized in Figure 1. The role of the newborns will become clear later, in Section 3, where we endogenize the demand for safe assets.

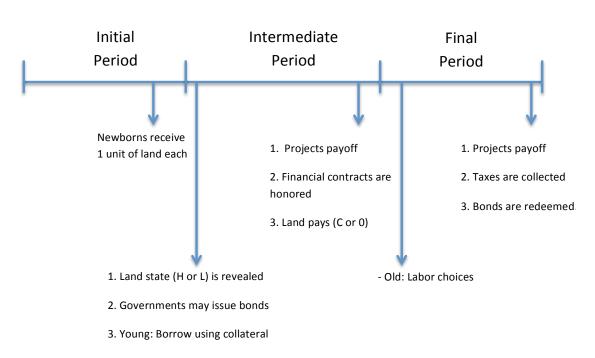


Figure 1: Timing

First we study the dynamics of the economy without a government and then we introduce a government that can issue one-period bonds, and study how these bonds can improve the economic situation.

2.1 Information Production about Collateral

At the beginning of each period there is no public information about which land is good and which is bad, only the average land quality is known. We assume that in normal times, each unit of land, which is believed to be good with probability p_H can sustain optimal borrowing in expectation:

 $p_H C > K^*.$

Following Gorton and Ordonez (2012), there are no incentives for lenders to acquire private information about the quality of each unit of land in order to speculate on finding good land, which in case the firm defaults is effectively obtained at a low price.³ This condition for no information acquisition can be written as:

$$(1-q)p_H[\frac{K^*}{p_H} - K^*] < \gamma$$

This says that the expected gains to lenders from privately producing information are lower than the cost. The cost is just γ . The benefits can be decomposed in the following way. With probability (1 - q) the lender will get the land rather than the loan repayment. In this case, if the lender finds out the land is good (with probability p_H) he obtains the collateral at a price K^* (the loan size) but obtains consumption equal to K^*/p_H .

Hence, in order to make the model interesting and to introduce a difference between "normal" and "low" times, we assume:

$$p_H > \max\left\{\frac{K^*}{C}, 1 - \frac{\gamma}{(1-q)K^*}\right\},\$$

which guarantees that all land sustains the optimal amount of borrowing K^* (first argument) without triggering information acquisition (second argument).

In contrast, we assume that during low states:

$$p_L < \max\left\{\frac{K^*}{C}, 1 - \frac{\gamma}{(1-q)K^*}\right\},\,$$

so the average quality of land is low enough such that either it does not sustain optimal borrowing or, in case of sustaining optimal borrowing, information acquisition is triggered at that level of borrowing. So, in the absence of government debt, the low state will correspond to a crisis, in which the production of the intermediate period is lower than the potential optimal production.

³Since borrowers do not have any numeraire, they are not capable of producing information about land quality. As shown by Gorton and Ordonez (2013), this assumption does not change the main results.

2.2 Normal Times and Crises

If times are normal, at the beginning of the period the rest-of-the-world lends the optimal amount of numeraire K^* to young individuals, without inducing any information acquisition about the quality of each individual unit of land. Old individuals work optimally.

At the end of the intermediate period, young individuals whose projects succeed repay K^* to foreign lenders from their production and young individuals whose projects failed hand over the fraction of land of expected quality p_H to foreign lenders. Old individuals consume their optimal production Y_2^* . Consumption in normal times (state *H*) is deterministic for the two generations and for foreign lenders:

$$U_Y^H = K^*(qA - 1) + E(p)C$$
$$U_O^H = Y_2^* - L_2^*$$
$$U_F^H = \overline{K}.$$

For these computations, recall that $U_F^H = \overline{K} - K^* + E(repayment)$ and $U_Y^H = K^*(qA - 1) + K^* - E(repayment) + E(p)C$. However, since debt is risk-free and the lenders are competitive, expected repayment for a loan of size K^* is exactly K^* .

If times are low, or what we call a crisis, only a fraction p_L of land is good. In this situation there are incentives for information acquisition about collateral quality. In this case, only a fraction p_L of land, the good land, supports the optimal borrowing level K^* , while the rest of the young generation, those with bad land, cannot get loans.⁴ Once information has been revealed in the economy, only the now known good land is able to obtain credit as long as the crisis persists. Hence, in a crisis (state L), consumption of the two generations and foreign lenders is also deterministic:

$$U_Y^H = p_L K^* (qA - 1) + E(p)C - \gamma$$
$$U_O^H = Y_2^* - L_2^*$$
$$U_F^H = \overline{K}.$$

We define crises as states where information is produced, and only a fraction of land is able to sustain borrowing and production by young individuals, as opposed to all of

⁴If $\frac{K^*}{C} > p_L > 1 - \frac{\gamma}{(1-q)K^*}$ lenders do not acquire information but only lend $K < K^*$, what we call a credit crunch in Gorton and Ordonez (2012).

them being able to produce at the optimal scale, as is the case under the informationinsensitive good state of the economy. Furthermore, there is a loss of numeraire spent on producing information that is deducted from the loans to young individuals.

We now turn the question of whether the government can effectively intervene during a crisis to prevent the decline in production and consumption by young individuals. Next, we study the effects of government debt in normal times and during a crisis. The bonds enter the economy in a very reduced form way and there is no discussion of government spending. These topics are taken up in the next section.

2.3 Government Bonds in Normal Times

We study a policy of the government that endows each young household with a government bond, and taxes those same households when old to payoff the bond. In Section 3 we will be more precise about how bonds enter the economy, how they are traded and how they are priced. The bond can be saved so that the old have it in retirement when their taxes are due. This implies that, as a benchmark, we guarantee Ricardian Equivalence in the absence of any collateral and informational frictions.

To determine the consumption of old individuals we have to specify how their income is taxed. If taxes are collected as a fraction of their total production of numeraire, then a member of the old generation has consumption of:

$$U_O^H = Y_2^*(\tau) - L_2^*(\tau) + B - \tau Y_2^*(\tau)$$
(1)

where $L_2^*(\tau) = [\alpha(1-\tau)]^{\frac{1}{1-\alpha}}$ is the distorted level of labor (declining in τ), then $Y_2^*(\tau)$ is the distorted level of production by the old, B is the face value of the bond which must be repaid and $\tau Y_2^*(\tau)$ are the tax revenues.

Since taxes are set to payoff any bonds outstanding, $B = \tau Y_2^*(\tau)$ and they are collected as a fraction of income, bonds reduce the consumption of the old generation by distorting their labor decision. We can show that, the utility of the old generation,

$$U_{O}^{H} = (1 - \alpha(1 - \tau))[\alpha(1 - \tau)]^{\frac{\alpha}{1 - \alpha}}$$

is monotonically decreasing in τ . Furthermore, the maximum revenue that the government can collect in this economy is subject to a Laffer curve. The tax rate that maximizes tax revenues, $\tau Y_2^*(\tau)$, subject to the constraint $B = \tau Y_2^*(\tau)$, is $\overline{\tau} = 1 - \alpha$. This implies the maximum volume of bonds that can be issued is $\overline{B} = \overline{\tau} Y_2^*(\overline{\tau}) = (1 - \alpha) \alpha^{\frac{2\alpha}{1-\alpha}}$.

In contrast, if taxes are collected by lump-sum transfers from the production of the old generation, such that T = B, then they do not introduce any distortion in labor supply by the old. Evidently, we have:

Proposition 1 Government bonds are never positive net wealth in normal times. If taxes are lump-sum then government bonds are neutral. If taxes are distortionary, then government bonds are negative net wealth.

Intuitively, old households can always pay their taxes with the government bonds that they saved from their youth, and there are no real effects of this unless those taxes are distortionary. If the bonds are just saved by young households because they do not have any impact on their production, then intervention in normal times is not desirable in this economy.

2.4 Government Bonds in Crises

We now consider a crisis period. A crisis means that there is a reduction in the average quality of land. If there is no government intervention, then in the crisis the rest-of-the-world lenders produce information and all agents learn which land is bad and which land is good. This production of information is not only costly in terms of consumption because of the information costs, but also costly in terms of production, since young households with bad land cannot use the land as collateral to back their borrowing needs to produce. In the absence of other considerations, the output of the economy falls, as we showed above when studying the case without the government.

When the government intervenes by introducing bonds, young individuals can offer those bonds to foreign lenders as collateral to back their loans. What is the amount of bonds the government needs to provide in order for young individuals to be able to borrow K^* without triggering information about who has good and who has bad land? From the assumption on p_L , and the condition for information acquisition, we know that:

$$(1-q)(1-p_L)K^* > \gamma.$$

This implies that in a crisis state the maximum that young individuals can borrow without triggering information production is $K = \frac{\gamma}{(1-q)(1-p_L)} < K^*$. So, the government needs to provide a bond for the difference in order for the young to be able to borrow K^* without triggering information production about the land quality. Then

$$B^* = K^* - \frac{\gamma}{(1-q)(1-p_L)}.$$
(2)

Suppose for the moment that this is acceptable to the foreign lenders. The fraction of the young that fail in their projects and default is (1 - q). This means that foreign lenders will receive (1 - q) bonds at the end of the period, which they will redeem at the beginning of the next period. Since bonds mature in one period, the government pays them off and the foreigners receive a windfall gain, which is financed by taxing the old, who suffer a loss on net since they have more liabilities than bonds. The average consumption of the old generation becomes:

$$U_O^H = Y_2^*(\tau) - L_2^*(\tau) + qB - \tau Y_2^*(\tau)$$
(3)

and, since $qB < \tau Y_2^*(\tau)$, the old must use some of their production to cover the shortfall, suffering more tax pressure than the case without government bonds captured by equation (1).

What are the costs and benefits of government bonds during crises? Their benefits are $(1 - p_L)K^*(qA - 1)$, which is the production that the bonds support by allowing borrowing by firms which have useless bad land. What about the costs? If taxes are collected by lump sum transfers from the old generation, then there are no distortions and the government can credibly pay the bonds promised amount as long as $Y_2^* > T - qB^*$ or, given $T = B^*$, as long as

$$Y_2^* = \alpha^{\frac{\alpha}{1-\alpha}} > (1-q)B^*.$$
 (4)

That is, the production of the old households must be high enough to cover the government bonds handed to the young households. This is just a parametric condition. In case this condition is not fulfilled, it implies that bonds can improve welfare, but not to the first best under which young individuals can produce at the optimal scale. In this case, since there are no distortions from bonds, they are always beneficial. **Proposition 2** *Government bonds are positive net wealth in times of crisis when they are financed through lump sum transfers to the old generation.*

If taxes are distortionary, it is necessary to balance the gains from bonds in increasing the production of young individuals against the costs from decreasing the production of old individuals. The optimal tax level, τ^* is given by the point at which $\tau^* Y_2^*(\tau^*) = B^*$, conditional on $B^* \leq \overline{B}$, i.e., the maximum number of bonds that can be sustained by taxation.

This level is feasible as long as the maximum possible revenues are enough to cover the bonds in the hands of the young generation. Since $\tau^* < \overline{\tau}$, where $\overline{\tau}$ was defined as the tax level that maximizes revenues, the lower bound on the production of the old is $Y_2^*(\overline{\tau}) = \alpha^{\frac{2\alpha}{1-\alpha}}$. Hence, the sufficient condition for feasibility is when $Y_2^*(\overline{\tau}) > (1-q)B^*$ or parametrically as long as

$$\alpha^{\frac{2\alpha}{1-\alpha}} > (1-q)B^*.$$

In the case where it is feasible for the government to provide the optimal amount of bonds, then the bonds are positive net wealth when the benefits for young individuals are higher than the costs imposed on old individuals, that is, when:

$$(1 - p_L)K^*(qA - 1) > U_O^H(\tau = 0) - U_O^H(\tau = \tau^*).$$
(5)

A parametric sufficient condition can be obtained for the highest possible cost to the old generation, $\overline{\tau} = 1 - \alpha$,

$$(1-p_L)K^*(qA-1) > \alpha^{\frac{\alpha}{1-\alpha}} \left[1-\alpha - (1-\alpha^2)\alpha^{\frac{\alpha}{1-\alpha}}\right].$$
(6)

Proposition 3 Government bonds are positive net wealth in times of crisis when they are financed through proportional taxes on the production of the old generation only if the benefits from higher production of young individuals compensates for the distortion from lower production of old individuals. The condition for this result is equation (5) and a sufficient condition on parameters is given by equation (6).

Finally, the next Proposition shows that governments are constrained in issuing bonds to improve the production of young individuals. This constraint is given by the taxing limits of the government, given by the production possibilities and incentives of the old generation. **Proposition 4** Governments are constrained in their ability to improve the output of the firms in times of crisis by the maximum revenue that can support the optimal amount of bonds B^* . This constraint is given by equation (2) if the government finances with lump-sum taxes and by equation (4) if the government finances with taxes proportional to the production of the old generation.

From condition (6), we can obtain a condition under which bonds are more likely to be positive net wealth during crises. The next corollary summarizes these points.

Corollary 1 The effectiveness of government bonds as collateral (given by the sufficient condition (6)) increases with the probability of success of the projects (q), the production gains for young households (A), the fraction of bad land $(1 - p_L)$, the optimal scale of production (K^*), and the production possibilities of the old generation.

The reasoning is natural. The smaller is q, the larger the tax bill that the old have to cover with their own output, without the possibility of using bonds. Since a fraction (1 - q) of bonds is a windfall gain to the rest-of-the-world, the redistribution of bonds towards foreign lenders who do not face the taxes, puts limits on the efficacy of government bonds to create net wealth.

As is clear from this corollary, it is more difficult for the government to improve welfare in times of crisis the lower is the probability of success of projects q. Hence, combining this result with the corollary of Proposition 3, a higher probability of success of young projects not only render government bonds more desirable (since they result in more efficient production), but also make the intervention by issuing bonds more feasible.

Two further points are noteworthy. First, note, from equation (2), that the smaller the cost of information production, γ , the greater the amount of bonds that need to be issued to avoid the effects of the crisis. In other words, when information is cheap to generate or easily accessible, it is more costly and it may not even be feasible to use bonds to alleviate the effects of a crisis.

Second, let *z* be the fraction of loans made by the rest-of-the-world. Note that the bonds available domestically to pay the taxes is not qB anymore but (q + (1 - z)(1 - q))B. That is, to the extent that the rest-of-the-world is important in lending against good collateral, the less feasible it will be for the government to intervene in a crisis.

In other words, there is too much tax pressure on the old generation when most bonds end up in the hands of the foreign lenders because they were used as collateral.

3 Supply and Demand for Safe Assets

So far we have assumed that there are no benefits of government bond issuance, other than mitigating the effects of crises, when those crises suddenly appear. Indeed, when bonds are financed through distortionary taxes, their effects are always negative during normal times. However, bonds are not typically introduced to deal with crises but rather to cover government expenditures. In this section we explicitly model a reason for government debt. The government sells bonds to optimally finance infrastructure that improves the output of the economy. We also have assumed that the newborns just receive land that can be used as collateral when young. However they may have the option between buying land or buying government bonds to use as collateral when young. We introduce this possibility, which determines the price of government bonds and the convenience yield on government debt.

More specifically, we consider the same economy as above, with two important modifications. First, the production function of old individuals will depend both on labor and a stock of infrastructure in the economy, which we call X, following a Cobb-Douglas production function $Y_2 = X^{\beta}L^{\alpha}$ of numeraire, with $\alpha + \beta < 1$. We assume the government is the only agent who can invest in infrastructure (to avoid, for example, free riding problems, or to avoid agents taking advantage of natural monopolies). It is possible to transform a unit of numeraire at the end of the initial period into a unit of infrastructure that is ready for production at the beginning of the final period. The government can raise the money to invest in infrastructure by issuing bonds in the initial period, which are claims on taxation revenues in the final period. These bonds pay no interest.

Second, newborns are not endowed with land at the end of the initial period, but instead with endowment \overline{K} of numeraire. Since the numeraire good is non-storable, individuals at the end of the initial period choose to buy bonds (possibly in limited supply) or an indivisible unit of land, which is expected to be good with probability \hat{p} , as defined above. As before, since the endowment accrues at the end of the initial period, newborns cannot act as lenders to their contemporaneous young households

with that numeraire, maintaining the simplifying assumption that loans only come from the rest of the world. Figure 2 provides a timeline for this extended model.

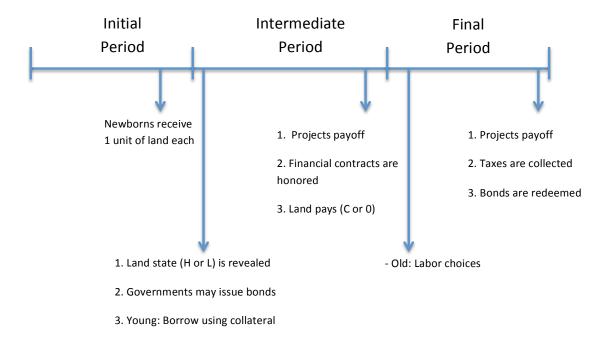


Figure 2: Timing Extended Setting

First, we characterize the government's optimal bond issuance when we only consider the effects of the government financing infrastructure in the economy. Then we discuss the additional benefits of those bonds in reducing the likelihood and size of potential crises, in terms of lower output.

3.1 The Supply of Bonds to Finance Infrastructure

When governments' only consideration to supply bonds is to finance infrastructure investment, then they trade-off the benefits of bonds in terms of increasing infrastructure and output during the final period, with the costs of bonds in reducing consumption in the initial period to invest in infrastructure which potentially reduces labor supply in the final period, depending on how the bonds are financed.

Here we focus on the case of taxes that are proportional to total output, since that was the case in which bonds were negative wealth during normal times in the previous section. When bonds are financed with lump sum taxes, their supply is larger since there are no distortions in the labor supply of the last period, and then the costs of investing in infrastructure is smaller. Regardless of the way of financing bonds, there will be an optimal supply if the government only consider their effects on infrastructure and larger output.

The target level of infrastructure, X, requires a government investment of the same magnitude, for which it is necessary to issue bonds B to sell to newborns at the end of the initial period. We also assume budget balance, which implies that B cannot exceed the revenues from taxing the output in the last period

$$X = B = \tau Y_2.$$

Proposition 5 Assume bonds are repaid using taxes τ proportional to the total production of the final period. The optimal tax rate is $\tau^* = \beta$ and the supply of bonds is

$$B^* = \left[\alpha^{\alpha}\beta^{1-\alpha}\right]^{\frac{1}{1-\alpha-\beta}}.$$

Proof Since $X = \tau Y_2$, we can express the government's problem as one of choosing τ to maximize

$$\max_{\tau} Y_2(\tau) - L_2(\tau) - X(\tau)$$
(7)

subject to the restriction that $X = \tau Y_2$, and that the old generation will choose its labor supply as a function of the taxes they face.

First we need to solve the labor supply decision in the last period as a function of the tax rate. Since $Y_2 = (\tau Y_2)^{\beta} L_2^{\alpha}$, then $Y_2 = [\tau^{\beta} L_2^{\alpha}]^{\frac{1}{1-\beta}}$ and the old generation's problem is:

$$\max_{L_2} (1-\tau) Y_2(L_2) - L_2 + B.$$

Taking first order conditions in the last period,

$$L_2^{\frac{1-\alpha-\beta}{1-\beta}} = \frac{\alpha}{1-\beta}(1-\tau)\tau^{\frac{\beta}{1-\beta}}$$

which determines the labor supply in the last period as a function of the tax rate

$$L_2(\tau) = \left[\frac{\alpha}{1-\beta}(1-\tau)\right]^{\frac{1-\beta}{1-\alpha-\beta}} \tau^{\frac{\beta}{1-\alpha-\beta}}.$$

Rewriting the total output in the last period solely as a function of the tax rate

$$Y_2(\tau) = \left[\tau^{\beta} L_2^{\alpha}\right]^{\frac{1}{1-\beta}} = \left[\frac{\alpha}{1-\beta}(1-\tau)\right]^{\frac{\alpha}{1-\alpha-\beta}} \tau^{\frac{\beta}{1-\alpha-\beta}}.$$
(8)

Using the previous two expressions, labor in terms of output in the last period is

$$L_2(\tau) = \left[\frac{\alpha}{1-\beta}(1-\tau)\right]^{\frac{\alpha}{1-\alpha-\beta}+1} \tau^{\frac{\beta}{1-\alpha-\beta}} = \frac{\alpha}{1-\beta}(1-\tau)Y_2(\tau).$$

Substituting this last expression into equation (7), and imposing $X(\tau) = \tau Y_2(\tau)$, the government problem can be written as:

$$\max_{\tau} \frac{1-\alpha-\beta}{1-\beta} (1-\tau) Y_2(\tau).$$

Taking first order conditions

$$(1 - \tau^*)Y_2'(\tau^*) = Y_2(\tau^*),$$

where

$$Y_2'(\tau^*) \equiv \frac{\partial Y_2(\tau|\tau^*)}{\partial \tau} = \frac{Y_2(\tau)}{1 - \alpha - \beta} \left(\frac{\beta}{\tau} - \frac{\alpha}{1 - \tau}\right)$$

which implies that

 $\tau^* = \beta.$

Since $B^* = \tau^* Y_2(\tau^*)$, plugging τ^* into equation (8), we obtain the optimal supply of bonds (and then the optimal investment in infrastructure), in the Proposition.

Q.E.D.

Naturally, when $\beta = 0$, the optimal tax rate is zero, since there are no gains from investing in infrastructure. This is effectively the assumption in the previous sections, under which there were no gains from the government from issuing bonds and there were costs from distorting labor supply in the last period. In what follows we take the governments fiscal policy as determined in this way and, to make it interesting, we assume that parameters are such that $B^* < K^*$, such that bonds will not be enough collateral in the economy.

3.2 The Demand for Bonds and the Convenience Yield

Land is indivisible and in infinite supply, which implies that its price is $P_L = 1$ unit of numeraire per unit of land that delivers one unit of numeraire in the last period. Given the indivisibility, if an individual buys a unit of land with expected value $\hat{p}C$, he has to pay $\hat{p}C$ (no less, otherwise land would not be sold, and no more, given the infinite supply of land).⁵

Bonds are divisible but in finite supply B^* , determined above by the need for infrastructure. A bond promises 1 unit of numeraire in the last period. This implies that the lower bound for the price of a unit of bond is $P_B = 1$. However, given it is a scarce resource there may be competition for it. However, the maximum an individual is willing to bid per unit of bond is $P_B = qA$, the expected gain from using the bond as collateral.

Define $K_R \equiv \overline{K} - \hat{p}C$, that is, the residual numeraire available to buy bonds or consume after buying a unit of land, and $K_X \equiv K^* - K(\hat{p})$, that is, the difference between the optimal level of capital and the maximum that can be borrowed just using land of expected quality \hat{p} as collateral.

Assume first that $B^* > K_X$, which implies the optimal level of government bonds in the economy is enough to cover the private collateral shortfall, that is, the difference between the optimal loan size and the lending that land can support.

Net of the output in the last period these are the following possible levels of an individual's utility under different decisions (recall that individuals are risk neutral: $U = C_0 + C_1 + C_2$):

1. Individuals just consume their endowment (autarky):

$$U_A = \overline{K} + 0 + 0 = \overline{K}.$$

2. Individuals buy a unit of land and bonds B_R , using residual endowment, K_R

$$U_L = (\overline{K} - \hat{p}C - P_B B_R) + (K(\hat{p}) + B_R)(qA - 1) + \hat{p}C + B_R$$

= $\overline{K} + (K(\hat{p}) + B_R)(qA - 1) - (P_B - 1)B_R$

⁵For a discussion of different determination of land prices (in particular land prices that incorporate the value of land as collateral), see Gorton and Ordonez (2012).

where $P_B B_R \leq K_R$ and $B_R \leq K_X$.

3. Individuals just buy bonds to borrow K^* (that is, $B_R = K^*$). Recall that we have assumed that $\overline{K} > qAK^*$, so the endowment is enough to buy bonds to completely fund the first period project, even at the highest possible price of bonds. Nothing changes assuming otherwise, except the equations for prices.

$$U_B = (\overline{K} - P_B K^*) + K^* (qA - 1) + K^*$$
$$= \overline{K} + K^* (qA - P_B).$$

It is clear from the previous equations that, if $P_B = 1$ all individuals would like to buy bonds ($U_B > U_L$). Since everybody wants bonds, that would force bond prices down. It is also clear that in the opposite extreme, if $P_B = qA$ all individuals would like to buy land ($U_B < U_L$) because the government keeps the whole surplus from using bonds as collateral. Since nobody wants bonds, that would force bond prices down. This implies that some individuals will buy land and some will only buy bonds, so they should be indifferent (this is, $U_B = U_L$), or:

$$(K(\hat{p}) + B_R)(qA - 1) - (P_B - 1)B_R = K^*(qA - P_B).$$

Hence, bond prices are:

$$P_B = \frac{K(\hat{p})}{K^* - B_R} + \frac{(K^* - B_R - K(\hat{p}))}{K^* - B_R} qA.$$
(9)

Naturally, $B_R = \min\left\{\frac{K_R}{P_B}, \max\{K_X, B^*\}\right\}$, then $P_B \in \left[1, \frac{K(\hat{p})}{K^*} + \frac{(K^* - K(\hat{p}))}{K^*}qA\right]$. If $K_R = 0$, then $B_R = 0$ and $P_B = \frac{K(\hat{p})}{K^*} + \frac{(K^* - K(\hat{p}))}{K^*}qA$. If K_R is very large and $B_R = K_X$, then $P_B = 1$. If K_R is very large and $B_R = B^*$, then $P_B \geq \frac{K(\hat{p})}{K^* - B^*} + \frac{(K^* - B^* - K(\hat{p}))}{K^* - B^*}qA$. In the intermediate range, the solution comes from substituting $B_R = K_R/P_B$ into the equation (9) for P_B and solving for the roots of:

$$K^* P_B^2 + [(K(\hat{p}) - K^*)qA - (K(\hat{p}) + K_R)]P_B + K_R qA = 0$$
(10)

which has a unique positive root in the range defined above.

To gain intuition about these prices, assume, for example, that $K_R = 0$, then $B_R = 0$ and so $P_B = \frac{K(\hat{p})}{K^*} + \frac{(K^* - K(\hat{p}))}{K^*} qA$. Since the individuals who buy land cannot buy bonds, then they should be indifferent between financing a small fraction of the project with land, or financing the whole project with bonds, but at a higher price.

At the other extreme, if K_R is large enough such that it is possible for individuals to buy land and extra bonds to finance the whole project, for any price $P_B > 1$ individuals would never buy only bonds to finance the whole project (since the price of land is 1). Then the only possibility is that $P_B = 1$.

What is the fraction of individuals who do not buy land but instead buy bonds to finance the project up to full scale? Call this fraction x. From the indifference condition we computed the price of bonds. Now, from the resource constraints we can compute the fraction of individuals who only buy bonds. We can compute $xK^* + (1 - x)B_R = B^*$, which implies that:

$$x = \frac{B^* - B_R}{K^* - B_R}.$$

At one extreme, if $K_R = 0$, then $B_R = 0$, and there is a fraction $x = B^*/K^*$ of individuals who use only bonds to finance the full amount of their projects and the rest only use land to finance a fraction of the project, but get a larger surplus from it.

At the opposite extreme, when K_R is sufficiently large such that $B_R = K_X < B^*$, then $P_B = 1$, a fraction of individuals $x \frac{B^* - B_R}{K^* - B_R}$ acquire only bonds to use as collateral, and the rest of individuals buy land and finance fully the rest of the project with bonds.

Finally, relaxing the first assumption and allowing $B^* < K_X$ means that the level of bonds in the economy is insufficient to cover the difference between the loan that land can support and the optimal loan size. In this case x = 0. For this to be an equilibrium it is required that $U_L > U_B$, which is fulfilled for:

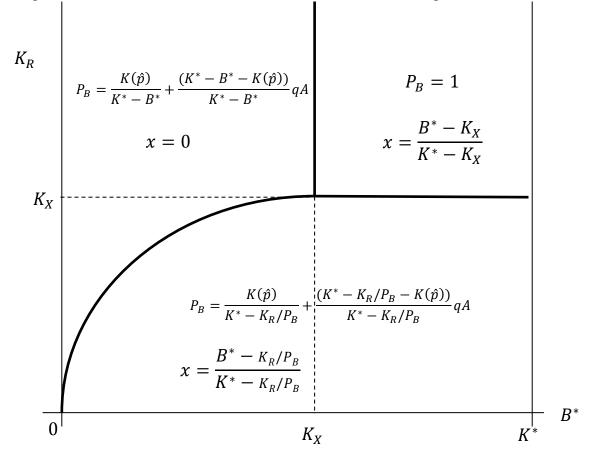
$$P_B \ge \frac{K(\hat{p})}{K^* - B^*} + \frac{(K^* - B^* - K(\hat{p}))}{K^* - B^*} qA.$$

Figure 3 shows graphically the regions of bond prices and the fraction of individuals just holding bonds (without land) in the economy, just as function of the parameters. To summarize,

Proposition 6 Bond prices and fraction of individuals just holding bonds.

1. When individuals are rich in endowment and there are many bonds in the economy, i.e., when $K_R > K_X$ and $B^* > K_X$, then all projects are fully financed. Bond prices are identical to land prices (there is no convenience yield).

Figure 3: Bond Prices and Fraction of Households not Using Land as Collateral



- 2. When individuals are rich in endowment but there are few bonds in the economy, i.e., when $B^* < K_X$ and $K_R > B^*P_B$ (where P_B is given by equation 10) then young agents holds land and finance just a fraction of the project. The bonds have a convenience yield that just depends on the volume of bonds and not on the endowment of individuals.
- 3. When individuals are relatively poor in endowment but there are enough bonds in the economy, i.e., when $K_R < K_X$ and $B^* > K_R/P_B$, then not all young agents hold land. Those with land only finance a fraction of the project and those with bonds finance projects fully. The convenience yield declines with endowment and the fraction of individuals just with bonds who finance the project fully increases with the quantity of bonds in the economy.

Note that, if land is good enough as collateral, this is \hat{p} is such that $K(\hat{p}) = K^*$, then $K_X = 0$, and the whole region is given by the first case, in which there is no conve-

nience yield of government bonds and a fraction $x = B^*/K^*$ of young households use only bonds as collateral to finance fully their projects.

3.3 The Probability of a Crisis

Assume, as above, that there are two possible states of the world, a normal state (a fraction p_H of land is good) and a low state (a fraction p_L of land is good). Assume also that the probability of normal times is η . Since the newborn decision of buying land or bonds happens before the state is realized (newborns buy bonds at the end of the initial period and the state is realized at the beginning of the intermediate period), this decision is based on $\hat{p} = \eta p_H + (1 - \eta)p_L$, such that $K_X > 0$.

In case the intermediate period is characterized by a low aggregate state, the fraction (1 - x) of households that hold land as collateral will suffer from a reduction in borrowing and a downsizing of their projects, while a fraction x of households who only hold government bonds as collateral for borrowing will not suffer from the shock and still would be able to borrow the optimal amount K^* .

Assume a level of endowment for newborns, and then a level of K_R . As can be seen from the analysis of the regions above, and increase in the supply of bonds in the economy B^* (for example there is a larger importance of infrastructure in the production function of the final period, captured by an increase in β), increases the fraction xof households who hold only bonds to finance the projects (except in region 2, where x = 0). This implies that the difference between being in normal times and being in crisis times in terms of output is given by:

$$(1 - x(B^*)) [K(p_H) - K(p_L)].$$
(11)

Clearly this loss is decreasing in the volume of bonds in the economy, which increases x. In other words, a potential shock in the expected value of collateral is more costly the lower the volume of bonds in the economy.

Proposition 7 *The decline in output during a crisis is lower to the extent that there are more government bonds outstanding, as given by equation (11).*

Since bonds can be effectively used as collateral, a larger fraction of bonds buffers the economy from potential shocks to the expected value of land that may reduce its role as collateral, inducing a lower probability that such shock translates into a financial crisis. This is consistent with the empirical findings of Krishnamurthy and Vissing-Jorgensen (2012a); an increase in Treasury debt decreases the probability of a financial crisis. In our setting this is because bonds can be used as superior substitutes for private collateral – they are independent of shocks to land.

Another interesting implication from our analysis is that, when the probability assigned to a crisis state is low enough (that is, high η), then \hat{p} is relatively high and K_X is relatively small. This basically implies that region 1 in our analysis is large and then for a given combination of K_R and B^* , it is more likely that government bonds do not have a convenience yield and a large fraction of households only hold bonds to fully finance their projects. This leads to the following Proposition.

Proposition 8 *The decline in output during a crisis for a given volume of government bonds outstanding declines with the probability assigned to crises.*

Intuitively, when land is good collateral in expectation, bonds do not offer a large convenience yield. Since they are not relatively expensive vis a vis land, then more households acquire bonds to use as collateral. In case a crisis state does happen, then the economy is buffered with those bonds to avoid a large decrease in output. In that sense an optimistic economy in terms of the quality of land is less exposed to crises.

4 Conclusion

Collateral plays an important role in the economy. There is a demand for safe assets. Since the private sector cannot produce riskless collateral, it relies, at least to some extent, on private collateral and then there is the possibility of a crisis. In a crisis, the use of government bonds as collateral, as in the Term Securities lending Facility, breaks Ricardian Equivalence. Even in normal times there is a demand for the safe assets produced by the government because they dominate the private assets, creating the presence of a convenience yield. But, if taxes to repay bonds are distortionary, it may be optimal for the government to issue debt in times of crisis, but not in normal times.

There are limits to how much debt the government can issue aside from distortionary taxes. Since collateral redistributes bonds and wealth across countries, it can turn out

that the generation liable for taxes does not hold the government bonds and has a limited income. This redistribution limits the possibility of using bonds as net wealth. If most bonds given to young households end up in the hands of foreign lenders, then young individuals cannot use those bonds to cover taxes next period, when old. So, the value of government bonds as collateral justifies their use as net wealth to facilitate production, but at the same time put limits on their own efficacy by redistributing such wealth to potentially foreign households.

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