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### **Liquidity as an Investment Style**

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## **ABSTRACT**

We present comprehensive evidence in support of giving liquidity equal standing to size, value/growth, and momentum as investment styles, as defined by Sharpe (1992). First, we show that financial market liquidity, as identified by stock turnover, is an economically significant indicator of long-term returns. Then, we show that liquidity, as a characteristic, is not merely a substitute for size, value, and/or momentum. Finally, we show that liquidity has historically been a relatively stable characteristic of stocks, and that changes in liquidity are associated with changes in valuations.

## 1. Introduction

William F. Sharpe suggested the idea of investment styles as early as 1978 in a general paper about investment (Sharpe 1978). He later refined the idea of style analysis in (Sharpe 1988) and applied it to asset allocation in (Sharpe 1992). The Morningstar Style Box popularized the size vs. value categorizations during that same year. Sharpe (1992) defined four criteria that characterize a benchmark style: 1) “identifiable before the fact,” 2) “not easily beaten,” 3) “a viable alternative,” and 4) “low in cost.”<sup>1</sup>

We propose that equity liquidity is a missing investment style that should be given equal standing as the currently accepted styles of size (Banz 1981), value/growth (Basu 1977; Fama and French 1992, 1993), and momentum<sup>2</sup> (Jegadeesh and Titman 1993, 2001). When assembled into portfolios, these styles define a set of betas which can only be beaten if portfolios provide an extra positive alpha.

The literature focusing on the relationship between liquidity and valuation in the U.S. equity market has grown dramatically<sup>3</sup> since Amihud and Mendelson (1988) used bid-ask spreads to show that less liquid stocks outperform more liquid stocks. Other researchers have confirmed the impact of liquidity on stock returns using various measures of liquidity. Despite this significant and multifaceted body of evidence, a recent survey of the last 25 years of literature on the determinants of expected stock returns found that liquidity is rarely included as a control (Subrahmanyam 2010)<sup>4</sup>.

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<sup>1</sup> We quote Sharpe’s original language for the criteria but re-order them here. In conversations, Sharpe does not claim to have invented the concept of style, since others were using the same terminology during the 1980s.

<sup>2</sup> We do not take a position here as to whether or not momentum is truly a style in the Sharpe framework. However, given that it is often included as a control in studies of the cross-section of returns, we treat momentum as a style in this article, in order to more thoroughly test liquidity as an independent style.

<sup>3</sup> See Amihud, Mendelson, and Pedersen (2005) for a review of the liquidity literature.

<sup>4</sup> According to Subramanyan, “In general, most studies use size, book/market, and momentum as controls, but it is quite rare for liquidity controls to be used.”

In our paper we use stock turnover, which is a well-established measure of liquidity that is negatively correlated with long-term returns in U.S. equity markets. Haugen and Baker (1996) and Datar, Naik and Radcliffe (1998) demonstrated that low turnover stocks on average earn higher future returns than high turnover stocks. We examine stock-level liquidity in a top 3,500 market cap universe of U.S. equities from 1971-2011 and subject it to the four style tests of Sharpe. Our empirical findings, which extend and amplify the existing literature, are that liquidity clearly meets all four criteria.

In the sections that follow, we individually examine each criterion in turn. Section 2 focuses on turnover as a stock-level metric that identifies liquidity “before the fact.” We then look at the long-run performance of liquidity and demonstrate that it is “hard to beat.” Section 3 examines cross-sectional returns for each style and demonstrates with double-sort portfolios that the liquidity style of investing is a distinctly “viable alternative” from the established styles of size, value, and momentum. In section 4, we further show that liquidity is additive to size, value, and momentum by creating a liquidity factor that is compared with the other three style factors. In section 5, we show that liquidity management is “low in cost,” since liquidity migration is not only relatively infrequent, but also is associated with sizable returns when it does occur. Section 6 offers discussion and concluding remarks, summarizing how liquidity meets the Sharpe criteria for an investment style. An appendix describes the datasets and stock universe used in our analysis.

## **2. Long-term Return Comparisons**

There are numerous ways to identify liquidity. Amihud and Mendelson (1986) used bid-ask spreads to explain a cross-section of stock returns. Brennan and Subramanian (1996) regressed price impact of a unit trade size from microstructure trading data. Amihud (2002) developed a metric using the average price impact relative to daily

trading volume of each security. Pástor and Stambaugh (2003) demonstrated that stock returns vary with their sensitivity to marketwide liquidity.

We use stock turnover as our “before the fact” measure of liquidity. It is a characteristic, but it can also be expressed as a covariance factor. Another frequently used liquidity metric is the Amihud (2002) metric which is also readily measured, although Idzorek, Xiong, and Ibbotson (2012) showed that turnover exhibits greater explanatory power for U.S. mutual fund returns. A single “perfect” measure of liquidity is unlikely to exist, since Brown, Crocker and Foerster (2009) found that liquidity measures may encode momentum and information effects in large-cap stocks.

We do not claim that turnover is the “best” way to measure liquidity, but we argue that it is a simple measure which works well. The other styles can also be measured in various ways. Value versus growth can be measured by price/earnings ratios (Basu 1977), by book/market ratios via Fama and French, by dividend/price, or by other fundamental ratios. Momentum can be measured over different horizons and weighting schemes. Even size can be measured over various capitalization ranges and universes. Our goal here is not to compare the various liquidity metrics but rather to show that a simple liquidity measure can match the results of the other styles, so that liquidity deserves to have equal standing to the accepted styles of size, value, and momentum.

Our methodology consists of a two-part algorithm for the selection (prior) year and the performance (current) year. For each selection year (1971—2010), we examine the top 3,500 U.S. stocks by year-end capitalization. From this universe, we record liquidity as measured by the annual share turnover (the sum of the twelve monthly volumes divided by each month’s shares outstanding), value as measured by the trailing earnings/price ratio (with lagged earnings because of reporting delays) as of year-end, and momentum as measured by the annual return during the selection year (i.e., 12-month momentum.) We rank the universe and sort into quartiles for each variable, so that each of the

selection-year portfolios receives a quartile number of the stocks for each of turnover, size, value, and momentum.

In each of the performance years (1972—2011), the portfolios selected are equally weighted at the beginning of each year and passively held. Delistings of any kind (liquidations, mergers) cause the position to be liquidated and held as cash for the remainder of the performance year. Returns at the end of the performance year are recorded for each portfolio selected during the selection year, so that the portfolios are “identifiable before the fact”.

Table 1 reports the long-term annualized geometric mean, arithmetic mean, and standard deviation of returns for each equal-weighted quartile portfolio in liquidity, size, value, and momentum. The annualized geometric mean is the compound annual return realized by the portfolios over the period, which, unlike the arithmetic mean, is not diminished by the variability of the returns. Liquidity appears to differentiate the returns about as well as the other styles.

Figure 1 depicts long-term cumulative returns of the 1<sup>st</sup> quartile portfolio in each style. The 1<sup>st</sup> quartile portfolios on value, liquidity, size, and momentum are all seen to outperform the equally weighted universe portfolio. The low liquidity quartile portfolio clearly outperforms both the small cap portfolio and the high momentum portfolio, producing returns that are indeed “hard to beat.” The strategies presented here are all passive, rebalanced once each year end. Thus we can characterize all these style portfolios as beta portfolios.

From Table 1, there is little evidence that styles are related to risk, at least as measured by standard deviation. For value and momentum, the 1<sup>st</sup> quartile portfolio is less risky than the 4<sup>th</sup> quartile portfolio. Only for size is there a clear risk dimension: the smaller the capitalization, the larger the standard deviation. For liquidity, there is an inverse

relationship between the returns and risk, with the low liquidity portfolio having the highest return, but the lowest risk. We believe that less liquid portfolios have higher returns in equilibrium, not because they are more risky, but rather because they have higher transactions costs.

We can, of course construct risk factors from any style or characteristic, using differences in returns across the quartiles. That is, styles can be presented as either metrics or risk factors. Lou and Sadka (2011) differentiated liquidity levels from liquidity risks. Li, Mooradian, and Zhang (2007) showed that commission costs can also be expressed as a metric or as a risk factor. The fact that we can make risk factors does not mean that there is a payoff for risk. Rather, there is a payoff for a factor that fluctuates, which is associated with the underlying characteristic. Indeed, as we have seen, low liquidity portfolios are not riskier than high liquidity portfolios.

In equilibrium, a style gives a payoff for taking on a characteristic that the market finds to be undesirable. For some factors, like size, it may be related to risk. But investors might not like small size stocks for other reasons as well, *e.g.* investors cannot take on big positions even though they may require extra analysis. Investors may dislike value as well, since the companies may be in a distressed state. Growth stocks are the more exciting and in more demand, because the companies have future potential.

Of all the styles, liquidity has the most obvious connection to valuation. Investors want more liquidity and wish to avoid less liquidity. Less liquidity has a cost, namely that stocks may take longer to trade and/or have higher transactions costs. In other words, if all else is equal, investors will pay more for more liquid stocks, and pay less for less liquid stocks. Fortunately trading costs can be mitigated by those investors who have longer horizons and do less trading. This translates into higher returns for the less liquid stocks, before trading costs. In a later section of this paper we consider whether a less liquid stock portfolio can be managed at low cost.



The idea that investors are willing to pay for liquidity is not the same as saying that less liquidity has more risk. Indeed, Table 1 shows that less liquid portfolios have *lower* standard deviations. Later we will see that less liquid portfolios also have low market betas, and long/short liquidity factors have negative market betas. It is of course possible to imagine less liquid portfolios as risky in a different sense. It may involve tail risk, or the risk of needing to quickly liquidate positions in a crisis. However, during the recent financial liquidity crises, stock liquidity actually increased. Furthermore, more passively held portfolios can largely mitigate this risk.

### **3. Liquidity versus size, value, and momentum**

We seek to show that liquidity is “a viable alternative” to the other well established styles. We focus on distinguishing turnover from size, value, and momentum by constructing double-quartile portfolios that combine liquidity with each of the other styles.

It is often presumed that investing in less liquid stocks is equivalent to investing in small-cap stocks. To determine if liquidity is effectively a proxy for size, we construct equally weighted double-sort portfolios in capitalization and turnover quartiles.

Table 2 reports the annualized geometric mean (compound) return, arithmetic mean return, and standard deviation of returns along with the average number of stocks in each intersection portfolio. Across the micro-cap quartile, the low-liquidity portfolio earned a geometric mean return of 15.36% per year in contrast to the high-liquidity portfolio returning 1.32% per year. Across the large-cap quartile, the low- and high-liquidity portfolios returned 11.53% and 8.37% respectively, producing a liquidity effect of 3.16%. Within the two mid-size portfolios, the liquidity return spread is also significant. Therefore, size does not capture liquidity, *i.e.* the liquidity premium holds

regardless of size group. Conversely, the size effect does not hold across all liquidity quartiles, especially in the highest turnover quartile. However, it is true that the liquidity effect is the strongest among micro-cap stocks and then declines from micro to small to mid to large-cap stocks. The micro-caps row contains both the highest return and the lowest return cells in the matrix.

Similarly, to address the question of how the liquidity style differs from value, we construct equally weighted double-sort portfolios on turnover and the earnings/price (E/P) ratio, with the understanding that E/P is highly correlated with the dividend/price and book/price ratios. In the next section we will construct a liquidity factor and compare it to the Fama-French book/market factor.

The annual return results are reported for the 16 value and liquidity portfolios in Table 3. In this case among the high-growth stocks, the low-liquidity stock portfolio has an annualized geometric mean (compound) annual return of 9.99% while the high-liquidity stock portfolio has a return of 2.24%. For high-value stocks, low-liquidity stocks have a 18.43% return, while high-turnover stocks have a return of 9.98%. Both value and liquidity are distinctly different ways of picking stocks. The best return comes from combining high-value with low-liquidity stocks, while the worst return comes from high-growth stocks with high-turnover stocks.

Finally, we show returns from equally weighted double-sort portfolios on turnover and 12-month momentum quartiles in Table 4. Momentum stocks are ranked by the previous year returns, with the winners placed in the 1<sup>st</sup> quartile and the losers placed in the 4<sup>th</sup> quartile. The highest annualized geometric mean (compound) return, 16.03%, is achieved by high-momentum low-liquidity stocks, while the lowest return, 3.03%, is for the low-momentum high-liquidity stocks. Again, momentum and liquidity are different stock-picking styles and not substitutes for one another.

Since the liquidity style differs from each of the established styles, one might expect to observe a synergistic effect when combining low liquidity with the other styles. This proves to be the case, as illustrated in Figure 2, which shows cumulative long-term returns of selected quartile and quartile-quartile portfolios from Tables 1 through 4. In all three cases, it is clear that liquidity mixes well with the higher performing portfolio, and adds incremental return.

#### **4. Liquidity as a Factor**

To further demonstrate that liquidity is “a viable alternative,” we can also express liquidity as a factor (i.e. a series of dollar-neutral returns) and attempt to decompose it as a linear combination of the other style factors. Most researchers refer to these series as risk factors, although we regard the “risk” label to be somewhat unsatisfactory, because our results show that less liquid stock portfolios appear to be less risky than more liquid portfolios, when measured either by standard deviation or market beta. This decoupling between factors and risk may, to some extent, also apply to some of the other style factors.<sup>5</sup> Nevertheless, it is mechanically possible to recast liquidity into a factor framework, so we do so here in order to further our case for establishing liquidity as a fourth investment style.

We construct monthly returns of a long-short portfolio in which the returns of the most liquid quartile are subtracted from the returns of the least liquid quartile. This series constitutes a dollar-neutral liquidity factor, which we proceed to regress upon the

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<sup>5</sup> An examination of the Fama-French value and momentum decile portfolios from 1972-2011 reveals that the risk profile of both factors is “U”-shaped; middle portfolios exhibit the least risk while extreme portfolios are higher risk. Only size has a clear risk dimension with smaller capitalization stocks being riskier than large capitalization stocks.

(extended) CAPM framework using dollar neutral factors for market, size, value<sup>6</sup>, and momentum obtained from Kenneth R. French's website.<sup>7</sup>

In the CAPM framework, the liquidity long-short (dollar-neutral) factor is regressed upon the excess returns of the market portfolio:

$$(1) \quad R_{it} = \alpha + \beta_{iM}(R_{Mt} - R_{ft}) + \varepsilon_{it}$$

In the standard Fama-French three factor model, the long-short liquidity factor is regressed upon the long market portfolio, and the long-short size and value portfolios:

$$(2) \quad R_{it} = \alpha + \beta_{iM}(R_{Mt} - R_{ft}) + S_i \times SMB + h_i \times HML + \varepsilon_{it}$$

Finally, we regress upon a four factor model which also includes the momentum factor:

$$(3) \quad R_{it} = \alpha + \beta_{iM}(R_{Mt} - R_{ft}) + S_i \times SMB + h_i \times HML + m_i \times WML + \varepsilon_{it}$$

We perform a similar analysis with the long only portfolios by regressing the least liquid quartile portfolio less the risk-free rate from U.S. Treasury Bills, upon the CAPM:

$$(4) \quad R_{it} - R_{ft} = \alpha + \beta_{iM}(R_{Mt} - R_{ft}) + \varepsilon_{it}$$

The Fama-French and four-factor regressions on the long-only portfolios less the risk free rate are done similarly to equations (2) and (3). (We note that it is unnecessary to subtract the risk-free rate from the size, value, and momentum factors, since they contain zero net positions.)

In Table 5 we present the results. In the CAPM variant, the long-short liquidity factors are negatively associated with the market, with a beta of -0.66. The low liquidity long

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<sup>6</sup> The Fama-French value factor is based upon the book to market ratio, instead of the earnings to price ratio that we used in the previous section. We do not take a position as to which method is better for forming value growth portfolios, but here we use the more commonly used Fama-French factors.

<sup>7</sup> French labels the factors for size (SMB) or small minus big, value (HML) or high minus low book to market, and momentum (WML) or winners minus losers.

portfolio has a low beta of 0.75. In both cases the monthly alpha is very positive and significant.

After including the size and value factors into the regression, we see that the liquidity factor is negatively related to size but positively related to value. The liquidity factor is also positively related to momentum in the four factor model. However, after adjusting for the market, size, and value in the Fama-French model or after also adding in momentum in the four factor model, we see that the less liquid alpha is still positive and significant.

Similarly for the low liquidity long portfolio, there is a positive and statistically significant alpha for the CAPM, Fama-French, and four-factor equations. This positive alpha exists, despite adjusting for the market size, value, and momentum.

We interpret the positive and significant monthly alphas for the long-short factor and the long less liquid portfolios as further evidence that less liquid portfolios are “not easily beaten.” An efficient portfolio should not have a significant alpha intercept left over; therefore, the size, value, and momentum styles together are not capable of completely describing the set of betas needed to put together an efficient portfolio.

The links between the liquidity long-short factor and the market, size, value, and momentum factors are also seen in the cross-correlations shown in Table 6. The liquidity factor has the largest negative correlations with the market and size factors, and a substantial positive correlation with value. Value and size are negatively correlated with each other. None of the other factors are as strongly negatively related to the market as is the liquidity factor.

Table 7 shows results from regressing the combined-style long (net of the risk-free rate) portfolios (i.e. the northwest corner portfolios of Tables 2, 3, and 4) upon the CAPM,

Fama-French, and four factor models. These portfolios all correlate with the market, but with low betas. They are again related to the size and value portfolios, but no longer positively related to the momentum factor, except of course for the high momentum portfolio. In all but two borderline cases, the monthly alphas are significant at the 5% level.

We have constructed a liquidity factor that the size, value, and momentum factors did not fully explain, because in almost every regression there was a significant alpha left over. Previous studies have established that the liquidity premium is not captured by the four-factor model, but the results in Table 7 go a step further in showing that the four-factor model does not explain the returns from the three liquidity combined-style portfolios.

Much of the liquidity literature uses stock sensitivity to a liquidity factor instead of measuring the impact of the characteristic itself. We now use the Daniel and Titman (1998) methodology to examine whether the turnover of a stock (characteristic) or the sensitivity to the turnover factor (covariance) has a larger impact on a stock's performance.

Table 8 contrasts characteristic vs. covariant liquidity metrics using double-sort portfolio returns. The characteristic cross-section (table columns) is based on ranked turnover rates from the selection year, just as in Tables 2, 3, and 4. To obtain the covariant cross-section (table rows), we regress the 12-month returns of each stock (less market universe returns), upon a modified turnover factor that uses only selection year returns. We then rank the stocks into liquidity-beta quartiles, as shown in the rows of Table 8. The returns vary strongly and directionally going across columns (characteristic) but vary weakly and non-dependently going down rows (covariance), thus supporting the hypothesis that liquidity characteristics have greater explanatory power for returns.

Since most high (low) turnover stocks will exhibit selection-year return patterns that correlate with those of the high (low) liquidity quartile, the stocks tend to cluster in the diagonal portfolios. However, as Daniel and Titman observed, the off-diagonal portfolios illustrate the relative importance of characteristics vs. covariances. Figure 3 shows longitudinal returns from the two extreme off-diagonal portfolios. Low-turnover stocks that exhibit high-turnover return patterns during the selection year outperform high-turnover stocks that exhibit low-turnover return patterns.

In summary, we find that despite the success of our liquidity factor, the data supports a liquidity characteristic model of stock returns as opposed to a liquidity covariance model. Our results concur with Daniel and Titman, who showed similar results for the value/growth style.

## **5. Liquidity Stability and Migration**

The remaining investment style criterion is that the style be “low in cost”. We will make this case by showing that the liquidity portfolios can be managed relatively passively. Our previous double-sort results already suggest that our portfolios are stable, since the rebalancing frequency is only once per year. We now examine directly the migration of stocks in the liquidity portfolios, which will also help to explain why investing in less liquid stocks pays extra returns.

Table 9a shows how the stocks in each liquidity quartile (in the selection year) migrate to other liquidity quartiles (in the performance year.) For the lowest liquidity quartile, 77.28% remain in the quartile the following year, while 22.72% migrate to higher liquidity quartiles. Overall, 62.93% of the stocks remain in the same liquidity quartile from the selection year to the subsequent performance year.

Tables 9b, 9c, and 9d show the corresponding year-to-year migration of stocks among size, value, and momentum quartiles. The fractions of stocks in these quartile portfolios that remain in the same quartile for the subsequent year are 78.73% for size, 51.63% for value, and 29.03% for momentum. Therefore liquidity is observed to be significantly more stable than 12-month momentum as a basis for portfolio formation, and comparably stable to the well-accepted styles of size and value.

That liquidity is observed to be a relatively stable characteristic of stocks has two implications in the Sharpe style framework. First, it further reinforces that a selecting a liquidity-based portfolio is “identifiable before the fact.” Second, it implies that the transaction costs associated with maintaining a liquidity-based portfolio are “low in costs.” Indeed, Idzorek, Xiong, and Ibbotson (2012) have analyzed U.S. equity mutual fund holdings and confirmed that the liquidity premium remains economically and statistically significant net of trading and all other costs.

Table 10 shows the mean arithmetic returns from our stock universe, by liquidity migration. The evidence is that as less liquid stocks become more liquid, their returns increase dramatically. Conversely, as more liquid stocks become less liquid their returns drop. Since migration is not known *a priori*, separation of the return components listed in each row is also not possible *a priori*. Nevertheless, these results demonstrate that changes in liquidity strongly correlate with changes in valuation. Thus the results suggest that liquidity may be related to a discount factor that could be used in valuation.

## **6. Discussion and Conclusion**



William F. Sharpe provided four criteria to identify an investment style. We summarize why we believe that liquidity as measured by stock turnover meet the criteria.

First, the previous year's turnover of the stock is "identifiable before the fact". Other liquidity measures could have met that criteria as well, but we chose turnover because it was simple, easy to measure, and has a significant impact on returns.

When we compared the 1<sup>st</sup> quartile returns of the various styles, they all outperformed the equally weighted market portfolio. The returns from the low liquidity quartile were comparable to the other styles, beating size and momentum, but trailing value. We consider all four styles to be "not easily beaten".

We examined double sort portfolios comparing liquidity with size, value, and momentum in four-by-four matrices. The impact of liquidity on returns was somewhat stronger than size and momentum, and roughly comparable to value. It was also additive to each style. Thus we determined that liquidity was "a viable alternative" to size, value, and momentum.

We also constructed a liquidity factor by subtracting the 4<sup>th</sup> quartile return series from the 1<sup>st</sup> quartile. This factor added significant alpha to all the Fama-French factors when either expressed as a factor, or as a low liquidity long portfolio. The existence of the significant positive alpha further confirmed that investors need liquidity to be included along with the other styles to form efficient portfolios.

Finally, we demonstrated that less liquid portfolios could be formed "at low cost." Our portfolios were formed only once per year, and 62.93% of the stocks stayed in the same quartile. The high-performing low quartile had 77.28% of the stocks stay in that quartile. Thus the liquidity portfolios themselves exhibit low turnover, which can keep their costs low.

Liquidity has perhaps the most straightforward explanation as to why it deserves to be a style. Investors clearly want more liquidity and are willing to pay for it in all asset classes, including stocks. Less liquidity comes with costs: it takes longer to trade less liquid stocks and the transactions costs tend to be higher. In equilibrium, this cost has to be compensated by less liquid stocks earning higher gross returns. The liquidity style rewards the investor who has longer horizons and is willing to trade less frequently.

Similarly to less liquidity, in equilibrium investors may wish to avoid and demand to be compensated to hold small stocks, value stocks, or high momentum stocks. But in many of these cases the underlying rationale is less clear. Small stocks are more risky, but high value stocks are not necessarily more risky than growth stocks. High momentum stocks appear to be less risky than low momentum stocks. These styles are often presented as risk premiums, but we are more convinced by the idea that the styles embody characteristics (other than or in addition to risk) that the market seeks to avoid.

Using the simple stock-level characteristic of turnover, we have shown that liquidity is “identifiable before the fact.” Through both single- and double-style portfolio returns, we have shown that liquidity is “not easily beaten.” Our regression and covariance results show that liquidity is “a viable alternative.” We also show that liquidity may be managed “low in cost” by employing a low portfolio turnover strategy. In conclusion, we have demonstrated that liquidity meets all four of Sharpe’s (1992) criteria for a benchmark style.

## **APPENDIX: Data and Methodology**

We measure U.S. stock returns over the period 1972 through 2011. Our sample is collected from the University of Chicago Center for Research in Security Prices (CRSP) and Compustat databases and accessed via Wharton Research Data Services (WRDS), and consists of firms listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and NASDAQ stock markets. Portfolios are formed at the end of December of each selection year (1971 – 2010), with the following filters: first, Real Estate Investment Trusts (REITs), warrants, American Depository Receipts (ADRs), Exchange Traded Funds (ETFs), American Trust Components, and closed-end funds are all excluded from the study. Second, a stock must have available information on trading volume and monthly total returns, earnings, number of shares outstanding, and stock price, for all 12 months of the selection year. Finally, the year-end share price must be at least \$2 and the market capitalization must both rank within the largest 3,500 for the year and also exceed \$5 million.

To ensure a sufficient stock universe for our analyses, we choose to focus on the period from January 1972 through December 2011. This period covers the oil crisis of 1973 and the resulting “bear market” in the mid 1970’s. It also includes the “bull” markets of the 1980s and 1990s, as well as the two recessions of the current century.

Table 1A reports summary statistics for the universe, including the number of stocks, along with the largest, average, median and minimum market capitalization for each year. The years listed in Table 1A lag the performance periods by 1 year, since portfolio selection is based on prior (selection) year metrics.

We measure the annual turnover of each stock by summing the 12 monthly turnovers, defined as the trading volume divided by shares outstanding. For the purposes of style comparisons, we measure the capitalization of each stock at year-end. Earnings data are

taken from the CRSP/Compustat merged database. We calculate earnings to price ratios (E/P ratios) for each company as the earnings per share (EPS) divided by the year-end price. Specifically, we use the four most recent quarters (or two most recent semiannual periods) of EPS, with the most recent quarter ending two months prior to the portfolio formation date. This avoids forward-looking bias as it usually takes several weeks for a company to report its recent quarterly earnings after the end of the quarter. Earnings in non-USD currencies are converted to USD before calculating E/P. We measure momentum from the prior year's return. After constructing the portfolios from selection-year metrics, returns are measured in the subsequent performance year.

For NASDAQ stocks, all reported trading volumes are divided by a factor to counter the relative overreporting of volume on that exchange. This factor is our weighted average of the correction factors from Anderson and Dyl (2005) based on a comparison of trading volumes of companies switching from NASDAQ to NYSE. We apply this correction factor for NASDAQ volume data throughout the time period covered by this analysis, since Anderson and Dyl (2007) find no evidence that the relative overreporting of NASDAQ volumes has lessened in 2003-2005 relative to 1990-1996, despite the regulatory and technological changes that took place at NASDAQ in the early 2000s.

We create a liquidity factor by selecting our lowest liquidity quartile returns and subtracting out our highest liquidity quartile returns. We compare our liquidity long-short factor to the factors on Kenneth R. French's website. Those factors include a market return which is the CRSP capitalization weighted average return of NYSE, AMEX, and NASDAQ stocks, a risk free rate which is the Ibbotson Associates' one-month U.S. Treasury Bill rate, and the three Fama-French long-short zero net exposure size, value/growth, and momentum portfolios.

**Table 1A: Summary Statistics of Stock Universe by Year**

This table reports summary statistics for NYSE, NASDAQ, and AMEX stocks that meet our criteria for data selection, including \$5 million minimum market capitalization, \$2 minimum per-share price, no REITs, no ETFs, no warrants, and no ADRs. Market capitalization is as of the end of the selection year.

Selection Year	# of Stocks	Market Capitalization (\$mm)			
		Mean	Median	Max	Min
1971	1,733	385	70	38,696	5.0
1972	1,875	432	70	46,701	5.0
1973	1,761	374	53	35,832	5.0
1974	1,611	289	47	24,979	5.0
1975	1,816	350	54	33,289	5.0
1976	1,770	443	78	41,999	5.0
1977	1,906	387	74	40,333	5.0
1978	1,894	404	82	43,524	5.0
1979	1,894	471	107	37,569	5.0
1980	1,867	610	135	39,626	5.0
1981	1,834	574	132	47,888	5.1
1982	1,848	655	146	57,982	5.2
1983	3,478	472	80	74,508	5.0
1984	3,500	444	75	75,437	5.8
1985	3,500	566	88	95,607	5.9
1986	3,500	632	83	72,711	5.3
1987	3,500	626	72	69,815	5.2
1988	3,500	687	85	72,165	6.6
1989	3,447	850	94	62,582	5.0
1990	3,105	856	95	64,529	5.0
1991	3,398	1,046	121	75,653	5.0
1992	3,500	1,119	146	75,884	12.1
1993	3,500	1,262	204	89,452	26.8
1994	3,500	1,271	230	87,193	44.3
1995	3,500	1,709	305	120,260	62.9
1996	3,500	2,080	383	162,790	77.4
1997	3,500	2,734	478	240,136	101.6
1998	3,500	3,405	427	342,558	81.0
1999	3,500	4,169	451	602,433	76.6
2000	3,500	3,920	401	475,003	48.2
2001	3,500	3,465	435	398,105	55.5
2002	3,500	2,720	323	276,631	35.7
2003	3,500	3,615	516	311,066	64.3
2004	3,500	3,965	614	385,883	66.4
2005	3,500	4,144	623	370,344	66.3
2006	3,500	4,566	669	446,944	76.3
2007	3,500	4,616	552	511,887	45.4
2008	3,228	3,013	375	406,067	5.0
2009	3,418	3,631	449	322,668	5.5
2010	3,386	4,212	564	368,712	5.2
Whole Sample	118,769	2,004	223	602,433	5.0

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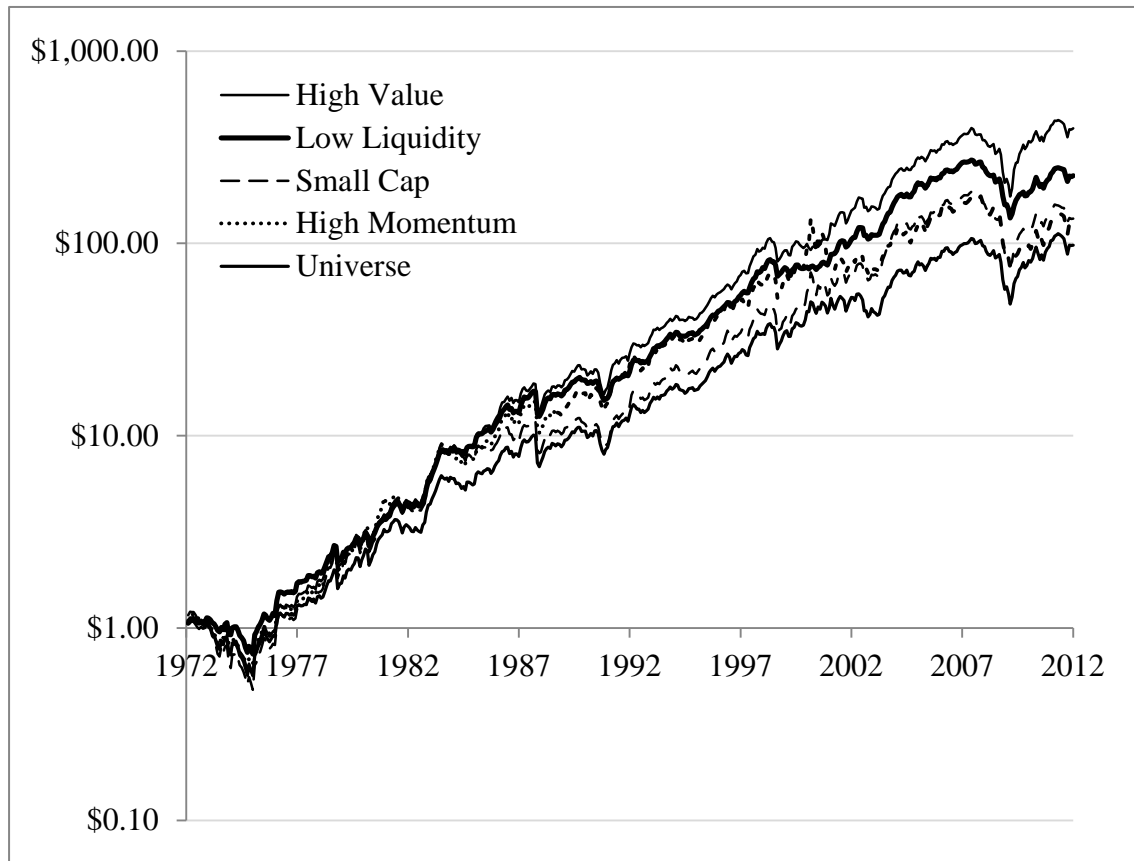
**Table 1: Cross-sectional Style Returns, 1972-2011**

For this table, the top 3,500 market-cap stock universe is independently and separately sorted into four quartiles based on year-end market capitalization (Size), trailing 12-month earnings-to-price ratios (Value), trailing 12-month return (Momentum), and trailing 12-month share turnover (Liquidity), at the end of each December from 1971 to 2010. Each December we equally weight the style quartile portfolios. Reported for each style quartile portfolio are the annualized geometric mean (compound) return, arithmetic mean return, and return standard deviation. Each style quartile portfolio contains an average of 742 stocks per year, or one-fourth of the universe aggregate average of 2,969 stocks per year.

Cross-Section	Result	Q1	Q2	Q3	Q4
<b><u>Size</u></b> Q1=small Q4=large	Geom. Mean	13.04%	11.93%	11.95%	10.98%
	Arithm. Mean	16.42%	14.69%	14.14%	12.61%
	Std. Dev.	27.29%	24.60%	21.82%	18.35%
<b><u>Value</u></b> Q1=value Q4=growth	Geom. Mean	16.13%	13.60%	10.10%	7.62%
	Arithm. Mean	18.59%	15.42%	12.29%	11.56%
	Std. Dev.	23.31%	20.17%	21.46%	29.42%
<b><u>Momentum</u></b> Q1=winners Q4=losers	Geom. Mean	12.85%	14.25%	13.26%	7.18%
	Arithm. Mean	15.37%	16.03%	15.29%	11.16%
	Std. Dev.	23.46%	19.79%	21.21%	29.49%
<b><u>Liquidity</u></b> Q1=low Q4=high	Geom. Mean	14.50%	13.97%	11.91%	7.24%
	Arithm. Mean	16.38%	16.05%	14.39%	11.04%
	Std. Dev.	20.41%	21.50%	23.20%	28.48%
Universe Aggregate	Geom. Mean	12.15%			
	Arithm. Mean	14.46%			
	Std. Dev.	22.39%			



**Figure 1: Comparison of Top-Quartile Style Portfolios 1972 – 2011**



**Table 2: Size and Liquidity Quartile Portfolios 1972-2011**

For this table, the top 3,500 market-cap stock universe is independently and separately sorted into four quartiles according to each stock's market cap and trailing 12-month turnover ratios (liquidity measure), at the end of each December from 1971 to 2010. Each December we equally weight the 16 size and liquidity intersection portfolios. Reported for each intersection portfolio are the annualized geometric mean (compound) return, arithmetic mean return, return standard deviation, and average number of stocks in each cell.

Quartiles		Low Liquidity	Mid-Low	Mid-High	High Liquidity
Micro-Cap	Geom. Mean	15.36%	16.21%	9.94%	1.32%
	Arithm. Mean	17.92%	20.00%	15.40%	6.78%
	Std. Dev.	23.77%	29.41%	35.34%	34.20%
	Avg. No. Stocks	323	185	132	103
Small-Cap	Geom. Mean	15.30%	14.09%	11.80%	5.48%
	Arithm. Mean	17.07%	16.82%	15.38%	9.89%
	Std. Dev.	20.15%	24.63%	28.22%	31.21%
	Avg. No. Stocks	196	193	175	179
Mid-Cap	Geom. Mean	13.61%	13.57%	12.24%	7.85%
	Arithm. Mean	15.01%	15.34%	14.51%	11.66%
	Std. Dev.	17.91%	20.10%	22.41%	28.71%
	Avg. No. Stocks	141	171	197	233
Large-Cap	Geom. Mean	11.53%	11.66%	11.19%	8.37%
	Arithm. Mean	12.83%	12.86%	12.81%	11.58%
	Std. Dev.	16.68%	15.99%	18.34%	25.75%
	Avg. No. Stocks	83	194	238	227

**Table 3: Value/Growth and Liquidity Quartile Portfolios 1972-2011**

For this table, the top 3,500 market-cap stock universe is independently and separately sorted into four quartiles according to each stock's trailing earnings/price ratios (value versus growth measure) and trailing 12-month turnover ratios (liquidity measure), at the end of each December from 1971 to 2010. The reported trailing earnings over the last 12 months are used, measured with a 2 month lag to correct for reporting delays. The lowest earnings/price quartiles are called high growth and mid growth, and the highest earnings to price quartiles are called high value and mid-value. Each December we equally weight the 16 value/growth and liquidity intersection portfolios. Reported for each intersection portfolio are the annualized geometric mean (compound) annual return, arithmetic mean annual return, return standard deviation, and average number of stocks in each cell.

Quartiles		Low Liquidity	Mid-Low	Mid-High	High Liquidity
High-Value	Geom. Mean	18.43%	16.69%	15.97%	9.98%
	Arithm. Mean	20.47%	19.00%	18.72%	13.37%
	Std. Dev.	21.69%	22.88%	24.75%	26.46%
	Avg. No. Stocks	232	182	172	156
Mid-Value	Geom. Mean	14.75%	14.44%	12.67%	11.76%
	Arithm. Mean	16.27%	16.07%	14.78%	14.67%
	Std. Dev.	18.60%	19.38%	21.65%	24.70%
	Avg. No. Stocks	210	204	184	144
Mid-Growth	Geom. Mean	12.53%	12.09%	9.96%	6.58%
	Arithm. Mean	14.27%	13.93%	12.20%	10.40%
	Std. Dev.	19.69%	20.15%	21.37%	28.16%
	Avg. No. Stocks	154	183	197	209
High-Growth	Geom. Mean	9.99%	12.32%	8.39%	2.24%
	Arithm. Mean	13.12%	16.08%	12.41%	7.58%
	Std. Dev.	25.70%	29.00%	29.98%	34.13%
	Avg. No. Stocks	146	173	189	234

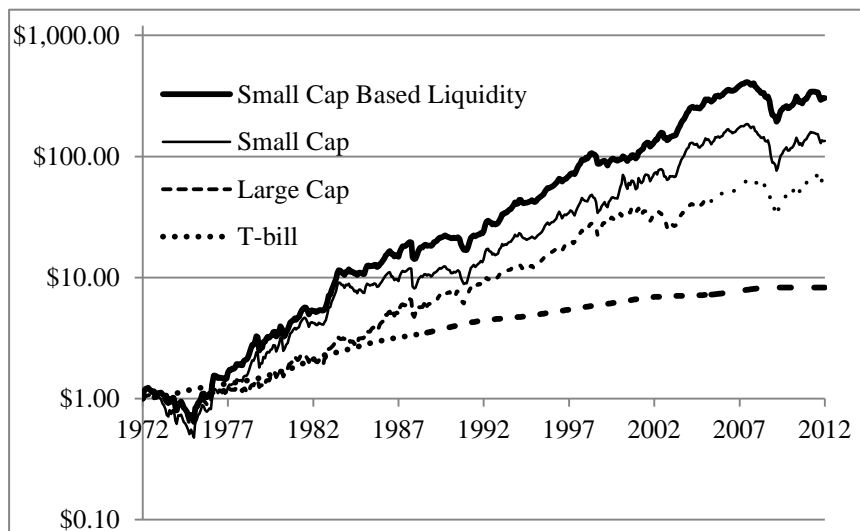
**Table 4: Momentum and Liquidity Quartile Portfolios 1972-2011**

For this table, the top 3,500 market-cap stock universe is independently and separately sorted into four quartiles according to each stock's trailing 12-month return (momentum measure) and trailing 12-month turnover(liquidity measure), at the end of each December from 1971 to 2010. Each December we equally weight the 16 Momentum and liquidity intersection portfolios. Reported for each intersection portfolio are the annualized geometric mean (compound) annual return, arithmetic mean annual return, return standard deviation, and average number of stocks in each cell.

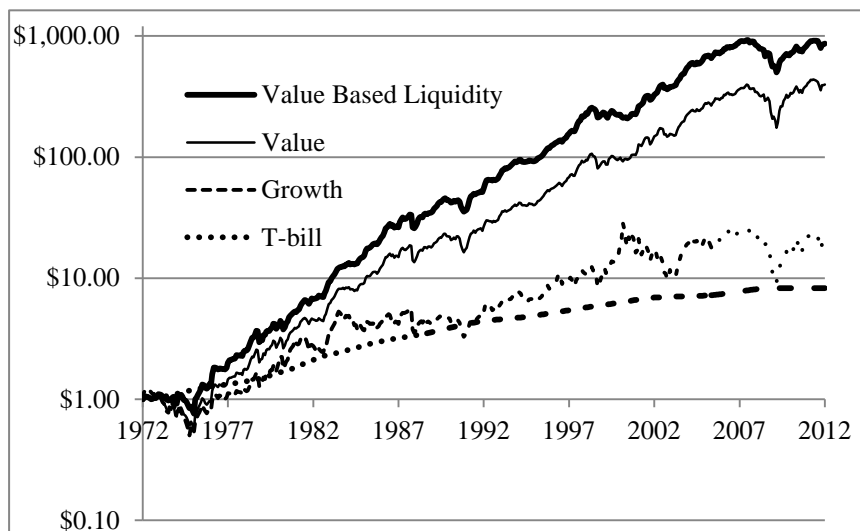
Quartiles		Low Liquidity	Mid-Low	Mid-High	High Liquidity
High-Momentum (winners)	Geom. Mean	16.03%	15.18%	12.97%	8.53%
	Arithm. Mean	18.08%	17.43%	15.42%	12.41%
	Std. Dev.	21.08%	22.69%	23.01%	29.33%
	Avg. No. Stocks	146	165	187	244
Mid-High	Geom. Mean	16.02%	15.31%	13.43%	9.05%
	Arithm. Mean	17.73%	16.99%	15.33%	12.15%
	Std. Dev.	19.53%	19.52%	20.39%	25.56%
	Avg. No. Stocks	215	205	186	137
Mid-Low	Geom. Mean	14.61%	14.65%	12.85%	7.97%
	Arithm. Mean	16.51%	16.50%	15.03%	11.45%
	Std. Dev.	20.84%	20.50%	22.07%	27.08%
	Avg. No. Stocks	225	206	181	131
Low-Momentum (losers)	Geom. Mean	10.30%	9.62%	7.52%	3.03%
	Arithm. Mean	13.24%	13.63%	11.87%	7.76%
	Std. Dev.	25.57%	30.07%	31.40%	32.18%
	Avg. No. Stocks	156	166	189	230

**Figure 2: Cumulative Investment Returns for Intersection Portfolios 1972 – 2011**

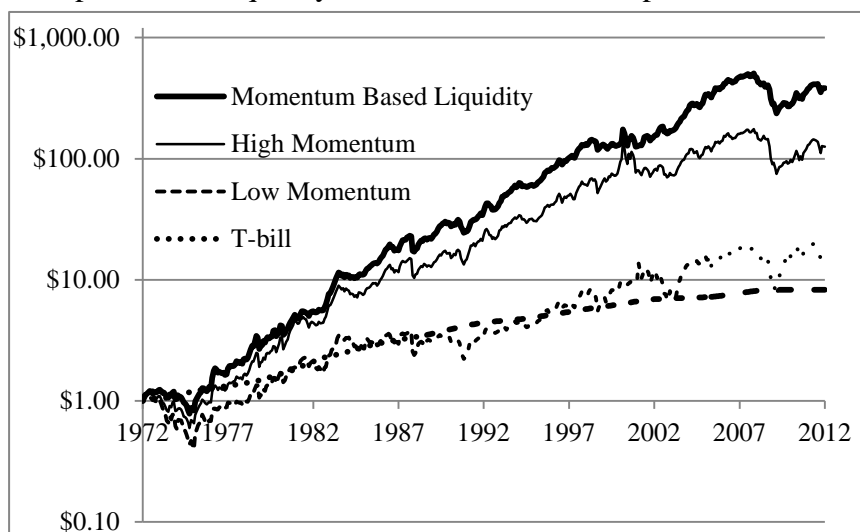
**A. Comparison for liquidity and market cap related portfolios**



**B. Comparison for liquidity and value/growth related portfolios**



**C. Comparison for liquidity and momentum related portfolios**



**Table 5: Regression Analyses of Dollar Neutral Liquidity Factor and Low Liquidity Long Portfolio 1972- 2011**

Our Liquidity Factor (LMH) is the dollar-neutral returns series of the liquidity long-short portfolio (i.e. low liquidity quartile minus high liquidity quartile.) The Low Liquidity Long portfolio consists of stocks in the low liquidity quartile. Market beta is calculated based on market returns minus the risk free rate. Market returns (Mkt-Rf) and the dollar-neutral size (SMB), Value (HML), and Momentum (WML) factors are all downloaded from Kenneth R. French's website. Results from regressing the Liquidity Factor and the Low Liquidity Long (net of risk-free rate) upon the CAPM, Fama-French, and Four-Factor models are shown below.

	Monthly Alpha	Market Beta	Size	Value	Momen- tum	Adj. R <sup>2</sup>	N
<b>Liquidity Factor</b>							
CAPM	0.66	-0.66				48.0%	480
t-stat	(4.52)	(-21.06)					
Fama-French	0.44	-0.47	-0.39	0.54		70.4%	480
t-stat	(3.93)	(-18.55)	(-10.53)	(14.05)			
Four-factor	0.31	-0.45	-0.39	0.58	0.14	72.2%	480
t-stat	(2.80)	(-17.66)	(-10.87)	(15.33)	(5.54)		
<b>Low Liquidity Long</b>							
CAPM	0.45	0.75				67.4%	480
t-stat	(3.97)	(31.47)					
Fama-French	0.16	0.73	0.56	0.44		88.2%	480
t-stat	(2.41)	(47.32)	(24.98)	(18.63)			
Four-factor	0.16	0.74	0.56	0.44	0.00	88.2%	480
t-stat	(2.30)	(46.40)	(24.95)	(18.24)	(0.25)		

**Table 6: Pearson Correlations of Monthly Liquidity Factor Returns with Other Factors 1972 - 2011**

Our Liquidity Factor (LMH) is the dollar-neutral returns series of the liquidity long-short portfolio (i.e. low liquidity quartile minus high liquidity quartile.) The market factor is the monthly market return minus the risk free rate. Market returns (Mkt-Rf) and the dollar-neutral size (SMB), Value (HML), and Momentum (WML) factors are all downloaded from Kenneth R. French's website.

Variable	Liquidity				
	Factor All	Market	Size	Value	Momentum
Liquidity Factor	1	-0.694	-0.503	0.594	0.139
Market	-0.694	1	0.281	-0.316	-0.137
Size	-0.503	0.281	1	-0.233	-0.005
Value	0.594	-0.316	-0.233	1	-0.160
Momentum	0.139	-0.137	-0.005	-0.160	1

**Table 7: Regression Analyses of Enhanced Liquidity Portfolios 1972 – 2011**

Small Cap Based Liquidity is the intersection portfolio of the smallest size quartile and the lowest liquidity quartile. Value Based Liquidity is the intersection portfolio of the value quartile and the lowest liquidity quartile. Momentum Based Liquidity is the intersection portfolio of the smallest size quartile and the lowest liquidity quartile. Market beta is calculated based on market returns minus the risk free rate. Market returns (Mkt-Rf) and the dollar-neutral size (SMB), Value (HML), and Momentum (WML) factors are all downloaded from Kenneth R. French's website. Results from regressing intersection portfolios (net of risk-free rate) upon the CAPM, Fama-French, and Four-Factor models are shown below.

	Monthly Alpha	Market Beta	Size	Value	Momen- tum	Adj. R <sup>2</sup>	N
<b>Small Cap Based Liquidity</b>							
CAPM	0.54	0.75				51.0%	480
t-stat	(3.41)	(22.34)					
Fama-French	0.21	0.70	0.78	0.47		78.2%	480
t-stat	(2.00)	(28.85)	(22.57)	(12.99)			
Four-factor	0.20	0.70	0.78	0.48	0.01	78.2%	480
t-stat	(1.85)	(28.36)	(22.55)	(12.78)	(0.46)		
<b>Value Based Liquidity</b>							
CAPM	0.75	0.71				56.5%	480
t-stat	(5.66)	(24.94)					
Fama-French	0.41	0.72	0.56	0.57		81.3%	480
t-stat	(4.59)	(35.73)	(19.63)	(18.87)			
Four-factor	0.44	0.71	0.56	0.56	-0.04	81.4%	480
t-stat	(4.88)	(34.77)	(19.68)	(18.09)	(-1.81)		
<b>Momentum Based Liquidity</b>							
CAPM	0.55	0.84				61.2%	480
t-stat	(3.85)	(27.53)					
Fama-French	0.36	0.74	0.74	0.21		81.5%	480
t-stat	(3.60)	(32.44)	(22.74)	(6.02)			
Four-factor	0.14	0.79	0.74	0.29	0.24	85.7%	480
t-stat	(1.52)	(38.58)	(25.89)	(9.25)	(11.94)		



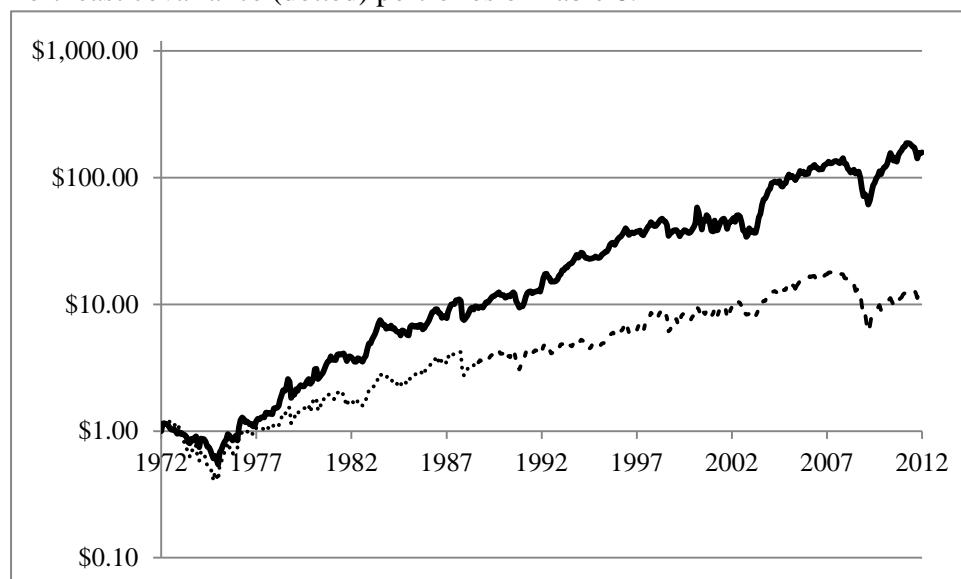
**Table 8: Characteristic vs. Covariance**

The columns show selection-year turnover quartiles, similarly to Tables 2, 3, and 4. We then regress each stock’s selection-year monthly returns (less the universe return) upon a modified selection-year long-short liquidity factor (Low Minus High), then rank and sort the resulting liquidity factor betas ( $\beta_{LMH}$ ) into quartiles, as shown in the table rows. Stocks cluster along the table diagonal because most high (low) liquidity stocks will exhibit return patterns that correlate with those of the high (low) liquidity quartile. The off-diagonal returns (in bold) reveal the relative importance of characteristics vs. covariances.

Quartiles		Low Liquidity	Mid-Low	Mid-High	High Liquidity
High $\beta_{LMH}$ (Correlates w/Low Liq.)	Geom. Mean	13.44%	13.05%	12.21%	<b>6.43%</b>
	Arithm. Mean	15.24%	14.91%	14.12%	<b>9.09%</b>
	Std. Dev.	20.28%	20.63%	20.77%	<b>23.54%</b>
	Avg. No. Stocks	293	204	146	<b>99</b>
Mid-high $\beta_{LMH}$	Geom. Mean	15.18%	13.94%	12.71%	9.95%
	Arithm. Mean	17.03%	15.61%	14.74%	12.62%
	Std. Dev.	20.29%	19.22%	21.13%	24.27%
	Avg. No. Stocks	232	215	184	112
Mid-low $\beta_{LMH}$	Geom. Mean	15.12%	14.65%	12.39%	8.89%
	Arithm. Mean	17.42%	16.95%	14.81%	12.06%
	Std. Dev.	22.07%	22.66%	22.82%	25.72%
	Avg. No. Stocks	147	194	217	185
Low $\beta_{LMH}$ (Correlates w/High Liq.)	Geom. Mean	<b>13.49%</b>	13.40%	9.30%	5.10%
	Arithm. Mean	<b>16.98%</b>	17.58%	13.67%	10.52%
	Std. Dev.	<b>29.32%</b>	31.02%	31.23%	34.33%
	Avg. No. Stocks	<b>70</b>	130	195	347

**Figure 3: Characteristic vs. Covariance**

Historical returns of the characteristic southwest (solid) and northeast covariance (dotted) portfolios of Table 8.



**Table 9: Evolution of Stocks' Style Quartiles, One Year after Portfolio Formation**

During the portfolio selection year (1971—2010), the stocks are sorted into four quartiles of liquidity, size, value, or momentum, as described in the Appendix. In the portfolio performance year (1972—2011) the same metrics are reassessed using the subsequent year's data. The resulting four by four migration matrices are indicative of the degree to which the style metrics remain stable from year to year. In the tables below, all rows sum (within rounding error) to 100%.

a. Liquidity evolution: 62.93% stay in the same quartile

		<b>Weights</b>	<b>Year <math>t+1</math> Liquidity</b>			
			1 Low	2	3	4 High
Year $t$ Liquidity	1 Low	<b>77.28%</b>	18.06%	3.54%	1.11%	
	2	18.80%	<b>53.11%</b>	22.29%	5.80%	
	3	2.96%	24.26%	<b>49.99%</b>	22.79%	
	4 High	0.77%	4.19%	23.70%	<b>71.33%</b>	

b. Size evolution: 78.73% stay in the same quartile

		<b>Weights</b>	<b>Year <math>t+1</math> Market Cap</b>			
			1 Low	2	3	4 High
Year $t$ Market Cap	1 Micro	<b>83.46%</b>	15.65%	0.87%	0.02%	
	2	19.85%	<b>64.75%</b>	15.19%	0.21%	
	3	1.20%	13.89%	<b>74.66%</b>	10.25%	
	4 Large	0.07%	0.22%	7.67%	<b>92.03%</b>	

c. Value evolution: 51.63% stay in the same quartile

		<b>Weights</b>	<b>Year <math>t+1</math> Value</b>			
			1 Low	2	3	4 High
Year $t$ Value	1 Low	<b>65.22%</b>	18.46%	7.55%	8.77%	
	2	21.01%	<b>44.47%</b>	23.85%	10.68%	
	3	9.92%	23.07%	<b>43.41%</b>	23.61%	
	4 High	12.73%	10.75%	23.09%	<b>53.43%</b>	

d. Momentum evolution: 29.03% stay in the same quartile

		<b>Weights</b>	<b>Year <math>t+1</math> Momentum</b>			
			1 Low	2	3	4 High
Year $t$ Momentum	1 Low	<b>37.29%</b>	21.49%	19.63%	21.60%	
	2	23.97%	<b>27.20%</b>	28.01%	20.82%	
	3	22.35%	27.86%	<b>28.23%</b>	21.56%	
	4 High	30.73%	23.50%	22.36%	<b>23.42%</b>	

**Table 10: Returns Associated with Migration in Liquidity Quartiles, 1972-2011**

Arithmetic mean annual returns by liquidity migration (as in Table 9a)

<b>Returns</b>		<b>Year <math>t+1</math> Liquidity</b>			
		1 Low	2	3	4 High
Year $t$ Liquidity	1 Low	<b>9.81%</b>	24.32%	60.98%	109.43%
	2	2.55%	<b>10.87%</b>	23.17%	65.36%
	3	-6.55%	2.70%	<b>12.18%</b>	29.45%
	4 High	-5.89%	-11.19%	1.22%	<b>14.41%</b>