Dynamic Competition, Valuation and Merger Activity

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We model the interactions between product market competition and investment valuation within a dynamic oligopoly. It is, to our knowledge, the first continuous time corporate finance model in a multiple firm setting with heterogeneous products. The model is tractable and amenable to estimation. We use it to relate current industry characteristics with firm value and financial decisions. Unlike most corporate finance models, it produces predictions regarding parameter magnitudes as well their sign. Estimates of the model's parameters indicate strong linkages between model-implied and actual values. The paper uses the estimated parameters to predict rivals' returns near merger announcements.

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Intuition and fundamental microeconomic theory tell us that product market dynamics should have a significant impact on valuation and financial incentives. Yet, directly testable models relating these issues have been largely absent from the corporate finance literature. This paper helps fill in some of these gaps by presenting a tractable framework for examining financial decision-making in a dynamic oligopoly with heterogeneous products. It shows that a firm's competitive position can both profoundly influence its financial decisions and impact how the firm is influenced by the decisions of others. The model's explicit closed form solution allows one to estimate its parameters with ease. This paper takes advantage of that to apply the model to two financial questions: (1) cross sectional valuation and (2) a horizontal merger's impact on rival firms. While other directly testable dynamic models (those that produce quantitative as well as qualitative forecasts) have been relatively rare in the corporate finance literature, notable exceptions include Leland (1994), Leland (1998), Goldstein, Ju and Leland (2001), Hennessy and Whited (2005, 2007), Strebulaev (2007), and Schaefer and Strebulaev (2008), Gorbenko and Strebulaev (2010). However, to our knowledge, this paper is the first continuous time corporate finance model that takes place in a multiple firm setting with heterogeneous products. The oligopoly setting allows us to derive predictions regarding the interaction between a firm's competitive position and how both its own and its rivals' decisions impact its immediate value and future responses.

This paper analyzes a differential game based upon a variant of the Lanchester (1916) "battle" model. In it n firms compete for market share (share of industry sales) by spending funds to acquire each other's customers. The model's continuous time setting allows for closed form solutions that would be very difficult to obtain in discrete time. The model's dynamic structure makes it straightforward to recover empirically unobservable parameters such as consumer loyalty and firm-level spending effectiveness. Identification in the model comes from market share evolution across firms and over time. Using accounting and financial data, one can use the model to generate estimates of these parameters and make predictions regarding the variation in firm values both within and across industries.

Although the model has several appealing features, its mathematical structure, which describes competition for market share, may not apply to all industries. However, it seems unlikely that any one model can properly describe every industry there is. The paper highlights both the model's empirical uses and limits by first presenting estimates of the ease with which firms can acquire market share. The industries, firms and years for which this is accomplished is not exhaustive. For example, given that the model describes an oligopoly, industries with too many firms are excluded before attempting to generate estimates. Nevertheless, they span a very broad array of 332 industries. While the model's structure inhibits it from accurately describing every existing industry, that limitation also opens up a way to see if the estimated parameters reflect actual economic forces or something else. This is done by comparing how the model's forecasts perform across industries where it should fit (mature ones) with those where it should not (high growth ones). We conduct this comparison. Our tests verify that, for high growth industries, the model's empirical estimates are less accurate when it comes to valuing the underlying firms relative to mature ones.

In the model, there are several drivers of firm value, all of which impact a firm's willingness to spend funds in an attempt to attract customers: consumer responsiveness (i.e., the ease with which a firm can steal consumers from rivals), firm level profitability per unit of market share, relative spending effectiveness, the number and capabilities of rival firms, industry growth and the discount rate. As an example of the model's ability to generate quantitative as well as qualitative predictions consider an innovation that increases the attractiveness of a firm's product by 10%. Based on the model's estimated parameters, an investment of this type increases the value of the average firm in the malt beverages industry by 29.7%. In contrast, the same investment in the line-haul railroad industry increases the value of the average firm by only 15.5%. The difference is partially due to how willing consumers are to switch brands in each industry, with it being relatively easier to lure away a competitor's customers in the malt beverage industry. Because of the model's structure, competitive responses of rivals to innovations are explicitly incorporated in these estimates. In principle, these figures, and others like them, can be used to test the model in a valuation context. Under appropriate conditions, one can potentially compare the market's immediate reaction to an innovation's revelation as well as the subsequent profitability and output of each competitor. In this way, the paper is related to the substantial empirical literature documenting intra-industry spillover effects near corporate events, including: initial public offerings (Hsu, Reed and Rocholl (2010)); mergers and acquisitions (Eckbo (1983, 1992), Fee and Thomas (2004), and Shahrur (2005)); dividend announcements (Laux, Starks, and Yoon (1998)); bankruptcies (Lang and Stulz (1992)); corporate security offerings (Szewczyk (1992)); and cash policy (Fresard (2010)). The advantage of the model in this paper is that it produces a testable structure for examining the cross-sectional variation in these spillover effects.

One important financial application of the model is that it can be used as a valuation tool for firms operating in oligopolistic product markets. As a starting point, we test whether model-implied firm values capture actual values for over 11,000 firm-year observations. This is done by taking parameters estimated with the model and then using them as inputs to the value functions derived in the paper. While market shares alone can only explain approximately 20% of the variation in firm values, the model explains over 43%. The fit of the model is driven by estimates of unobservable parameters such as industry-level consumer responsiveness, the company's profitability and ability to attract customers.

The model is also capable of generating forecasts of each firm's eventual market share and how long it will take to reach it. We use the model to project 3- and 5- year ahead changes in market shares and find correlations between actual and predicted market share changes of more than 0.08 and 0.15, respectively. These are highly statistically significant. That the magnitudes of these correlations are substantially less than one suggests some of the limitations of the empirical implementation. However, as a benchmark, it is worth comparing the explanatory power of the model's predicted market share changes to other variables that have been used in the empirical corporate finance literature to describe the behavior of oligopolies (e.g., Eckbo (1983) and Shahrur (2005)). The three candidate variables that we examine are industry concentration (*HHI*), change in *HHI* and the (log) number of firms in the industry. While these variables do offer additional explanatory power when added to the predictive market share regressions, the model-implied market share changes remain statistically significant. Moreover, when we run a "horserace" among these variables using stepwise model selection based on the Schwarz Bayesian Information Criterion, we find that the model implied changes in market share ranks highest of the four candidate variables. When a version of the model with stochastic market shares is applied to the empirical implementation we find that the predictive power of the model is very robust. The market share prediction exercise in this paper is, to our knowledge, novel and may enhance current approaches to valuation.

The model's flexibility allows it to be applied in many corporate finance settings.[1] We present one example that revolves around a horizontal merger (M&A). In this setting, conflicting forces vie to determine the ultimate impact on rival firms. Rivals benefit from the reduced number of competitors. But they are hurt if the combined firm is a much stronger competitor than were the stand alone firms. We take advantage of the structural model to disentangle these two effects. The model also shows how mergers between one pair of rivals can trigger profitable mergers among other pairs. This may prove useful in future research on how merger waves start.

Estimates based on the M&A model indicate that it does help to explain the cross sectional pattern of rival returns in response to a horizontal merger. Regressing actual merger announcement period returns against the model's (out of sample sample) forecast yields parameter estimates showing that a 1% change in the model's return is associated with about a 1% change in actual returns. The R^2 statistics are also quite reasonable for an exercise of this type, coming in at about 9%. Furthermore, this is accomplished by the model with the help of only two data series: revenues and cost of goods sold. By comparison, the purely empirical 11 variable model of customer and supplier returns in Fee and Thomas (2004) generates an R^2 of 1.4% while Shahrur's (2005) model of rival returns, with 10 explanatory variables, generates one of 9%. These analyses fit observed returns using a variety of explanatory variables that are potentially correlated with returns. Here the exercise is forward-looking. Our forecasts are based on the estimated model parameters using only data that was available prior to the forecast date.

The model's structure also allows one to decompose the effect of a merger on industry rivals in ways that are impossible with a static model as guidance. In particular, it can be used to estimate the gain from a reduction in the number of competitors versus the loss from facing a potentially stronger rival. Based on the empirical estimates, if within industry mergers did nothing but soften competition through the reduction in the number of firms then the median rival's value in our data would increase by about 2.52%. Similarly, if the only effect was to generate a stronger competitor, the rivals would lose about 0.30% in value. This makes intuitive sense. Prior studies show that mergers create considerable value for the combining companies. Betton, Eckbo and Thorburn (2008) report abnormal returns to the combined firm of more than 2%. Yet, studies going back to Eckbo (1983, 1992) also show that rival returns are small. The model reconciles this by providing estimates of the two competing forces. It also shows that nonlinearities may matter. On average, the model forecasts a rival return of 0.36%. (In actuality, firms in our database earn a mean return of about 0.61% and a median return of 0.47%.) This comes from a 2.22% gain due to the reduction of the number of firms in the industry, which is partially wiped out by 1.86% reduction in rival firm value, caused by the reduction in competitors along with a new stronger firm. This offsetting effect occurs because the reduction in the number of competitors along with the creation of a stronger rival creates an interaction effect that works to the newly created firm's relative advantage.

The paper is organized as follows. Section I presents the basic model, including the solution to the infinite horizon case. Section II presents results from estimating key parameters in the model. Section III presents the M&A application. Section V concludes. Finally, the Appendix contains details regarding the derivation of the model's equilibrium.

I. Basic Model

A. Players, Timing, Dynamics and Strategies

The Lanchester (1916) battle model was originally designed to study military strategy. Since then variants have been widely used in the marketing literature to examine advertising strategies (see e.g., Erickson (1992); Erickson (1997); Fruchter and Kalish (1997); Bass et al. (2005) and Wang and Wu (2007); for a review, see Dockner, Jørgensen, Van Long, and Sorger (2000)), although, to our knowledge, not in the form presented here. This paper's adaptation creates a differential game where competition among oligopolists selling heterogeneous goods can be explored.

Consider *n* risk neutral value maximizing firms battling for market share. Let $u_i(t) \geq 0$ represent the dollars spent by firm *i* on gaining market share at instant t. Let s_i denote the effectiveness of spending. Note that spending to acquire a competitor's customers (u_i) can imply a wide range of activities including advertising, new product design, opening new stores and R&D. The s_i parameters can represent the relative attractiveness of each firm's product and/or the relative quality of their marketing campaigns.

The market share of firm *i* at time *t* is denoted $m_i(t)$. Time is continuous and there is a finite starting point at $t = 0$. Given the initial condition $m_i(0)$, m_i evolves as follows:

$$dm_i = \frac{\phi\left[(1-m_i)u_i s_i - m_i \sum_{j \neq i} u_j s_j\right]}{\sum_{j=1}^{n} u_j s_j} dt \qquad (1)$$

where $\phi \geq 0$ represents the speed with which consumers react to each firm's entreaties and can be interpreted as consumer disloyalty. (High values imply consumers are easily lured away from one firm to another.) Intuitively, (1) says that the variation in Firm *i*'s market share is simply the difference between what it gains from the market share held by its competitors and what it loses to them. For now, (1) is deterministic. Later, the analysis will generalize it to include a stochastic term.

Equation (1) is the driving force behind the model. According to Equation (1), the market share of firm *i* increases with its own spending and effectiveness (u_i and s_i, respectively) and decreases with the spending and effectiveness of its competitors. Note that a high current $m_i(t)$ gives Firm *i* "more to lose" to its rivals and as a result makes it easier for competitors to gain market share. Thus, there are diminishing returns to being large.

Since this paper seeks to examine economic outcomes within industries that are natural oligopolies an assumption about consumer behavior is needed. If the industry is characterized by positive network externalities then it is a natural monopoly. In this case, once a firm's market share reaches a tipping point, it eventually acquires all of the market. In a natural oligopoly such as the one described in this paper, that is not the case. Instead, it must be that every firm has some consumers that find its offerings exceptionally attractive even if most people use a rival's products. For example, McDonalds is the largest fast food restaurant chain in the U.S. Nevertheless, many consumers only eat at Burger King. Equation (1)'s structure captures this general property: firms produce heterogeneous products and consumers have heterogeneous preferences. This formulation also implies there are diseconomies of scale in spending to attract customers. Differentiating (1) shows that it is monotonically decreasing in u_i. Essentially, the first dollar a firm spends on its customer acquisition program does more to attract buyers than does the second, and so on. This is natural since customers that have a strong preference for a firm's product line should be easy to bring in. As one moves further away in preference space, the firm is then forced to spend even more to acquire new customers. For example, Burger King's loyal customers will probably continue to eat there, no matter how much McDonald's spends to attract them. On the other hand, the converse is true too – there are fans of McDonalds that Burger King cannot attract.

Related to the issue of ensuring that the model describes an oligopoly is the assumption that spending effectiveness and actual dollars spent are multiplicative. That is, the relative value of a dollar spent by any two firms is constant. Other formulations like a power relationship, for example, $u_i^{s_i}$ will alter that. In this case, the relative value of each dollar spent would either increase (s_i>1) or decrease (s_i<0) with a firm's own spending. In equilibrium, we suspect that with s_i<1 the results would be qualitatively similar to what the current setting yields. But, of course, tractability would suffer. For s_i>1, however, as spending increases the firm becomes even more effective in attracting consumers. In the end, this produces an industry with what amount to network externalities and thus a natural monopoly.

The last element in Equation (1) is ϕ. This is a consumer "stickiness" parameter. High values imply that customers are easy to move in a short period of time from one firm's product line to another's. Low values imply the opposite. Thus, one imagines that ϕ has a high value in the fast food industry since people purchase meals several times a day and purchases do not have to be made repeatedly from the same firm. Conversely, it is likely that ϕ is low in industries that sell heavy equipment like backhoes. These are durable goods that are only replaced every few years. Furthermore, once a firm has committed itself to a product line, it may be costly to switch vendors if the products interact with each other.[2]

Before specifying the profit function, three additional observations from the formulation of *dm* (Equation (1)) are worth noting. First, the discussion in the paper assumes $u_i \geq 0$ in equilibrium. However, the equations are solved unconstrained and in principle there exist exogenous parameter sets such that one would need to solve a constrained problem instead. Since this paper seeks to focus on mature stable industries, where exit is of secondary importance, it is useful to restrict attention to cases where the unconstrained equilibrium values of *u* are always strictly positive. Later on sufficient exogenous parameter conditions needed to do this are laid out.

A second point regarding *dm* is that it is discontinuous whenever a firm "gives up" and sets its u_i to zero. This is a result of the model's assumption that it is relative spending that matters and ensures that the model is unit free. Beyond that, the *dm* equation's behavior when a firm sets $u_i = 0$ also generates one particularly useful statistic, which the paper calls the "industry half-life." That is, if a competitor sets u_i equal to zero, one can estimate the length of time it takes that firm to lose half of its customers when the other firms continue to compete for them.[3] Third, the law of motion shown in Equation (1) differs from the marketing literature, which typically examines a duopoly model with either: (1) $dm/dt = u_1(1-m) - u_2 m$ or (2) $dm/dt = u_1\sqrt{1-m} - u_2\sqrt{m}$ (Dockner et al. (2000)). One advantage of using Equation (1) instead is that it is unit free. This eliminates the problem that changing the unit of currency also changes the rate at which *m* changes over time. Another important advantage to this formulation is that relative (rather than absolute) measures of spending are likely to be most relevant for within-industry dynamics.

Returning to the model, instantaneous profits are assumed to be proportional to market share and include a fixed operating cost. Let α_i denote the revenue generating ability of firm *i* per unit of market share. Profits π equal revenues minus both spending on market share competition and a fixed operating cost f_i:

$$\pi_i(t) = e^{gt} \left(\alpha_i m_i(t) - u_i(t) - f_i \right) \tag{2}$$

The term g represents the industry's rate of growth. It is assumed that as the industry grows larger, profits and costs grow proportionately.[4] Note that spending by each firm does not impact the industry growth rate. Thus, the model should be thought of as applying to an industry in which innovations tend to change customer loyalties rather than increase overall demand. For example, an easier to swallow aspirin will probably cause consumers to switch brands but seems unlikely to lead to an overall increase in pill consumption. One can modify the model to allow g to depend on the u_i but at the cost of a closed form solution. We therefore leave g as an exogenous parameter; making the model better suited for an analysis of lower growth industries, as in the aspirin example. Still the model is quite flexible in its ability to describe differences across industries. If one thinks that it is easier to acquire market share in faster growing industries this can be accommodated by simply setting ϕ to a larger value if g is larger. In terms of the mathematics it does not matter if market share growth comes from taking in newly entering consumers or stealing existing ones from rivals.

The profit function in Equation (2) is similar to that used in many applications; for example the standard Cournot oligopoly model. Because profits are linear in market share (sales), the firm's production function exhibits constant variable costs. At the same time, the fixed operating cost (f_i) implies that there are economies of scale. (If f_i equals zero then total production costs are simply proportional to sales. There is nothing in the model's analysis that requires a strictly positive value of f_i.) For many, although not all, industries these seem like reasonable assumptions. For example, a fast food chain purchases raw materials (beef, potatoes, cleaning supplies, ovens, etc.) in a competitive environment. In cases like this, variable costs should be approximately proportional to sales.

To help streamline the exposition, details regarding the derivation the model's equilibrium conditions can be found in the Appendix. There a general version is solved. The main text then employs that general solution to discuss various special cases. Thus, in the main body of the paper, equilibrium conditions are simply stated without proof except for occasional references back to the Appendix.

B. The Equilibrium Value Functions

Let r denote the instantaneous discount rate. Assume that the discount rate exceeds the industry rate of growth ($r>g$). Define $\delta = r-g$. Firms choose u_i to maximize expected discounted profits:

$$\int_0^{\infty} \left(\alpha_i m_i(\tau) - u_i - f_i\right) e^{-\delta\tau} d\tau \tag{3}$$

Assume the parameters are such that no firm ever exits. Following standard practice in the literature on differential games, the analysis seeks a Nash equilibrium in which the players use Markovian strategies (see Dockner, et al. (2000)). The Appendix solves for the pure strategy equilibrium of this game and shows that each firm's value function V_i at time t (i.e., the present discounted value of each firm's profit stream conditional on the equilibrium strategies) can be written as:[5]

$$V_i\,(m,t) = a_i + b_i m_i \tag{4}$$

within the scenarios considered in this paper.[6] As shown in the Appendix

$$a_i = \frac{\phi\alpha_i\left[\alpha_i s_i z - (n-1)\right]^2}{\delta(\phi+\delta)(\alpha_i s_i z)^2} - \frac{f_i}{\delta}, \tag{5}$$

and

$$b_i = \frac{\alpha_i}{\phi+\delta}. \tag{6}$$

In Equation (5) above, z equals:

$$z = \sum_{j=1}^{n} \frac{1}{\alpha_j s_j} \tag{7}$$

and can be thought of as a measure of the competitive strength of firms within an industry. Later on it will also be useful to define its mean as $\overline{z} = z/n$. Intuitively, a firm is a strong competitor if it can both profit from gaining market share (α_j) and economically attract customers (s_j). Since the units of measure are arbitrary (dollars or euros for α and some measure of effective marketing s) what matters is z, the ratio of one firm's competitive strength relative to that of each rival. As a result, the term z appears repeatedly throughout the model's solution.

C. Equilibrium

C.1. Spending on Customer Acquisition and Retention

In the Appendix it is shown that the V_i' has as its solution $\alpha_i/(\phi+\delta)$. Then using the definition of z and some algebra one can show that

$$u_i = \frac{\alpha_i \phi (n-1)[\alpha_i s_i z - (n-1)]}{(\phi+\delta)(\alpha_i s_i z)^2} \tag{8}$$

which characterizes the equilibrium strategies being sought. Observe that if there are no fixed costs (f_i=0) equilibrium spending remains unchanged. This is because V_i' is a function of only α_i, ϕ, and δ.

Since the focus of this paper is on an ongoing oligopoly, we need to assume the exogenous parameter values are such that no firm wishes to exit the industry. This naturally requires setting each firm's fixed costs low enough that it is worth more if it operates than if it closes down. A sufficient condition to guarantee this is to select f_i small enough that (5) is strictly positive. However, that will only hold in the steady state if firms actually compete for market share and sufficiently weak firms will not. So long as a firm's spending on market share is strictly positive, the law of motion (1) guarantees a strictly positive market share along all possible paths. However, if spending (u_i) is negative this need not be the case. Thus, in keeping with this paper's focus, assume every firm is strong enough that $\alpha_i s_i z_i-(n-1)>0$.[7]

From Equation (8), one can examine how competitive forces impact equilibrium spending. It is straightforward to show that spending is strictly increasing in consumer responsiveness, ϕ. When there is a greater incentive to spend money to attract customers, firms do so. Spending is strictly decreasing in the discount rate net of industry growth (δ) since it lowers the present value of the revenue a new customer brings in. As one might expect, firms that earn a greater profit per sale (higher α_i) spend more on customer acquisition because they are worth more to acquire. The derivative $\partial u_i / \partial \alpha_i$ is positive as long as $\alpha_i s_i z > 2 - 2/n$. As noted above, if every firm in the industry is competing for customers then $\alpha_i s_i z_i > n-1$ and for $n \geq 2$ this implies that $\alpha_i s_i z_i > 2-2/n$ as well. However, the impact of spending effectiveness (s_i) on equilibrium spending depends on a firm's competitive strength. The derivative $\partial u_i / \partial s_i$ is positive as long as $\alpha_i s_i z > 2(n-1)$. Thus, if the firm is strong enough, higher values of s_i will lead to higher value of u_i, otherwise u_i goes down.[8] There are two forces at work here. One is the standard tradeoff. Firm's with higher values of s_i can spend less and still attract customers. Weak firms find that the best option is to "split the difference" when s_i increases by reducing u_i. For strong firms, the gain in market share from spending yet more on customer acquisition is just too strong to pass up for the offsetting gains a reduction in u_i would bring. But there is a second factor at play that only becomes apparent in a multiple firm model. When firms are relatively weak, increases in spending are met with more aggressive spending by rivals, which decreases the incentives to spend more. This can be seen by an examination of $\partial u_i / \partial(\alpha_{j \neq i} s_{j \neq i})$, which is also positive as long as $\alpha_i s_i z > 2(n-1)$. Comparative statics using Equation (8) are summarized in Table I.

C.1.a. Entry and Exit: Impact on Customer Acquisition and Value

With the model's solution and restrictions on the exogenous parameter values in place, one can now analyze the equilibrium responses to changes in $\overline{z}$; the competitive environment. Since the paper's empirical section examines the impact of a merger between rival firms it is useful to begin by seeing how a change in the number of competitors (n) alters equilibrium spending across firms. In a standard Cournot model, adding competitors decreases the equilibrium quantity produced by each firm. Firms "accommodate" the new entrant. Is the equivalent true here? Does adding a firm to the industry cause its competitors to reduce their spending on customer acquisition, with the incumbents all settling for smaller market shares? Because the model allows for heterogeneous firms this question cannot be answered until one first specifies what type of competitor is being added. A natural choice is to assume the new firm is "average" in that it leaves $\overline{z}$ unchanged. Assuming that is the case then differentiating (8) leads to the following Proposition.

PROPOSITION 1: *Increasing n while holding $\overline{z}$ constant leads to the following spending change by firm i.*

$$\frac{du_i}{dn} = \frac{\phi\left[n\alpha_i s_i \overline{z} + 2(1-n)\right]}{n^3(\phi+\delta)\alpha_i(s_i\overline{z})^2}. \tag{9}$$

This change in spending is strictly negative for average and below average firms (where $\alpha_i s_i \overline{z} \le 1$) whenever n is greater than 2. For these weak firms, increasing spending to attract customers in response to an increase in the number of competitors is relatively futile. Stronger firms have larger values of α_i and s_i and thus react differently from weaker firms. It is easy to show that firms which obtain more value per unit of market share (large α_i) will spend more relative to their less profitable rivals in response to entry (i.e. $\partial^2 u_i / \partial n \partial \alpha_i > 0$). For firms that are particularly good at customer acquisition (high s_i) the cross derivative is ambiguous. But for an average or below average competitor in the industry (i.e. $\alpha_i s_i \overline{z} \le 1$) one can show it is strictly positive. Thus, if the model captures the competitive features of an industry, then looking across companies from weaker to stronger the response to an increase in n should be increasing in the data.

Compare the result in Proposition 1 to its analog within a standard homogenous product Cournot model. In a Cournot model, entry induces every firm to accommodate the new firm by cutting back on production. Whether a firm is strong or weak it scales back on the control variable. In this paper, that is generally false for very strong firms. Empirically, this dichotomy may look like "predatory behavior" on the part of an industry's leaders as these are the firms most likely to have large values of α_i, s_i, and thus $\alpha_i s_i \overline{z}$.

Based on the above results, intuitively one might now expect to find that very strong firms actually gain market share if a new firm of average competitive ability enters the market. However, it turns out that is not true. To begin the analysis, the next proposition derives the steady state market share of each firm ($\bar{m}_i$) in terms of the model parameters.

PROPOSITION 2: *The steady state market share of each firm equals:*

$$\bar{m}_i = 1 - \frac{n-1}{\alpha_i s_i n \bar{z}}. \tag{10}$$

Proof: Steady state occurs when m_i is such that dm_i/dt=0, or

$$m_i = \frac{u_i s_i}{\sum_{j=1}^{n} u_i s_i}. \tag{11}$$

To generate (10) substitute (8) into (11). For the denominator of (11) this produces $\sum_{j=1}^{n} u_j s_j = 1/z$,

after using $z = \sum_{j=1}^{n} 1/\alpha_i s_i$. Some simple algebra then yields (10). Q.E.D.

Two somewhat obvious empirical implications arise immediately from Proposition 2. The first is that in the steady state stronger competitors (those with higher values of $\alpha_i s_i$) obtain a larger share of the market. Second, adding a new competitor of average competitive strength (thus leaving $\bar{z}$ unchanged) reduces the market share of every firm.

A more interesting set of empirical predictions arises from a closer examination of the model's cross sectional attributes. Proposition 1 showed that very strong competitors increase their spending on market share acquisition in response to entry. Proposition 2 however demonstrates that in the end they still lose some customers. But the additional spending is not in vein. The increased spending by stronger firms causes them to lose fewer customers than their weaker rivals to the new entrant. Some minor algebra shows that the cross derivative of a firm's steady state market share to the number of competitors and its own competitive ability ($\partial^2 \bar{m}_i / \partial n \partial \left(\alpha_i s_i \right)$) is strictly positive. Thus, one has the empirical hypothesis that following the entry of a new firm into an industry, the weaker firms will lose a greater fraction of the market than the stronger firms. This happens even though the weaker firms begin with smaller fractions of the market to begin with. Given their relative inability to compete effectively, their best response is to essentially cede market share and not fight to retain it.

C.1.b. **Consumer Responsiveness and Corporate Values**

Another variable impacting long run industry values is the degree to which consumers respond to corporate entreaties (ϕ). By plugging $\bar{m}_i$ into V_i yields firm *i*'s steady state value:

$$V_i(\bar{m}_i) = \frac{\alpha_i[\alpha_i s_i z - (n-1)][(\phi+\delta)(\alpha_i s_i z) - \phi(n-1)]}{\delta(\phi+\delta)(\alpha_i s_i z)^2} - \frac{f_i}{\delta}. \qquad (12)$$

Differentiating (12) with respect to the consumer responsiveness parameter shows that, in the steady state, firms are worth less if they are in an industry where consumers are easily drawn away. The reason for this can be found in the equilibrium values of u_i and the fact that a firm's steady state market share ($\bar{m}_i$) does not depend on ϕ. An examination of equilibrium spending to attract customers (Equation (8)) shows that firms spend less if consumers become less responsive. Thus, every firm in an industry benefits from ϕ's reduction because they earn the same steady state revenue stream while wasting fewer resources trying to lure away each other's customers.

The effect of consumer responsiveness on corporate policy as outlined above is easily seen in real industries. For example, if beer drinkers exhibited greater loyalty to particular brands brewers would undoubtedly advertise less, and collectively earn higher profits. From 1981 to 2008 per capita beer consumption in the U.S. fell from 24.6 to 21.7 gallons despite heavy product advertising (USDA, 2010). But, no one brewer can reduce its own spending without losing customers to competitors. Thus, in equilibrium, they end up advertising just to retain their current market shares even amid stagnant sales. Compare this to the situation in, for example, natural gas distribution where consumers are locked into a single supplier and thus these firms do relatively little advertising.

Additional economic intuition can be gained by looking at what can be called the industry "half-life." This represents the time it would take a firm to lose half its customers if it stopped working to keep them by setting u_i to zero. This value can be calculated from (1) and turns out to equal $h = \ln(2)/\phi$.

While the half-life (h) is the time that it would take a firm to lose half of its market share if it stopped spending to attract customers, it can also be used to analyze the growth of new firms. If a firm enters an industry its rivals will not passively let it grow. This slows the entrant's growth making it impossible to capture half the market in the interval h described above. How long would it take? The model can be used to provide a quantitative answer. Plugging the equilibrium values of u_i into (1) yields

$$\frac{dm_i}{dt} = \frac{\phi n\alpha_i s_i \overline{z} - (n-1)}{n\alpha_i s_i \overline{z}} - \phi m_i \tag{13}$$

which has a solution for $m_i(t)$ of

$$m_i\left(t\right) = \frac{n\alpha_i s_i \overline{z} - \left(n-1\right)}{n\alpha_i s_i \overline{z}}\left(1 - e^{-\phi t}\right) \tag{14}$$

Thus the new entrant can be expected to capture a quarter of its steady state market share after t=h years. In this example, the incumbent firms' reaction to the entrant cuts the entrant's rate of growth in half.

Growing, newly public firms can be difficult to value, in part because of challenges associated with forecasting their future cash flows. Suppose the entrant in the above example goes public upon reaching a quarter of its steady state market share. Again solving the ODE one finds that the entrant's growth decelerates from its pre-IPO levels. While the firm gained a quarter of its long run market share in the first h years of its life it will now take the same amount of time to go from a quarter to three-eighths. What this means is that if one can estimate the value of h associated with a particular industry and each firm's s and α the model can be used to make predictions about each firm's long run market share, profitability, and spending on customer acquisition. In addition, it offers predictions about the time it takes new entrants to reach particular market share levels. These estimates should also help predict cash flows and improve valuation in IPO studies.

C.2. Stochastic Market Shares

Prior to applying the model to real world data, it is useful to generalize the law of motion governing the change in market shares (Equation (1)) to include a stochastic component. This allows one to expand the interpretation of the error term in empirical work from that of measurement error alone to one that also includes randomness in the underlying economy.

Since market shares always add to one, the structure of any error term must not pull them off of the unit simplex. At the same time, intuition suggests that there should exist a general symmetry in the error structure. For example, a natural restriction is that rearranging the order in which the firms are numbered should have no economic impact. One way to do that is by starting with the idea that competition in an industry is in some sense always bilateral. If a customer of firm *i* randomly walks into firm *j*'s store then *i* has lost that customer to *j*. Thus, one can think of a random process governing the change in market shares between two different firms *i* and *j* as $\iota_{ij}\sigma\sqrt{m_i m_j}\,dw_{ij}$, where dw_{ij} is a standard Weiner process, and ι_{ij} is an indicator variable that equals +1 if $i<j$ and -1 if $i>j$. The indicator variable guarantees that a customer gained by one firm is also a customer lost by another, thus insuring that the market shares will always add to one.

Each firm in an industry competes with *n*-1 others. It therefore faces $n-1$ stochastic processes relating where its customers may arrive from or depart to. Combined with the discussion above, the original law of motion describing firm *i*'s market share formulated in Equation (1) becomes:

$$dm_i = \frac{\phi\left[(1-m_i)u_i s_i - m_i \sum_{j\neq i} u_j s_j\right]}{\sum_{j=1}^{n} u_j s_j} dt + \sigma\sqrt{m_i}\sum_{j\neq i}\iota_{ij}\sqrt{m_j}\,dw_{ij}. \qquad (15)$$

Using (15), the instantaneous variance for each firm's market share is $\sigma^2 m_i(1-m_i)$ and its covariance with firm *j* is $-\sigma^2 m_i m_j$. Overall then, the variance-covariance matrix governing the change in market share can be written as:

$$\sigma^2\begin{bmatrix} m_1(1-m_1) & -m_1 m_2 & \cdots & -m_1 m_n \\ -m_1 m_2 & m_2(1-m_2) & \cdots & -m_2 m_n \\ \vdots & \vdots & \ddots & \vdots \\ -m_1 m_n & -m_2 m_n & \cdots & m_n(1-m_n) \end{bmatrix}. \qquad (16)$$

While adding a stochastic term to Equation (1) induces uncertainty in the value of each firm going forward, it does not change the solution to its optimization problem. The solution to the original deterministic problem involves a value function (V) that is linear in the state variable m. Thus, the second order term ($\partial^2 V_i / \partial m_i^2$), which interacts with the variance-covariance matrix (16), equals zero in the HJB equation for the new optimization problem. This implies that the solution to the deterministic problem is also the solution to the one with stochastic elements.[9]

Even though adding stochastic terms leaves the solution to the control problem unchanged, it does add new elements to the model's properties. With the addition of the Weiner processes, market shares no longer follow deterministic paths and thus neither do firm values. Combining Equations (4), (13) and (15) implies

$$dV_i = \frac{\alpha_i}{\phi + \delta} \left\{ \left[\frac{\phi n \alpha_i s_i \overline{z} - (n-1)}{n \alpha_i s_i \overline{z}} - \phi m_i \right] dt + \sigma \sqrt{m_i} \sum_{j \neq i} \iota_{ij} \sqrt{m_j} dw_{ij} \right\}. \tag{17}$$

Thus, the instantaneous variance in V_i is proportional to that of m_i.

II. Estimation

A. Outline

A primary goal in this paper is to incorporate market share dynamics resulting from product market competition into the analyses of firm valuation and financial decision-making. One important advantage of the main model is that it is well-suited for empirical analysis. As Table II shows, there are several readily available empirical proxies that can be used. In this section, we take the first of the three possible approaches to the estimation of ϕ that are suggested in Table II. Equation (1) provides a mechanism through which the consumer responsiveness parameter ϕ can be estimated for each industry. Substituting equilibrium values of spending from Equation (8) into Equation (1) and using Equation (11) to define steady state market share, $\overline{m}_i$, gives:

$$dm = \phi(\overline{m}_i - m_i(t))dt. \tag{18}$$

which has a solution for $m_i(t)$ of:

$$m_i(t) = \bar{m}_i + (m_i(0) - \bar{m}_i)e^{-t\phi} \tag{19}$$

Because the stochastic component in (15) is white noise, albeit with a volatility that depends on the vector of current market shares, Equation (19) applies whether or not the stochastic term is included in the dm_i equation. We rely on Equation (19) to estimate both ϕ and $\bar{m}_i$ using nonlinear least squares. As described in more detail in Section I.B below, identification in the model comes from market share evolution across firms and over time.

Recall that ϕ captures consumer responsiveness, and is expected to be greater than zero. If consumers are unresponsive to spending, then they continue to purchase from their current firm, no matter how much is spent to attract them. In estimation, the only restriction that we impose on ϕ is that it is non-negative and less than 25 (in our annual estimation, 25 would correspond to a customer half-life of just 10 days, implying extreme disloyalty). This rarely binds in the data. Recall from *Proposition 2* that $\bar{m}_i = 1 - \frac{n-1}{\alpha_i s_i n \bar{z}}$. Thus, parameter estimates from Equation (19) also provide estimates of each firm *i*'s competitive strength, $\alpha_i s_i z$.

B. Identification

For each industry and year, we estimate the model using data from rolling 10-year intervals and assume that ϕ remains constant over each interval. Since there are *N* firms and market shares have to add to one, there are *N*-1 independent observations in each period. To ensure that annual market shares do not add to 1, we eliminate the *j* smallest firms (i.e., those with *t*=0 market shares of less than 3%, or the smallest firm in the industry if there are no firms with market shares of less than 3%). There are 10 years in the estimation window, and therefore $10(N_t\text{-}j)$ observations are used to estimate the *N-j+1* unknown parameters ($\bar{m}_i$ for each of the *n-j* firms, plus ϕ). In reality, firms enter and exit industries, so there are actually $10(N_t$-j) observations for each industry. The requirement that *t*=0 market shares are greater than 3% reduces noise in parameter estimates due to small firms moving in and out of the sample (because we focus on larger firms that are in the sample at the beginning of the estimation window). Assuming ϕ remains constant over the sample period we estimate the parameters in Equation (19) by minimizing the total sum of squared errors via nonlinear least squares.

Identification in the empirical estimation comes from changes in market share over time (to identify $\overline{m}_i$) and across firms (to identify ϕ). To illustrate how this is done, one can think of it as a two-step iterative process. The first starts with initial values $m_{i,0}$ from the data and a starting guess for the industry value of ϕ (1 is used in the estimation). These are then plugged into (19) to produce a set of errors over time for each firm *i*. An initial estimate of $\overline{m}_i$ is then produced by finding the value that minimizes the sum squares for that firm *i*'s errors. This procedure is then repeated across all firms in the industry to produce an initial vector of $\overline{m}_i$. Given a set of $\overline{m}_i$, the second step then finds the industry value of ϕ that minimizes the cross sectional panel of squared errors in (19). Using the new ϕ as a starting value this two-step process can be repeated until convergence is obtained for ϕ and all of the $\overline{m}_i$. In practice, the nonlinear least squares estimation of both ϕ and the $\overline{m}_i's$ is done simultaneously, using an iterative process (Gauss-Newton method) and given starting values for all unknown parameters.

Given the estimated steady state market shares, $\overline{m}_i$, and Proposition 2, the firm-specific variable $\alpha_i s_i z$ comes directly from the estimation described above. One benefit of the model and our estimation approach is that we do not need to estimate the firm-specific parameter *s*. This is because it is only a firm's relative competitive strength (combined $\alpha_i s_i z$) that impacts equilibrium spending and value.

C. Data and Parameter Estimates

C.1. Data and Sample Selection

The only data required to estimate ϕ and $\overline{m}_i$ are the market shares of all firms in the industry. Market share, $m_{i,t}$, is defined as firm *i*'s sales divided by the sales all U.S. headquartered

CRSP/Compustat firms in the Compustat 4-digit SIC code during year *t*.[10] We choose 4-digit codes to mimic the model's industry setting as closely as possible. Compustat codes are used due to findings in the literature (e.g., Guenther and Rosman (1994)) that linkages among firms based on these codes are higher than with CRSP SIC codes.

The initial sample consists of all firms for which there is non-missing information on annual sales and all 4-digit SIC industries in which there are fewer than twenty firms. The oligopolistic structure described in the main model makes industries with a large number of firms inappropriate in the context of this paper. We also exclude all industries with fewer than two publicly traded firms during the entire sample period. As noted earlier, we estimate the model annually. Estimates for year *t* are obtained using rolling 10-year data intervals covering years *t* through year *t+9*. We restrict our attention to firms for which we have data for more than 5 of the 10 years of each estimation interval. The estimation period begins in 1980 and ends in 2004.[11] Given the model's assumptions that $r > g$ and that there is no entry or exit, we exclude industries that are growing (or shrinking) at very high rates. To do this, we impose a filter that $r > |g|$ where g is the average sales growth by all firms in the industry during the estimation window and r is the expected rate of return on the stock market at the beginning of the estimation window.

The observations are pooled for each industry and then the industry and firm-specific parameters ϕ and $\alpha_i s_i z$, respectively, are estimated according to (19). For each 10-year rolling window, we also estimate firm-level parameters α_i and fixed cost f_i based on OLS estimation of a modified Equation (2): $\pi_i(t) = \mathrm{e}^{gt}(\alpha_i m_i(t) - u_i(t) - f_i)$. Rather than using proxies such as advertising or capital expenditures to capture spending to attract customers (which can vary substantially in form given our large cross section of industries), we instead let $\pi_i(t) + \mathrm{e}^{gt}\, u_i(t) \equiv \hat{\pi}_i(t)$ = (Revenue – Cost of Goods Sold). This is equivalent to estimating pre-spending profitability (i.e., adding $u_i(t)$ back to both sides of Equation (2)). We explicitly subtract cost of goods sold from revenue in estimating pre-spending profitability ($\hat{\pi}_i(t)$) because this type of spending is tied to the production of the good, not spending to attract customers (e.g., advertising, investments in PP&E etc). Importantly, the parameters of interest are of α and f, which do not depend on the investment in market share, u_i. Therefore, Equation (2) becomes: $\hat{\pi}_i(t) = e^{gt}(\alpha_i m_i(t) - f_i)$.[12] For each firm, there are two unknown parameters: α, which multiplies market share, and fixed cost f (an intercept). To obtain estimates of these parameters, we need only one explanatory variable: market share (m_{it}). Estimating α_i and f_i now simply requires estimation of the regression coefficient on current market share and the intercept, respectively.

The e^{gt} term is calculated from industry sales. It equals the ratio of total current period industry sales to total industry sales in the first year of the sample (all values are in real 2007 dollars).

We obtain estimates for consumer responsiveness (ϕ), competitive strength ($\overline{m}_i$ and $\alpha_i s_i z$) and profitability (α_i) for 2,033 unique firms in 332 industries. There are a total of 12,643 valid firm-industry-year estimates, representing the majority of the possible 14,678 firm-industry-year observations that meet the initial data filtering requirements. Table III reports summary statistics on the estimated ϕ's, $\overline{m}_i$ and α_i. The mean (median) ϕ is 0.423 (0.191). This corresponds to an industry half-life of about 1.6 (3.6) years. That is, it would take a firm in the average industry 1.6 years to lose half of its customers if it completely stopped spending to acquire market share. The mean (median) steady state market share of firm i is 19.0% (11.1%). These magnitudes for $\overline{m}_i$ are expected given that a minimum current market share of 3% is required for inclusion in the sample.[13] Finally, the α parameter represents the annual profitability (in millions of 2007 U.S. dollars) per unit of market share. The mean (median) estimated α is \$3,831.6 (\$724.6) and is interpreted as the profitability of a firm with 100% of all industry sales. While the summary statistics in Table III provide a useful overview of the estimates, the discussion below highlights some potentially important between- and within-industry variation.

C.1.a. Phi (ϕ) and Industry Half Life Estimates

Table III lists estimated ϕ's at the industry level along with half-lives (*h*) based on the point estimates for ϕ. The half-lives are expressed in years (calculated as $\ln(2)/\phi$). Individual ϕ's, $\overline{m}_i$, and α_i are estimated at the four-digit SIC code level; however Table IV shows the median ϕ within each two-digit level (for brevity). Despite the aggregation, useful observations can be made from the table.

Economically, the question is whether the half-lives in Table IV are "reasonable." Recall that setting u_i to zero does not imply that the firm ceases operations, maintenance, or eliminates all customer service. Rather, it means that it does not actively compete for customers through things like advertising, R&D, and the construction of new outlets. In this light the estimates seem plausible. For example, within the transportation industry, rail has a half-life of 4.9 years and air 2.1 years. Given the fixed nature of rail track, this difference expected. While there are clearly some industries in Table IV with estimated half-lives that appear to be either too high or too low, most seem to lie within the ranges one would expect.

We obtain parameter estimates for firms in all 332 industries at the 4-digit level. For illustrative purposes, Table V presents firm-level estimates for five of these industries for the year 2000 (the most recent year for which we have full data for the year *t* to *t+9* estimation period). These industries reflect significant between-industry variation in consumer responsiveness (ϕ of 0.025 to 0.561), as well as within-industry variation in both steady state market shares and profitability of individual competitors. As in Table IV, many of the estimates appear very plausible. For example, the estimated half-life for SIC Code 5731, Radio, TV and Consumer Electronics Stores is 1.2 years, whereas the half-life for SIC code 3523, Farm Machinery and Equipment is more than six times that number, at 6.9 years. Here, the intuition is that modern storefronts, aggressive advertising or improvements to enhance the electronics shopping experience will make customers more likely to patronize a given electronics store. In contrast, consumers' established comfort with the features of a particular brand of farm equipment and the delay between replacement cycles will make them slower to switch brands in response to an improvement in, for example, tractor steering capabilities. This variation in consumer loyalties would seem to make these reasonable estimates. A dataset containing the estimated parameters shown in Table V for all firms and years in the sample is available in the Internet Appendix to this paper.

C.2. Calibration: Dynamics of Values and Market Shares

While the illustrative examples in Table V provide useful intuition, a natural question to ask is whether the value functions described in Equations (4)-(6) are consistent with observed firm value dynamics. We can use the parameter estimates obtained in the previous section to provide a more powerful test than the "reasonableness" checks above, used as a starting point in evaluating the validity of the model. We first calculate actual firm values, defined as the equity market capitalization, plus book values of debt at the end of year *t*. The estimation procedure for the parameters ϕ's, $\bar{m}_i$, and α_i used to calculate model-implied *V*(*m*) is described in Section I.C.1. The final input to the value functions is the cost of capital minus the growth rate (δ), which we define in two ways, using industry-level and market wide δ's. Industry δ_{It} is defined as the average (unlevered) cost of capital, minus the average 5-year sales growth rate for all firms in the 4-digit SIC code. The market-wide δ_{Mt} is defined as the long-run (1926 through period *t*) historical market risk premium plus the risk-free rate, minus the long-run GDP growth rate.[14] The market-wide measure captures overall equity market returns during our sample period.

Table VI presents results from regressing (log) actual firm values on the (log) model-implied V(*m*) from Equations (4)-(6) using ordinary least squares and allowing for clustering of standard errors at the firm level. Because the regressions are log-log regressions, the coefficients are interpreted as elasticities. Results from a benchmark regression of (log) actual firm values on market shares are also given in Table VI, for comparison.

Panel A of Table VI contains the main results: model-implied V(*m*) captures actual valuation. The coefficients on this variable are statistically significant and range from 0.497 to 0.583. Thus, a 1% increase in model-implied value corresponds to a 0.497% to 0.583% increase in actual firm value. This is to be expected given that the model does not fit the data perfectly; the standard regression towards the mean argument. However, it may also be due in part to the market anticipating the news in accounting releases. For the model, any change in an accounting variable is indeed "new news." Market participants, though, may have foreseen such changes in values well in advance.[15] The positive intercepts in Panel A of Table VI are further evidence of this; market values tend to rise over time for reasons outside the model and data it employs.

Finally, note that the Table VI adjusted R^2 in the regressions using V(m) alone are substantial. Their value ranges from 0.439 to 0.494 depending on the definition of δ used. These are large compared to the R^2 of 0.197 in the benchmark case which uses market share as the sole explanatory variable. Here, again, the model seemingly brings to the data information beyond what a standard linear regression might.

The model assumes that $r<g$ and also assumes no entry or exit. Because the model is not intended to explain value dynamics in industries exhibiting rapid growth or contraction, the initial data filters excluded such industries from the sample. In order to check the validity of this filter, in Panel B of Table VI, we allow high growth industries in the sample and introduce *lowgrowth*, a dummy variable equal to 1 if the firm is in a stable industry (i.e., where $r<|g|$, as in the Panel A regressions). We interact *lowgrowth* with $V(m)$ and test the hypothesis that the model does a better job for the stable industries for which it was intended. That is, we expect to observe a positive coefficient on the *lowgrowth* and $V(m)$ interaction. This is exactly what we observe. While the model is still important in explaining values of all firms, the estimated coefficient on $V(m)$ drops relative to Panel A. The estimated coefficient on the interaction is around 0.21, implying that for every 1% change in model-implied values, actual values increase by 0.21% more for firms in stable industries than firms in less stable ones. This validates the initial sample selection criteria and also suggests the types of industries for which the model does and does not perform well.

As mentioned previously, we define industries using 4-digit SIC codes. While this is the finest level of SIC categories, it is possible to examine even narrower industry definitions based on other classification systems. To check that our main results are not driven by the choice of industry definition, we re-estimate all parameters and the regressions shown in Table VI but replace SIC codes with 6-digit NAICS codes. Although we obtain estimates for a smaller set of firms using this industry definition, the main findings regarding the ability of the model to explain actual firm valuations remain. Detailed results are available in the Internet Appendix (Table IA.I).

In addition to predictions about value, the model also provides clear predictions regarding the evolution of market shares within industries. Subtracting $m_i(0)$ from Equation (19) yields $m_i(t) - m_i(0) = (\overline{m}_i - m_i(0))(1 - e^{-\phi t})$. Given initial condition $m_i(0)$, we can use estimates from the model to forecast *t*-period ahead market share changes. We first use data from year *t*-9 to year *t* to estimate the model's parameters. We then use these estimates to generate out-of-sample predictions of changes in market shares from year *t* to years *t*+3 and *t*+5. Table VII shows results from regressing actual 3- and 5- year ahead changes in market share on model-implied (predicted) changes for all firms and industries for which we are able to obtain estimates. These results are based on data from all the industries for which we have parameter estimates (i.e., those in Panel A of Table VI). Because the estimates require data for the 10 years prior to the forecast period as well as the forecast period itself, the number of observations is substantially smaller than in Table VI. Still, there are 4,417 observations in the 3-year ahead regressions and 3,871 observations in the 5-year ahead regressions. Both sets of regressions show significant predictive power of the model-implied market share changes.

In the theory presented in this paper, the model-implied market share change is the only relevant explanatory variable and is therefore the only variable in the main predictive regression specification, shown in the leftmost columns of Table VII. Further, the estimated coefficients on the model-implied market share changes are 0.10 for 3-year ahead changes and 0.21 for 5-year ahead changes, with *t*-statistics that are greater than 2.5. Thus, the model not only captures variation in firm values, but predicts within-industry market share dynamics as well. As a benchmark, it is worth comparing the explanatory power of the model's predicted market share changes (based, as previously noted, only on revenue and COGS) to other variables that have been used in intra-industry studies of returns near major events (e.g., Eckbo (1983) and Shahrur (2005)). In the right hand side columns of Table VII, we introduce industry concentration (*HHI*), change in *HHI* and number of firms in the industry as alternative explanatory variables. While these variables do offer additional explanatory power when added to the regressions, the coefficients on the predicted market share changes from the model remain statistically significant. Moreover, when we run a "horserace" among these variables using stepwise model selection based on the Schwarz Bayesian Information Criterion, we find that the model-implied market share changes ranks highest of the four candidates in these out-of-sample predictive regressions.

The benchmark market share analysis in Table VII assumes that the world is deterministic and that all errors in the estimation are due to measurement error. In reality, industries are likely to be hit by random shocks. We use the framework presented in Section I.C.2 to explicitly account for stochastic shocks in the estimation of market share evolution. Rather than simply allowing for industry clustering of standard errors as we do in Table VII, we introduce the structure of the variance-covariance matrix defined in (16) to the empirical implementation. We do this in two ways. In the first approach, we use ordinary least squares to obtain coefficient estimates and then use (16) and year *t* market shares to estimate standard errors (and industry-specific σ_i). In the second approach, given that we have information on market shares, we initially set all σ_i equal to the those obtained using OLS and use year *t* market shares in (16) as a weight matrix to estimate coefficients using GLS. Residuals from the equation using GLS parameter estimates are then used to estimate new σ_i and the coefficient standard errors. In both cases, the industry-specific σ_i are estimated as in a standard a random effects model (see e.g., Greene, 1997), except that the residuals are pre-multiplied by the square root of the inverse of the diagonal terms in (16). Results are in Table VIII. Similar to Table VII, we find that model-implied market share changes have substantial predictive power.[16] In fact, the significance of the estimated coefficients increases relative to the Table VII analysis once we explicitly account for stochastic market shares. This is due, at least in part, to the fact that market shares add to one, introducing negative correlation within each industry in a given time period.

One interesting question that arises is how industry ϕ might be related to the variance of its market share shocks (σ in Equation (15)). One might expect that when customers are very loyal (low ϕ), intrinsic market share volatility would be low as well. Similarly, disloyal customers might be associated with high levels of intrinsic market share volatility. This is precisely what we find. The correlations between estimated σ_i and ϕ are between 0.23 and 0.26, depending on the estimation window, and are statistically significant.

Since the results in Tables V through VIII indicate that the model fits actual value and market share dynamics, one can potentially use it to forecast corporate returns to investments that improve fixed costs, profitability and effectiveness. These are all very different types of investments and will generate different competitive responses. The advantage of the value functions produced in this paper is that they explicitly account for the competitive response to such improvements. To illustrate this idea, Table IX shows estimates of the average percentage change in value given an opportunity to improve α, *s,* and *f* by 10 and 25% for the five sample industries from Table V. Observe that investments in α provide the highest benefit. Not only does an increase in profitability make each customer more valuable, but also leads the firm to garner more of them (i.e. increase its market share). Decreases in fixed cost *f* can also substantially improve value. The benefit of this type of innovation is that there is no competitive response to improvements in *f* (the equilibrium spending described in Equation (8) is independent of *f*). Spending effectiveness has a somewhat smaller impact on value than the other two parameters. This is because improvements in *s* can increase a firm's market share, but will not improve value from each unit of market share. In fact, if there are more than two firms in the industry, it is possible that improvements in *s* by weaker than average firms are met with such aggressive responses from competitors that value is actually reduced.

C.3. Limitations and Possible Extensions

The analysis presented in this section is intended to provide an example of how the model in this paper can be applied in empirical research. Because we are the first to implement this type of model in the finance literature, we have chosen to estimate parameters for a broad set of industries and a large number of firms, rather than engage in a very specific industry study. This also helps to draw out the model's performance across industries. For example, the model is not intended to capture the dynamics of industries undergoing structural changes due to the entry and exit of significant firms, or shifts in regulation.[17] Consider the entrance of Amazon.com to the retail book industry. While the underlying product remains books, Amazon's entrance provided a new medium for attracting customers, capable of changing industry growth *g*. The tobacco industry, which has been the subject of advertising and labeling regulations over the past few decades, provides another scenario in which an industry's structural changes may limit the model's applicability. One way that we mitigate the impact of these types of structural changes is that we estimate the model parameters over rolling 10-year periods. This decreases the odds of more than one competitive regime existing in a given estimation window. Moreover, it allows the parameters of the model to change over time.

Entry, exit, regulatory changes and innovation are additional real-world factors that might impact market share evolution in ways that are not captured by the model and could bias parameter estimates. Entry would cause us to overstate steady state market shares of all of the firms because the model is estimated only for those firms that are in the industry at the beginning of the estimation window. But it would not bias ϕ. Exit would not impact estimates of steady state market shares (if the exiting firm's long run market share should be zero) and would give ϕ an upward bias because market shares would appear to increase very quickly as a result of the exiting, zero-spending firm. Regulatory changes or innovation would change the entire industry structure and could produce biases in all parameters (in either direction depending, on the nature of the change). For example, if one firm develops a new product and obtains a patent in the middle of the estimation period, steady state market shares for non-patent firms are likely to be upward biased and the market share for the patent firm is likely to be downward biased. Consumer responsiveness (ϕ) could be over-estimated due to the immediate shift in shares when the patent is introduced. Importantly, as noted above, we estimate the model over rolling windows to minimize the impact of these potential biases. We do not expect important bias across all time periods.

Another limitation is that the model is not intended for high growth industries. The model describes market share competition, not sales competition (which would be more appropriate for high growth industries). Indeed, when we condition on low growth industries in the analysis, we observe significant improvements in the fit of the model to the data. Future researchers should be cautious about applying the model or drawing inferences based on data from high growth industries.

Specific industry analysis using richer datasets than the *Compustat* tapes (e.g., detailed brand-level advertising and sales information; higher frequency data; international industries and private firms) would provide more realism and even more precise estimates of the impact of industry dynamics on values and investment incentives. Narrower industry definitions might also help. One option might be to use a firm's own statements regarding who they believe their competitors are along the lines of those used in Hoberg and Philips (2010a and 2010b). Similarly, applications that can use a specific industry as a case study might also increase the precision of model's estimated parameters.

Finally, following the literature using Lanchester battle models of advertising competition, we do not explicitly model consumer preferences or utility functions. Therefore, we cannot speak to welfare. We can, however, say that spending to attract customers is wasteful in the sense that it reduces firms' values without having an impact on the total quality or quantity being produced.

Despite the model's limitations, the evidence presented thus far shows that it captures at least some new information about the variation in firm values and the evolution of future market shares. Given the industry parameter estimates such as those presented in Table V and the value functions in functions in Equations (4)-(6), one can now potentially incorporate product market dynamics in a wide range of investment and financing decisions. The next section applies this potential to the issue of horizontal mergers and acquisitions.

III. Application: Horizontal Mergers and Acquisitions

A. Theory

Horizontal mergers offer the potential to benefit every firm in an industry by reducing the number of competitors. Indeed it is true that reducing n while holding $\overline{z}$ constant increases the value of every firm. However, that is only part of what a merger does. It also alters the structure of the competitive environment. The newly merged firm may have a different α and with two product lines a different (and presumably larger) s. This means that it will likely behave more aggressively, and may have a larger steady state market share. These competing influences imply that the impact on rival firms will depend on what the merger accomplishes.

In order to discuss how a merger impacts rival firms it is useful to establish what the union of two firms can do. Within the model there are three parameters that come to the forefront: the profit per unit of market share (α_i); the ability of the now joint product line to attract customers (s_i); and the fixed operating costs (f_i). It is useful to begin by establishing a baseline for each.

In order to simplify the exposition assume firms 1 and 2 in the industry merge. The parameters for the merged firm will be designated with a subscript 1+2. As a baseline, consider a merger that simply "glues" the two firms together without offering any operational or competitive advantages. Call this a "synergy free" merger: (1) no reductions in overhead, (2) no improvement in consumer response to the product line and (3) no production cost benefits. The first condition implies that firm 1+2 has the same total fixed costs as 1 and 2 or $f_{1+2} = f_1 + f_2$. The second implies that if every firm spends as much post-merger as it does pre-merger on marketing then firm 1+2 will acquire customers at the same rate as 1 and 2 do. This can be translated, via the dm_i Equation (1), into requiring that $u_1s_1 + u_2s_2 = (u_1+u_2)s_{1+2}$ holds. Using the equilibrium values for u_i (Equation (8)) the consumer responsiveness condition can be written, after some algebra as:

$$\frac{\alpha_1 s_1 z-(n-1)}{\alpha_1 s_1}\left(1-\frac{s_{1+2}}{s_1}\right)+\frac{\alpha_2 s_2 z-(n-1)}{\alpha_2 s_2}\left(1-\frac{s_{1+2}}{s_2}\right)=0. \qquad (20)$$

The first two conditions for a synergy free merger restrict f and s. The third restricts α and requires that $\alpha_1 m_1 + \alpha_2 m_2 = \alpha_{1+2}(m_1 + m_2)$. All three conditions, when taken together, imply that absent changes in market share or spending on customer acquisition the merged firm's total profits will equal the sum of the two underlying companies. Thus, in this case, any gains from the merger arise because of the oligopoly's stronger market power and not productivity improvements.

Clearly mergers that produce operating efficiencies of one sort or another will lead to consolidations. But what about those which do not? Is there a natural tendency for industries to form into monopolies? Or, as in the standard linear demand-constant marginal cost Cournot model, is there an incentive to break apart into smaller and smaller companies? Part of the answer is provided within the next proposition.

PROPOSITION 3: *Consider a merger between two identical firms ($f_1 = f_2 = f$, $\alpha_1 = \alpha_2 = \alpha$, $m_1 = m_2 = m$, and $s_1 = s_2 = s$) in which there are no synergies. Upon the merger's completion, the value of the merged firm will be higher than the sum of the standalone companies if $\alpha sz\left(\sqrt{2}-1\right) < \sqrt{2}$ and less if the inequality is reversed.*

Proof: See the Appendix.

Proposition 3 establishes that synergy free mergers between identical large rivals will not be profitable, while those between small ones may be. Relative to other firms in an industry, large rivals must have relatively large values of αs. This is how they generate a large steady state market share. Now, consider an industry with two weak (and thus small) firms with values of αs equal to 1 and three strong (and thus large) firms with values equal to 3. In this case z equals 3. For the two small firms $\alpha sz\left(\sqrt{2}-1\right)$ equals 1.24<1.41 and a synergy free merger is profitable.[18] For the large ones, however, $\alpha sz\left(\sqrt{2}-1\right)$ equals 3.73>1.41 and consolidating, absent synergies, would not generate additional value to the firm's investors.

Based on Proposition 3, unfettered mature industries may structure themselves into forms that look like oligopolies with several large firms and few if any small ones. However, it is unlikely they will end up as monopolies. Consider an industry of n identical firms. In this case αsz equals n for every firm and synergy free mergers are only profitable if $n\left(\sqrt{2}-1\right)<\sqrt{2}$. Plugging values of n into the inequality shows that it will not hold if there are four or more firms. Thus, after the very small firms consolidate, the M&A process should cease.

What discourages consolidation in the model and why is it more likely that small firms can profitably merge absent synergies? Economic intuition suggests that such horizontal mergers should increase market values, if only due to having fewer competitors to battle against. However, this ignores the influence of a countervailing force; increased rival aggressiveness. Rivals view the reduction in n as a reason in and of itself to ramp up their spending on customer acquisition. (One can see this by observing that the equilibrium values of u_i and $\overline{m}_i$ both depend on n.) Intuitively, fewer competitors mean larger market shares become easier to procure, so firms respond by increasing u_i. For would be acquirers, this means facing a tougher competitive environment post-merger. This discourages acquisitions.

Despite the response to a reduced value of n from a merger, potentially offsetting it is the impact on z. Lower values of z result in less aggressive competitors. This means the strength of the eliminated firm from a merger matters. In the case of a merger of two identical firms, z drops by $1/\alpha s$. Since large firms have large αs values a merger by them does little to change z. In this case the impact from reducing n is all there is, making a synergy free merger look unattractive. In contrast, removing a small firm has a relatively large impact on z leading to a less robust competitive response and making such acquisitions potentially worthwhile. Intuitively, the result is similar to what one might expect in an athletic tournament. If one of the top athletes is removed the others have a stronger incentive to compete for the top spots. On the other hand, removing a weak competitor leaves the incentives to compete for the top spots relatively unchanged. In the model, the former is like buying out a large firm and thus leading to large increases in u_i by everybody else. The latter is like buying out a small firm which has essentially the opposite effect.

Based on Proposition 3, one can also create settings that produce merger waves. A modified version of the prior 5 firm example shows how. As before assume that the three large companies have α*s* values of 3. This time, however, assume the two small ones have values of 2. In this case *z* equals 2 and absent synergies none of the firms will wish to merge. Suppose something changes and that two of the large firms can profitably merge because they can consolidate their back offices. In the model this means f_{1+2} is substantially lower than $f_1 + f_2$, but there are no other direct gains ($\alpha_{1+2}s_{1+2}= \alpha_1 s_1= \alpha_2 s_2$). Post-merger *z* drops to 1.67. Now it will pay for the two small firms to merge even absent synergies since their α*sz* values fall to 1.38. Here, one merger suddenly makes others look attractive. A full analysis of the potential for merger waves within the model takes the discussion far afield, thus the paper leaves it for future research.

Another empirical implication of the model has to do with post-merger announcement returns across the rival firms. Stronger firms (large values of α*s*) will see a larger negative percentage change in their values if $(n-1)/z<(n-2)/z_{1+2}$.

PROPOSITION 4: *If* $z_{1+2}/(n-2)<z/(n-1)$ *then rival firms all see a reduction in their values. On a percentage basis, the reduction is increasing in* $\alpha_i s_i$.

Proof: The rival values of α, *s*, and *f* are unaffected by the merger as are their current market shares. Thus, the only change in value comes from the part of Equation (5) that does change: *z* and *n*. The first statement in the proposition then follows directly from (5) after some minor algebra. The second and third claim can be verified by differentiating the percentage change in value by α_i and s_i. Q.E.D.

Proposition 4 offers a potentially testable set of hypotheses that appear to be new to the literature. However, it also indicates that some previous studies that examined the issue of horizontal mergers on rival firms may be able to draw clearer distinctions if they further parse the set of rivals by their competitive strength. The next section will implicitly test Proposition 4's predictions by examining how well the model fits actual rival returns upon the announcement of a horizontal merger in their industry.

B. Empirical Estimates

B.1. Employing the Structural Model

This section examines the model empirically within an M&A setting. We begin by using the parameter estimates (summarized in Table III) for rival firms, based on the prior estimation of Equations (2) and (19). While a merger may lead to the creation of a newly empowered or weakened firm, there is no reason to believe it should impact the firm specific attributes of others in the industry. Within the model this translates into assuming that if two firms combine and firm i is not involved then its α_i, s_i and f_i remain unchanged. Similarly, while m_i may eventually drift up or down as a result of the merger, on the announcement date it too should be unchanged. Finally there is no reason to believe a merger will dramatically impact the industry parameters related to discount rates (δ) or consumer responsiveness (ϕ).

Based on the above assumptions, a merger's impact on a rival firm's announcement day return occurs through two channels: (1) there is a reduction in competition via the change in n to $n-1$ and (2) the newly combined firm may be either stronger or weaker than the stand alone companies were, leading to a change in z. With these assumptions in place, Equations (4), (5), and (6) imply that the expected return to rival firm i from the merger equals:

$$r_i = \frac{\phi\left\{\dfrac{\phi\alpha_i\left[\gamma\alpha_i s_i z-(n-1)\right]^2}{\delta(\phi+\delta)(\gamma\alpha_i s_i z)^2}-\dfrac{\phi\alpha_i\left[\alpha_i s_i z-(n-1)\right]^2}{\delta(\phi+\delta)(\alpha_i s_i z)^2}\right\}}{\dfrac{\phi\alpha_i\left[\alpha_i s_i z-(n-1)\right]^2}{\delta(\phi+\delta)(\alpha_i s_i z)^2}+\dfrac{\alpha_i m_i}{\phi+\delta}-\dfrac{f_i}{\delta}} \tag{21}$$

where γ represents the change in $\alpha_i s_i z$ due to the merger. Given estimates of the firm parameters α_i, f_i and industry parameters δ, ϕ, and z the only unknown is γ. Thus using each rival's return from the merger one can use (21) to estimate γ via nonlinear least squares. The advantage of this approach is that it only requires pre-merger information to estimate the firm and industry parameters and only post-merger return information to estimate γ.

Recall that z equals the sum of the $1/\alpha_i s_i$. Thus, even if a combination of firms 1 and 2 implies that $1/\alpha_{1+2}s_{1+2} \neq 1/\alpha_1 s_1 + 1/\alpha_2 s_2$, the total impact on z should diminish with the number of firms in the industry. To help specify a functional form, note that if all n firms are identical then prior to the merger one has $z_{pre}=n/\alpha s$. (Here the subscripts have been dropped since they are redundant.) Now suppose the merger is synergy free, the case analyzed in Proposition 3. Then $1/\alpha_{1+2}s_{1+2} = 1/\alpha s$ and post-merger $z_{post}=(n-1)/\alpha s$. Let γ represent a parameter relating the pre and post-merger values of z such that $z_{post}=\gamma z_{pre}$. Substituting out for the values of z one has $(n-1)/\alpha s=\gamma n/\alpha s$. After some rearranging $\gamma=1-1/n$. With this backdrop the empirical model thus assumes γ can be parameterized as

$$\gamma = a_0 + a_1 / n_{\text{pre}} \tag{22}$$

where the a_i are estimated parameters, and n_{pre} equals the number of firms in the industry prior to the merger.

B.2. Sample Selection and Estimates

In order to be included in the M&A sample both the target and acquiring firm have to be in the same 4-digit SIC industry. As in the Table VII analysis, the main goal is to conduct an out-of-sample test of the model. Therefore, we use data from the 10 years preceding the announcement to estimate all relevant parameters. As in the main analysis, we filter on low growth industries since the model is more likely to be appropriate for these types of industries in the merger setting. Just as we need a fairly stable environment to estimate the model's parameters pre-merger, post-merger we assume that the change occurs within the newly combined firm is a once-and-for all event. Rivals must have valid firm specific parameter estimates using data from the 10-year period leading up to transaction year t. These filters, in addition to those described earlier, result in final a sample of 66 horizontal mergers that occurred during the 1990-2009 period with accompanying data on 183 rivals.

Table X Panel A contains the abnormal returns analysis of our sample of M&A transactions. Each observation represents a rival's stock return. As can be seen from the table, the actual announcement period rival abnormal returns are small. The median market adjusted return is 0.47%

while the beta adjusted return is 0.60% over 3-day windows. These relatively small values are consistent with prior findings such as Shahrur's (2005). However, there is substantial variation, with standard deviations that are close to 4%.

Table X Panel B provides statistics summarizing the estimated model parameters. Notice that the parameter on $1/n$ is negative in both specifications. Since the model is highly nonlinear, it is useful to examine the bootstrapped distribution of the parameters. The dark shaded row in the table creates bootstrapped distributions by drawing with replacement the data on a firm-by-firm basis to create new samples of the same size as the original. After each sample is drawn, the model is estimated and the parameter values recorded. The table displays the resulting values across various percentiles. The light shaded row repeats this exercise but this time industries are drawn with replacement to create samples with the same number of industries prior to estimation.

The firm-by-firm bootstrapped distributions using market adjusted returns show that the $1/n$ parameter lies somewhere between −1.32 and −1.24 with 95% confidence. At the 1% level, the range increases to between -1.34 and -1.22. When drawing by industry the results are comparable. Using beta adjusted returns yields nearly identical results which, given the short return window, is to be expected.

Panel C in Table X examines how well the model's resulting return forecasts fit the actual data. It does so by regressing the actual rival announcement returns against a constant and the model's forecast. Overall the results are economically and statistically significant. The parameter on the model forecast return is significant at the 1% level under either bootstrap procedure. The median value of the coefficient is about 0.94 when using market adjusted returns and 0.92 when using beta adjusted returns. This suggests that for every 1% increase in forecasted returns, observed rivals' returns increase by more than 0.9%. Perhaps more importantly, the R^2 statistics are in line with those generated by purely empirical models that use far more explanatory variables. The median R^2 value is about 9% with both market adjusted and beta adjusted returns. As noted in the introduction, this compares well with the purely empirical 11 variable model of customer and supplier returns in Fee and Thomas (2004) which generates an R^2 of 1.4% and with Shahrur's (2005) 10 variable model of rival returns that generates one of 9%.

These analyses fit observed rival returns by using a variety of potential explanatory variables, such as industry concentration (Herfindahl-Hirschman Index) and the observed abnormal returns to the merged firm, that are likely to be correlated with rivals' returns. By contrast, the exercise in this paper is forward-looking. It generates return forecasts, based only on the parameters estimated from the model which themselves derive from just two accounting variables: revenues and cost of goods sold.[19]

The model's quantitative structure allows it to break down the rivals' returns in ways that are impossible in models that yield only forecasts about whether the returns should be positive or negative. Consider the median sample firm. This is a hypothetical firm with the sample median estimated parameter values in a hypothetical industry with the median number of firms. Plugging these values into (21), along with the estimates for (22) from Table X Panel B for the market and beta adjusted returns, yields a forecasted return for the median sample firm of r_{median}= 0.36%. [20]

By using the model's structure it is also possible to break down the merger's impact from both the reduction in competition due to the decrease in n and the increase in the newly merged firm's competitive ability (via the estimated increase in z). The overall return can be split into three components: the change in the number of firms in the industry (Δn); the change in the general level of industry competition due to the merger ($\Delta\alpha sz$); and the total return minus the first two components (r_{median}- Δn- $\Delta\alpha sz$). The first two return computations simply require reproducing the calculation for r_{median} but holding either n or αsz in (21) constant.

Consider what would happen to the average rival's value if the merged firm were no stronger than its individual components ($\Delta\alpha sz$=0). Based on estimates from the market adjusted return model, the rivals would gain 10.21% in value from the reduction in n by one. But, this figure overstates what a rival might truly expect solely from a reduction in the number of competitors. The median sample firm competes in an industry with 14 firms. A typical firm in this industry has to have a 7.14% share of industry sales and profits. If one firm randomly drops out, then holding industry profits constant, the remaining 13 firms will gain 0.55% in market share on average. This represents a 7.69% increase in size and thus presumably value. The estimated value of 10.21% from reducing n by 1 really consists of two parts. The 7.69% arises from the fact that the now missing firm's profits have to be reallocated among the remaining competitors. In reality, it is unlikely that a merger will reallocate the target's profits to anybody other than the bidder. Removing this 7.69% from the 10.21% generated by simply reducing n leaves a 2.52% forecasted rival return. This 2.52% figure is thus the model's forecast of a rival's return when a horizontal merger fails to yield a stronger or weaker competitor and instead creates a tighter oligopoly.

While a 2.52% gain in value from just the reduced competition is substantial, it is partially wiped out by the fact that the newly combined firm is actually a somewhat stronger competitor. That should not be surprising. Studies typically report that target companies see their values increase by 20% to 30% on average when mergers are announced. That value has to come from somewhere. Based on nearly any oligopoly model, the reduction in n should increase industry profits. If, however, the increase does not show up in rival returns then the natural explanation is that some of the value increase seen by the target comes from gains in the competitive ability of the combined entity. That gain, whatever it is, must then come at the expense of rivals. Holding n constant but allowing $\Delta\alpha sz$ to equal its estimated value from (22) generates a loss to the median rival of 7.99%. As with the analysis of how reducing n impacts rival values, this number appears large but actually combines two affects. In this case, the change in z is being driven largely by the fact that a new firm with twice the customer drawing power has been created. If one wants to understand whether or not the two merged firms make for a stronger or weaker rival than when they were independent, one should first add back the 7.69% from the simple reduction in n. Doing so leads the model to imply that the newly merged firm is indeed a stronger rival; strong enough that it would reduce the values of others in the industry by about 0.30% even if other competitive pressures remained unchanged.

According to the above estimates, the gain to the median rival firm from reducing n by 1 and the loss from the increased competitive ability of the newly merged firm still leaves a net gain of about 2.22% (2.52% minus 0.30%). Since the total forecasted return is only 0.36% it is natural to ask what element within the model eliminates the difference. The answer lies in the interaction that arises from the increased competitive ability of the merged firm that is now in an industry with fewer competitors. Not only is the merged firm stronger than the two stand-alone firms, the industry now has one stronger firm with a smaller set of rivals. Since it is better to be a stronger competitor with $n-1$ firms than with n, rival values will be further reduced. The model allows one to see just how important this interaction is: 2.22%−0.36%, or 1.86%. This is the loss to the rival firms in the industry from facing a stronger competitor in an environment with fewer firms.

The analysis shows that the model produces statistically significant forecasts regarding rival returns after a horizontal merger in their industry. However, as before, this brings up the issue of how well the structural model does relative to the empirical instruments others have used. Table XI examines this issue. The forecast from the structural model yields an adjusted R^2 of 9%. Adding HHI, the change in the HHI from the merger, and the number of competitors to the regression actually *reduces* the adjusted R^2 to 8.9%. As one might thus expect, in a "horserace" between model-implied returns and the alternative variables based on the Schwartz Bayesian Information Criterion, the model-implied return is the only one selected for inclusion.

B.3. M&A: Limitations and Possible Extensions

The empirical estimates in the prior section reflect an implicit assumption that mergers occur between firms that are, in many ways, typical representatives of the industry. Before delving into the possible ways in which violations of this may lead to biases, it is worth starting with those that do not. There are no expected biases if, for example, mergers take place between firms with a particular level of productive efficiency. The cross sectional variation in this trait should be captured by the estimated profit functions. For the same reason there are no restrictions on how appealing the merging firms' products may be. All that matters is whether or not our estimates capture this properly. Finally, there is no bias if firms that merge are generally growing faster or slower than their competitors. The only requirement is that our estimates of each company's steady state market share is sufficiently accurate.

While both the theoretical and empirical work offer many ways to handle data problems, one can think of a number of elements in the M&A analysis that may ultimately bias the reported estimates beyond those already discussed in the earlier limitations section I.C.3. Mergers do not occur between random firms and this may lead to systematic errors with a particular sign. The following should be taken as a partial list of potential caveats and opportunities for future research in the area.

As noted earlier, structural industry changes will likely lead to biased estimates. For mergers, this may present a particular problem as such changes may be the catalyst that drives them. Technological innovations will lead to an industry's consolidation if, for example, efficiency requires firms to increase the ratio of their fixed to marginal costs per unit of output. Our estimates, in contrast, assume that the production technology has not changed near the time of the merger. In this case, post-merger the estimated fixed costs will be too low and marginal costs too high. This will lead to biased estimates of how the merger impacts rivals. From Equations (4), (5) and (6) the cross derivative of a firm's value with respect to α_i and z is positive. At the same time the cross derivative with respect to fixed costs (f_i) and z is zero. Under the hypothesized technological change, any strengthening of the rival firm from the merger, which *decreases* z, will therefore have a bigger marginal impact than what our estimates indicate. In this case, we will then systematically overestimate the impact from reduced competition to equate the model's returns to those in the data. One can potentially evaluate the importance of this conjecture by trying to estimate rival production functions pre and post-merger and see to what degree the values of α and f do or do not change and in what direction.

In the model all of the firms compete in a well-defined industry. Thus, for example, consumers pick among brands of potato chips. But, do makers of pretzels also compete with these same firms? They undoubtedly do to some degree, although probably not to the degree one potato chip producer competes with another. However, the industry definitions used here are based on rules that may not reflect these subtleties. In many industries it may be the case that firms A and B are close together in product space (they produce potato chips in the example) as are C and D (they produce pretzels). However, A and B are not as close to C or D. Within the model that would imply that increased spending by A on customer acquisition will draw more from B than C or D. Currently, neither the model's setting nor the parameter estimates allow for this. If in general mergers occur between close rivals (A with B) then our estimates will overestimate the impact on rivals relative to what would happen if two random firms combined. The converse holds if in general more distant competitors tend to merge (A with D). In that case, the model estimates will understate the impact on rivals relative to what a random merging of firms would produce. Theoretically, this problem can be addressed by allowing firms to draw customers at different rates from different rivals. Using the modified model, one can then potentially estimate a model allowing for these differences.

IV. Conclusions

The paper's main goal has been to present an estimable model that addresses the following questions: First, how do product market dynamics impact firm valuation? Second, how do these dynamics impact M&A activity? Third, what are the value implications for rivals? In the context of a dynamic oligopoly, we provide closed form solutions for the values of n competing firms. These solutions allow us to estimate the values of innovations in fixed costs, profitability and spending effectiveness, explicitly incorporating the current state of the industry and rivals' competitive responses to such investments.

The model's formulation makes it amenable to empirical estimation. We estimate the main parameters of the model for a broad cross-section of firms and industries. We find strong evidence that the model-implied value functions presented in the paper capture actual values. We also use the estimated parameters to estimate the potential value-implications for investments in various types of innovations. Because the model explicitly incorporates competitive responses to innovation, these calculations can enrich standard valuation analyses of corporate investment decisions.

While structural models help pin down empirical specifications, they do have limits. Mathematical tractability requires placing restrictions on the properties of the industry the model seeks to depict. In this paper, the goal has been to describe competition in a mature oligopoly. As the empirical work also shows, that comes at the expense of the model's relatively poor fit for both very high and low growth industries. A possible solution to this problem is the development of a battery of structural models, each of which is crafted for a particular industry type. Hopefully, future research will determine if this avenue proves productive.

Finally, the paper exploits the model's flexibility to analyze M&A activity. We provide evidence that rivals likely benefit from a reduction in the number of competitors and are simultaneously harmed from the stronger firm a merger typically creates. Just as importantly, the model allows us to break down the estimates quantitatively as well as qualitatively. Overall, we find that for the median sample firm the gain from the reduction in the number of competitors comes to about 2.5% but that gain is largely lost due to the increased strength of the merged firm.

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Appendix Proofs

A. *The Basic Model and its Solution*

In order to find a solution to the model, it is useful to begin by guessing that firm *i*'s value function depends only on its own market share, and not on the distribution of market shares across its rivals. Combining this assumption with the fact that the problem described in Section I is time independent yields a value function for each firm *i* equal to:

$$V_i\left(m_i\right) = \int_0^\infty (\alpha_i m_i(\tau) - u_i - f_i)\, e^{-\delta\tau} d\tau \tag{23}$$

The analysis seeks a Nash equilibrium in which the players use Markovian strategies. For each firm, the instantaneous value functions given by (2) imply that in a Markovian Nash equilibrium the following Hamilton-Jacobi-Bellman (HJB) equations must hold:

$$0 = \max_{u_i} \quad \alpha_i m_i - u_i - f_i + \frac{\partial V_i}{\partial m_i}\left\{\frac{\phi\left[(1-m_i)u_i s_i - m_i \sum_{j\neq i} u_j s_j\right]}{\sum_{j=1}^{n} u_j s_j}\right\} - \delta V_i. \tag{24}$$

Letting $V_i' = \partial V_i / \partial m_i$ the first order condition for Firm *i* is:

$$V_i'\phi s_i \sum_{j\neq i} u_j s_j = \left(\sum_{j=1}^{n} u_j s_j\right)^2. \tag{25}$$

To solve for each u_i independently of the others begin by noting that the right hand side of (25) is identical across all the firms. Use this to write for each firm $k \neq i$

$$V_i' s_i \sum_{j\neq i} u_j s_j = V_k' s_k \sum_{j\neq k} u_k s_k \tag{26}$$

which can be rewritten to isolate the u_i terms as:

$$\frac{V_i' s_i}{V_k' s_k}\left(\sum_{j\neq i} u_j s_j\right) = u_i s_i + \sum_{\substack{j\neq i \\ j\neq k}} u_j s_j \tag{27}$$

for each firm *k* not equal to *i*. Summing Equation (27) across the $k \neq i$ firms yields

$$V_i' s_i \left(\sum_{j \neq i} u_j s_j \right) \sum_{j \neq i} \frac{1}{V_j' s_j} = (n-1) u_i s_i + (n-2) \sum_{j \neq i} u_j s_j \tag{28}$$

which can be used to solve for the summation of the

$$\sum_{j \neq i} u_j s_j = \left[V_i' s_i \sum_{j \neq i} \frac{1}{V_j' s_j} + 2 - n \right]^{-1} (n-1) u_i s_i. \tag{29}$$

Plugging (29) into (26) yields an equation for the u_i without the u_k terms for $k \neq i$. After some minor algebra one has from (25) for each u_i

$$u_i \left\{ 1 + (n-1) \left[2 - n + V_i' s_i \sum_{j \neq i} \frac{1}{V_j' s_j} \right]^{-1} \right\}^2 = V_i' \phi (n-1) \left[2 - n + V_i' s_i \sum_{j \neq i} \frac{1}{V_j' s_j} \right]^{-1}. \tag{30}$$

Importantly, note that the solution to the u_i in (30) does not depend on m_i so long as V_i' does not. As will be seen shortly, it is this feature of the problem that ultimately allows for a closed form solution.

The next question is whether or not the functional form guess in (4) can be used to satisfy the system of equilibrium Equations (24) after using (30) to eliminate the u_i terms. Assuming so, then $V_i' = b_i$. Using this, substitute b_i for V_i' in (24) and (30). Note that after this change, all of the terms in (24) are either constants or linear in m_i.

If values for a_i and b_i can now be found that set both groups to zero for all m_i, a solution to the problem will have been found. Collect the terms multiplying m_i and set them equal to zero to yield

$$\alpha_i - b_i \phi m_i - \delta b_i = 0 \tag{31}$$

which yields a solution for each b_i of (6). Next one can use (6) to substitute out for the b_i terms in (24) and collect the terms independent of m_i to solve for the a_i. Doing so produces the following that a_i must satisfy:

$$0 = -f_i - \frac{\phi \alpha_i (n-1)[\alpha_i s_i z - (n-1)]}{(\phi + \delta)(\alpha_i s_i z)^2} + \frac{\phi \alpha_i}{\phi + \delta} \left[\frac{\alpha_i s_i z - (n-1)}{\alpha_i s_i z} \right] - \delta a_i. \tag{32}$$

Rearranging and solving for a_i produces (5) yielding a solution to the system of equations. Q.E.D.

A.1. Solving for u_i

To solve for each firm's optimal spending on customer acquisition substitute out b_i for V_i' in (30) and then use (6) to substitute out b_i in terms of the model's parameters. This yields:

$$u_i\left\{1+(n-1)\left[2-n+\alpha_i s_i\sum_{j\neq i}\frac{1}{\alpha_i s_j}\right]^{-1}\right\}^2=\frac{\alpha_i\,(n-1)\phi}{\phi+\delta}\left[2-n+\alpha_i s_i\sum_{j\neq i}\frac{1}{\alpha_i s_j}\right]^{-1}. \quad (33)$$

Next use the relationship:

$$\alpha_i s_i\sum_{j\neq i}\frac{1}{\alpha_j s_j}=\alpha_i s_i\sum_{j=1}^{n}\frac{1}{\alpha_j s_j}-\frac{\alpha_i s_i}{\alpha_i s_i}=\alpha_i s_i\sum_{j=1}^{n}\frac{1}{\alpha_j s_j}-1=\alpha_i s_i z-1 \quad (34)$$

in (33) to yield

$$u_i\left\{1+(n-1)[1-n+\alpha_i s_i z]^{-1}\right\}^2=\frac{\alpha_i\,(n-1)\phi}{(\phi+\delta)(1-n+\alpha_i s_i z)}. \quad (35)$$

Some simple algebra then generates (8).

B. Proofs for the Propositions in the Merger and Acquisition Section

PROPOSITION 3*: Consider a merger between two identical firms ($f_1=f_2=f$, $\alpha_1=\alpha_2=a$, $m_1=m_2=m$, and $s_1=s_2=s$) in which there are no synergies. Upon the merger's completion the value of the merged firm will be higher than the sum of the standalone companies if* $\alpha sz\left(\sqrt{2}-1\right)<\sqrt{2}$ *and less if the inequality is reversed.*

Proof: If the firms are identical then the no value added conditions imply that $f_{1+2}=2f$, and $\alpha_{1+2}=\alpha$. Use

these equalities in Equation (20) to show that $s_{1+2}=s$ after recalling that if $u_i\geq 0$ then $\alpha_i s_i z-(n-1)\geq 0$ as well.

In general, if two identical firms merge then the pre-merger value minus the post-merger firm value equals: $a_{1+2} + 2b_{1+2}m$. Filling in the solutions for *a*, *b*, pre-merger and then a_{1+2}, and b_{1+2} for the post-merger firm shows that $bm=b_{1+2}m$. Thus, determining whether or not the merger adds or subtracts value reduces to determining if *a* is greater than or less than a_{1+2}. Based on (5), and after using the condition $f_{1+2}=2f$, the value added from the merger (ΔV) equals:

$$\Delta V = \frac{\sqrt{2}\left[\alpha s z - (n-1)\right]}{z} - \frac{\sqrt{2}\left[\alpha s z_{1+2} - (n-2)\right]}{z_{1+2}}, \tag{36}$$

where z_{1+2} is the post-merger value of *z*. To finally prove the proposition note that under its assumptions one can write *z* and z_{1+2} as:

$$z = \frac{1}{\alpha s} + \frac{1}{\alpha s} + \sum_{j=3}^{n} \frac{1}{\alpha_j s_j}, \quad z_{1+2} = \frac{1}{\alpha s} + \sum_{j=3}^{n} \frac{1}{\alpha_j s_j} \tag{37}$$

and thus their relationship as

$$z_{1+2} = z - \frac{1}{\alpha s}. \tag{38}$$

Using this in (36) proves the proposition. Q.E.D.

Table I: Change in the Equilibrium Spending u_i from Equation (8)			
Derivative w.r.t.	Economic Interpretation	Sign	Condition
ϕ	The impact of an increase in consumer responsiveness increases spending to acquire customers.	+	All firms.
δ	An increase in the discount rate reduces spending to acquire customers.	-	All firms.
α_i	The impact of an increase in firm profitability per unit market share on spending to acquire customers.	+	Large firms.
s_i	The impact of an increase in the attractiveness of a firm's products on spending to acquire customers.	+	Large Firms.
$(\alpha_{j\neq i} s_{j\neq i})$	The impact of an increase in the competitive strength of a rival on spending to acquire customers.	+	Large firms.

Table II: Possible Empirical Proxies for the Model's Parameters

Parameter	Description	Possible Empirical Proxies
m	Market share	Share of total industry: ▪ Sales ▪ Assets
u	Spending to gain market share	▪ Advertising ▪ R&D ▪ Capital Expenditures ▪ Coupons ▪ Loyalty Programs
ϕ, αsz	Consumer responsiveness and relative competitive strength.	▪ Estimation based on Equation (1), using equilibrium spending given in Equation (8) and equilibrium market shares in Equation (10) to obtain ϕ, $\bar{m}_i$, and αsz (see Equation 19). ▪ Estimation based on the discrete time version of Equation (1) to obtain ϕ and s: $$m_{i,t+1} - m_{i,t} = \phi\left(\frac{u_i s_i m_t - (1 - m_t)\sum_{j\neq i}^{N} u_j s_j}{\sum_{j=1}^{N} u_j s_j}\right)$$ ▪ Estimation of ϕ, α, s based on Equation (4)
A	Revenue-generating ability	▪ Operating profit ▪ Estimation based on Equation (2) or (4)
f	Costs of operations (fixed)	▪ Operating expenses (net of proxy for market share spending) ▪ Estimation based on Equation (2) or (4)
δ	*r-g*: discount rate minus industry growth rate	▪ Industry cost of capital ▪ Industry growth rate ▪ Estimation based on Equation (4)
V	Value of the firm	▪ Equity Market Capitalization + Debt

Table III Summary of Estimated Parameter Values

This table presents summary statistics for the estimated parameter values for the 12,643 firm-year observations. Estimates for industry ϕ and firm $\bar{m}_i$ are based on non-linear least squares estimation of Equation (19), which is based on the law of motion for market share *dm.* Firm i's market share*, $m_i(t)$*, is defined as the share of sales of all CRSP/Compustat firms in the industry. The initial sample consists of all 4-digit SIC codes with fewer than 20 firms for the period 1980 through 2009. Estimates are obtained for firms with market shares greater than 3%. Industries with r<|g| are excluded from the estimates. The individual firm profitability parameters (α_i) per unit market share (millions of 2007 dollars per year) are estimated via OLS estimation of $\hat{\pi}_i(t) = e^{gt}(\alpha_i m_i(t) - f_i)$, where $\hat{\pi}_i(t)$=(Revenue – Cost of Goods Sold) and e^{gt} is the ratio of period t industry sales to industry sales in the first year of the sample.

	ϕ	$\bar{m}_i$	α_i
Mean	0.438	0.215	\$2,020.0
25th Percentile	0.061	0.054	\$138.8
Median	0.192	0.132	\$577.9
75th Percentile	0.548	0.305	\$1,832.8
Std. Dev.	0.630	0.231	\$6,042

Table IV Estimated Market Share Half Lives By Two Digit SIC Industry

This table presents estimated industry ϕ and market share half-lives, by two digit SIC code using data for the years 1980-2009. The ϕ parameter is estimated for each 4 digit SIC industry using Equation (19), which is based on the law of motion for market share *dm*. The estimated values of ϕ and $\bar{m}_i$ are chosen to minimize the sum of squared errors, ε_i. Each 2-digit estimate is based on the median of the individual 4-digit industry parameter estimates. Industries with more than 20 firms or with $r<|g|$ are excluded from the estimates. Individual firms' steady state market shares, $\bar{m}_i$ are estimated but not reported. The half lives listed use the point estimates for ϕ. Based on Equation (1) the half life (h) equals $\ln(2)/\phi$. This half life represents, in years, the time it would take a firm that spends nothing on customer recruiting to lose half its current market share.

SIC Code (2-Digit)	Industry	ϕ	Half Life
1	Agricultural Production Crops	0.061	11.453
7	Agricultural Services	0.707	0.981
8	Forestry	0.010	68.087
10	Metal Mining	0.236	2.941
13	Oil And Gas Extraction	0.108	6.425
14	Mining And Quarrying Of Nonmetallic Minerals, Except Fuels	0.264	2.630
15	Building Construction General Contractors And Operative Builders	0.365	1.898
16	Heavy Construction Other Than Building Construction Contractors	0.284	2.444
17	Construction Special Trade Contractors	0.414	1.675
20	Food And Kindred Products	0.204	3.403
21	Tobacco Products	0.040	17.441
22	Textile Mill Products	0.203	3.408
23	Apparel And Other Finished Products Made From Fabrics And Similar Materials	0.153	4.517
24	Lumber And Wood Products, Except Furniture	0.079	8.745
25	Furniture And Fixtures	0.176	3.936
26	Paper And Allied Products	0.152	4.561
27	Printing, Publishing, And Allied Industries	0.097	7.175
28	Chemicals And Allied Products	0.153	4.542
29	Petroleum Refining And Related Industries	0.154	4.510
30	Rubber And Miscellaneous Plastics Products	0.225	3.087
31	Leather And Leather Products	0.122	5.693
32	Stone, Clay, Glass, And Concrete Products	0.271	2.559
33	Primary Metal Industries	0.171	4.050
34	Fabricated Metal Products, Except Machinery And Transportation Equipment	0.241	2.876
35	Industrial And Commercial Machinery And Computer Equipment	0.183	3.797
36	Electronic And Other Electrical Equipment And Components, Except Computer Equipment	0.195	3.549
37	Transportation Equipment	0.306	2.265

Table IV Estimated Market Share Half Lives By Two Digit SIC Industry Estimated Market Share Half Lives By Two Digit SIC Industry (cont'd)

SIC Code (2-Digit)	Industry	ϕ	Half Life
38	Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	0.201	3.457
39	Miscellaneous Manufacturing Industries	0.199	3.484
40	Railroad Transportation	0.143	4.861
41	Local And Suburban Transit And Interurban Highway Passenger Transportation	0.087	7.975
42	Motor Freight Transportation And Warehousing	0.454	1.528
44	Water Transportation	0.565	1.226
45	Transportation By Air	0.457	1.516
46	Pipelines, Except Natural Gas	0.008	87.833
47	Transportation Services	0.146	4.738
48	Communications	0.118	5.879
49	Electric, Gas, And Sanitary Services	0.141	4.906
50	Wholesale Trade-durable Goods	0.222	3.126
51	Wholesale Trade-non-durable Goods	0.226	3.068
52	Building Materials, Hardware, Garden Supply, And Mobile Home Dealers	0.085	8.202
53	General Merchandise Stores	0.076	9.086
54	Food Stores	0.373	1.856
55	Automotive Dealers And Gasoline Service Stations	0.073	9.448
56	Apparel And Accessory Stores	0.187	3.715
57	Home Furniture, Furnishings, And Equipment Stores	0.122	5.701
59	Miscellaneous Retail	0.079	8.738
60	Depository Institutions	0.071	9.765
61	Non-depository Credit Institutions	0.293	2.363
62	Security And Commodity Brokers, Dealers, Exchanges, And Services	0.316	2.193
63	Insurance Carriers	0.122	5.698
64	Insurance Agents, Brokers, And Service	0.104	6.683
65	Real Estate	0.615	1.127
67	Holding And Other Investment Offices	0.279	2.483
72	Personal Services	0.146	4.763
73	Business Services	0.282	2.456
75	Automotive Repair, Services, And Parking	0.146	4.742
76	Miscellaneous Repair Services	0.792	0.876
78	Motion Pictures	0.181	3.829
79	Amusement And Recreation Services	0.170	4.086
80	Health Services	0.236	2.934
82	Educational Services	0.153	4.525
83	Social Services	0.420	1.649
87	Engineering, Accounting, Research, Management, And Related Services	0.580	1.195
99	Nonclassifiable Establishments	0.104	6.685

Table V: Select Industry Parameter Estimates
Selected Industries: Estimated Consumer Responsiveness, Steady State Market Shares and Profitability, 2000

This table presents parameter estimates at the individual firm level for a sample of five industries for the year 2000 (the last year in the sample for which we have the full data for years *t* through *t*+9, used for estimation). The parameters ϕ, $\bar{m}_i$, and $\alpha_i s_i z$ used to calculate model-implied *V*(*m*) are estimated via non-linear least squares estimation of Equation (19), which is based on the law of motion for market share *dm*. Firm *i*'s steady state market share, $m_{i,t}$ is defined as the share of sales of all CRSP/Compustat firms in the industry. The individual firm profitability parameters (α_i) are estimated via OLS, based on Equation (2) in the text.

SIC	Industry Name	Company	Est. ϕ	ϕ s.e.	Est. $\bar{m}_i$	$\bar{m}_i$ s.e.	Est. α	α s.e.
2082	Malt Beverages	Anheuser Busch	0.388	0.344	0.721	0.036	7667.9	1053.5
2082	Malt Beverages	Molson Coors	0.388	0.344	0.227	0.022	2783.4	55.9
2731	Books: Pubg, Pubg & Printing	McGraw Hill Corp	0.276	0.134	0.391	0.013	2190.3	143.1
2731	Books: Pubg, Pubg & Printing	Readers Digest Inc	0.276	0.134	0.149	0.014	3142.7	847.8
2731	Books: Pubg, Pubg & Printing	Scholastic Corp	0.276	0.134	0.150	0.007	1587.49	563.6
3523	Farm Machinery and Equipment	Deere & Co	0.105	0.038	0.657	0.021	5509.1	2028.7
3523	Farm Machinery and Equipment	Toro Company	0.105	0.038	0.032	0.012	3424.7	220.1
3523	Farm Machinery and Equipment	A G C O Corp	0.105	0.038	0.252	0.030	607.5	146.8
4011	Railroads,Line-Haul Operatng	Union Pacific	0.025	0.002	0.163	0.024	17418.5	4936.0
4011	Railroads,Line-Haul Operatng	Burlington Northern Santa Fe	0.025	0.002	0.473	0.027	3099.9	2356.3
4011	Railroads,Line-Haul Operatng	C S X	0.025	0.002	0.000	0.000	2544.8	2051.5
4011	Railroads,Line-Haul Operatng	Norfolk Southern	0.025	0.002	0.215	0.020	23578.6	5867.6
5731	Radio, TV, Cons Elect Stores	Radioshack Corp	0.561	0.157	0.096	0.011	1181.9	61.5
5731	Radio, TV, Cons Elect Stores	Circuit City Stores	0.561	0.157	0.211	0.012	686.9	63.5
5731	Radio, TV, Cons Elect Stores	Best Buy Co.	0.561	0.157	0.604	0.011	324.8	35.4

Table VI Predicted Dynamics: Model-Implied V(m) and Actual Firm Values

The dependent variable is the (log) market value of the firm, defined as market capitalization of equity plus the book value of debt, in 2007 dollars, at the end of year *t*. The explanatory variables are (log) model-implied firm value (as specified in Equations 4 through 6), industry growth and the firm's share of industry sales. The parameters $\phi,\ \alpha_i s_i z,\ \alpha_i,\ and\ f_i$ used to calculate model-implied *V*(*m*) are estimated via non-linear least squares estimation of Equation (19), which is based on the law of motion for market share *dm*, and OLS estimation of the firm profitability equation, Equation 2. The δ_t parameter is estimated in two ways: industry-by-industry, and a market wide estimate. Industry δ_{It} is defined as the average (unlevered) cost of capital, minus the average 5-year sales growth rate for all firms in the 4-digit SIC code. The market-wide δ_{Mt} is defined as the long-run (1926 through period *t*) historical market risk premium plus the risk-free rate, minus the long-run GDP growth rate. Market-wide δ_t is identical for all firms. Panel A contains results of estimating the model for the sample of stable industries (industries with $r<|g|$ are excluded from the estimates). Panel B contains results of the sample selection validation exercise, in which we use all industries for which we are able to obtain estimates and introduce a *lowgrowth* dummy variable equal to 1 if $r<|g|$ and interact it with *V(m)*. We test the hypothesis that the model does a better job estimating actual values for these industries. All firm-year observations of actual and predicted market values (V_{it} and $V(m_{it})$) are pooled and the model is estimated via OLS, with standard errors clustered at the industry level.

Panel A: Model Implied V(m) and Actual Firm Values

	V(m) calculated using Industry δ_{It}			V(m) calculated using market-wide δ_{Mt}		
Intercept	5.932	2.311	2.436	6.038	2.936	3.043
t-value	*51.38*	*11.16*	*11.68*	*57.33*	*13.13*	*14.08*
Market Share	3.773		1.646	3.818		1.779
t-value	*17.03*		*8.27*	*18.48*		*8.75*
Model-Implied V(m)		0.583	0.519		0.570	0.497
t-value		*22.89*	*17.51*		*19.22*	*14.91*
R-Sq (Adj)	0.197	0.494	0.525	0.197	0.439	0.474
N	8,943	8,943	8,943	11,231	11,231	11,231

Panel B: Sample Selection Check

Intercept	5.084	3.517	3.456	5.382	3.911	3.799
t-value	*51.29*	*15.39*	*15.65*	*51.48*	*17.43*	*17.74*
Market Share	3.312		2.116	3.111		1.879
t-value	*15.32*		*9.63*	*15.81*		*9.55*
Model-Implied V(m)		0.367	0.3023		0.364	0.312
t-value		*10.64*	*8.13*		*11.02*	*8.94*
Lowgrowth	0.848	-1.206	-1.020	0.656	-0.975	-0.757
t-value	*6.72*	*-5.02*	*-4.29*	*6.05*	*-3.95*	*-3.20*
Lowgrowth* Share	0.461		-0.471	0.707		-0.100
t-value	*1.67*		*-1.84*	*3.13*		*-0.47*
Lowgrowth*V(m)		0.217	0.217		0.207	0.184
t-value		*6.14*	*5.47*		*5.98*	*4.96*
R-Sq (Adj)	0.241	0.474	0.510	0.219	0.421	0.457
N	12,417	12,417	12,417	16,194	16,194	16,194

Table VII: Forecasting Market Share Changes

This table presents results of predictive regressions in which 3- and 5- year changes in market share are regressed on model-implied market share changes. Summary statistics are given in Panel A. $m_i(0)$ is the initial market share of firm *i*. $m_i(t)$ is the *t*-year ahead market share. $\bar{m}_i$ is steady state market share and ϕ is the consumer responsiveness parameter, both estimated using Equation (19). $\Delta m_i(0,t)$ is the change in market share from the current year *0* to year *t*. $Pred\_\Delta m_i(0,t)$ is defined as $(\bar{m}_i - m_i(0)(1-e^{-\phi t})$ and is obtained by subtracting $m_i(0)$ from Equation (19). OLS regression results are given in Panel B. All standard errors are clustered by industry. *HHI, Change_HHI* and *Num_Competitors* are added in the extended specification. *HHI* is the industry Herfindahl-Hirschman Index. *Change_HHI* is defined as HHI_t-HHI_{t-1}. *Num_Competitors* is the natural log of the number of firms in the industry. *Rank* is the rank of the variable in model selection based on the Schwartz Bayesian Information Criterion. *F value* is the F statistic for variable inclusion.

Panel A: Summary Statistics

	Mean	Std. Dev.	25th Pctl.	50th Pctl.	75th Pctl.
3-Year Horizon (N=4517)					
$m_i(0)$	0.264	0.224	0.092	0.183	0.378
$m_i(3)$	0.267	0.234	0.087	0.184	0.385
$\bar{m}_i$	0.219	0.226	0.060	0.135	0.308
$\Delta m_i(0,3)$	0.003	0.097	-0.029	-0.002	0.030
$Pred\_\Delta m_i(0,3)$	-0.025	0.081	-0.029	-0.007	0.006
5-Year Horizon (N=3,871)					
$m_i(0)$	0.266	0.224	0.093	0.187	0.384
$m_i(5)$	0.273	0.238	0.088	0.189	0.399
$\bar{m}_i$	0.223	0.227	0.062	0.137	0.313
$\Delta m_i(0,5)$	0.011	0.126	-0.037	-0.001	0.045
$Pred\_\Delta m_i(0,5)$	-0.028	0.088	-0.039	-0.009	0.009

Panel B: Regression Results

Dependent Variable= $\Delta m_i(0,3)$

	Coefficient	t-stat	Coefficient	t-stat	*Rank*	*F-Value*
Intercept	0.005	2.24	0.050	2.87		
$Pred\_\Delta m_i(0,3)$	0.100	2.53	0.088	2.06	1	18.76
HHI			-0.048	-2.43	Exclude	
Change_HHI			0.127	3.37	2	15.91
NumCompetitors			-0.013	-2.34	Exclude	
R-Square	0.007		0.013			

Dependent Variable= $\Delta m_i(0,5)$

	Coefficient	t-stat	Coefficient	t-stat	*Rank*	*F-Value*
Intercept	0.013	3.22	0.090	3.13		
$Pred\_\Delta m_i(0,5)$	0.208	3.98	0.183	3.13	1	60.34
HHI			-0.094	-3.04	2	14.72
Change_HHI			0.175	3.39	3	19.98
NumCompetitors			-0.022	-2.27	4	18.83
R-Square	0.021		*0.030*			

Table VIII: Stochastic Market Share Model

This table presents results of regressions in which 3- and 5- year changes in market share are regressed on model-implied market share changes using the stochastic market share model from Section I. $\Delta m_i(0,t)$ is the change in market share from the current year 0 to year *t*. $Pred\_\Delta m_i(0,t)$ is defined as $((\overline{m}_i - m_i(0))(1-e^{-\phi t})$ and is obtained by subtracting $m_i(0)$ from Equation (19). *HHI, Change_HHI* and *Num_Competitors* are added in the extended specification. *HHI* is the industry Herfindahl-Hirschman Index. *Change_HHI* is defined as HHI_t-HHI_{t-1}. *Num_Competitors* is the natural log of the number of firms in the industry. OLS coefficients are estimated using ordinary least squares. GLS coefficients are estimated using weighted least squares, where the weight matrix is from Equation (16). All standard errors are estimated based on the variance-covariance matrix specified in (16).

Dependent Variable= $\Delta m_i(0,3)$ **= 4,517**

	OLS Coefficients				GLS Coefficients			
	Coefficient	t-stat	Coefficient	t-stat	Coefficient	t-stat	Coefficient	t-stat
Intercept	0.005	4.47	0.050	4.98	0.000	-0.04	0.001	0.07
$Pred\_\Delta m_i(0,3)$	0.099	5.42	0.088	3.61	0.120	6.18	0.124	4.97
HHI			-0.048	-5.27			0.003	0.29
Change_HHI			0.127	5.11			0.005	0.19
NumCompetitors			-0.013	-3.97			-0.001	-0.31
R-Square	0.007		0.013		0.003		-0.003	

Dependent Variable= $\Delta m_i(0,5)$ **N=3,871**

	OLS Coefficients				GLS Coefficients			
	Coefficient (OLS)	t-stat	Coefficient (OLS)	t-stat	Coefficient (GLS)	t-stat	Coefficient (GLS)	t-stat
Intercept	0.013	7.52	0.090	6.39	0.001	0.42	0.002	0.16
$Pred\_\Delta m_i(0,5)$	0.208	7.64	0.183	5.61	0.173	6.35	0.170	5.34
HHI			-0.094	-7.46			-0.003	-0.23
Change_HHI			0.175	4.27			-0.037	-0.91
NumCompetitors			-0.022	-4.67			0.000	-0.04
R-Square	0.021		0.030		0.013		0.007	

Table IX Selected Industries: Estimated Values of Opportunities to Improve α_i, s_i and f_i

This table presents the estimated values of investments in technologies to improve spending effectiveness, profitability per unit market share and fixed costs (s, α and f, respectively) for firms in a sample of five industries for the year 2000 (the last year in the sample for which we have the for all years from t through t+9, used for estimation). Value change, as a percentage of current firm value, is calculated as the mean change in value of all firms with greater than 3% market share in each industry. These calculations hold constant the effectiveness, profitability and fixed costs of rivals.

	Malt Beverages	**Books: Publishing and Printing**	**Farm Machinery and Equipment**	**Railroads – Line-Haul Operating**	**Radio, TV, Cons Elect Stores**
SIC Code	2082	2731	3523	4011	5731
ϕ	0.388	0.275	0.105	0.025	0.561
Mean change in value due to:					
10% improvement in:					
S	29.7%	29.1%	33.9%	15.5%	29.9%
α	63.6%	41.3%	75.7%	48.2%	32.4%
F	22.7%	6.0%	32.6%	27.9%	28.6%
25% improvement in:					
S	71.8%	78.5%	105.7%	40.4%	74.2%
α	167.0%	121.3%	228.0%	128.3%	81.2%
F	56.7%	15.1%	81.6%	69.7%	71.6%

Table X: Mergers and Acquisitions (Low Growth Industries)

This table presents the actual and predicted announcement returns of 183 industry rivals (141 unique rivals) near 66 M&A events. Panel A shows summary statistics, where Market-Adjusted returns are mean rivals' announcement returns minus the CRSP value-weighted returns, unlevered to reflect differences in capital structure. Beta-Adjusted returns are based on a market model where beta is calculated using monthly return data during the past 60 months (also unlevered). All CARs are calculated over a three day window from day -1 to +1 relative to announcement date 0. Predicted returns are calculated as the model-implied value changes, resulting from the change in the number of competitors from n_{pm} to $n_{pre}-1$ and the estimated change in the industry's competitive intensity, z. Panel A displays several summary statistics describing the M&A data. The estimated parameters in the Panel B come from the model Equations (21) and (22). In Panel C the realized returns are regressed against the predicted returns. Mean parameter estimates are in black (variable name row). Values in dark shaded cells (top row) are bootstrapped percentiles where data is drawn with replacement on a firm-by-firm basis. Values in light shaded cells (bottom row) are bootstrapped percentiles where industries are drawn with replacement. Firm specific parameters for year t are estimated using Equation (19) and 10 years of historical sales data, from year *t-9* through year t.

	Panel A: Summary Statistics									
	Market Adjusted Returns					Beta-Adjusted Returns				
Mean	0.618%					0.634%				
Median	0.466%					0.601%				
S.D.	4.198%					4.047%				
Obs.	183					183				
	Panel B: Nonlinear Least Squares Model Estimates									
Const.	1.0144					1.0148				
	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	1.0109	1.0122	1.0157	1.0193	1.0212	1.0111	1.0124	1.0159	1.0195	1.0212
	1.0096	1.0117	1.0156	1.0197	1.022	1.0102	1.0121	1.0159	1.0198	1.022
$1/n_{pre}$	-1.2572					-1.2624				
	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	-1.3417	-1.3185	-1.2739	-1.2355	-1.2193	-1.3413	-1.3193	-1.2758	-1.2389	-1.2235
	-1.3526	-1.3236	-1.2739	-1.2291	-1.2033	-1.3499	-1.3226	-1.2763	-1.2341	-1.2122
	Panel C: Bootstrapped Results from OLS Regression of Actual Against Model Returns									
Const.	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	-0.00427	-0.00215	0.002725	0.007837	0.010095	-0.00388	-0.00192	0.002882	0.007797	0.009912
	-0.00608	-0.00346	0.00271	0.009198	0.011845	-0.00538	-0.00293	0.00286	0.009095	0.011514
r_{model}	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	0.34182	0.59013	0.94109	1.2215	1.4542	0.27471	0.53561	0.91683	1.1023	1.2674
	0.26372	0.5837	0.94703	1.2614	1.54	0.1989	0.49936	0.92138	1.1246	1.2933
R^2	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	0.003836	0.013245	0.087526	0.22636	0.28739	0.002769	0.011312	0.093134	0.2475	0.31025
	0.002183	0.012132	0.088337	0.24124	0.31585	0.001538	0.009691	0.095339	0.26207	0.33519

Table XI: Mergers and Acquisitions, Rivals' Announcement Period Returns

This table presents results of regressions of rivals' actual announcement window returns on model-implied returns near the sample of M&A events. Market adjusted CARs are calculated over a three day window from day -1 to +1 relative to announcement date 0. Predicted returns, r_{model}, are calculated as the model-implied value changes resulting from the change in the number of competitors from n_{pm} to $n_{pre}-1$ and the estimated change in the industry's competitive intensity, *z*. *HHI, Change_HHI* and *Num_Competitors* are added in the extended specification. *HHI* is the industry Herfindahl-Hirschman Index. *Change_HHI* is defined as HHI_t-HHI_{t-1}. *Num_Competitors* is the natural log of the number of firms in the industry. The regressions are estimated via OLS, with standard errors clustered by industry. *Rank* is the rank of the variable in model selection based on the Schwartz Bayesian Information Criterion. *F value* is the *F* statistic for variable inclusion.

Dependent Variable=Rival CAR (-1,1)

	Coefficient	*t-stat*	*Coefficient*	*t-stat*	*Rank*	*F-Value*
Intercept	0.003	0.62	0.051	1.45		
r_{model}	0.941	3.74	0.959	3.71	1	18.76
HHI			-0.051	-1.76	Exclude	
Change_HHI			0.029	0.30	Exclude	
NumCompetitors			-0.015	-1.31	Exclude	
Adj. R-Square	0.090		0.089			

[1] There is a literature on finance and product market interactions; however, most models focus on the strategic implications of leverage (see e.g., Brander and Lewis (1986); Maksimovic (1988); Bolton and Scharfstein (1990); Hennessy and Livdan (2008); and the survey by Maksimovic (1990)). On the empirical side of the literature, Chevalier (1995) and Leach, Moyen, and Yang (2006) provide evidence on the interaction between leverage and corporate behavior, but reach opposite conclusions. It may be that as yet unmodeled industry characteristics influence the degree to which the predictions in Brander and Lewis (1986) and Maksimovic (1988) are borne out in the data.

[2]Further intuition can be garnered by looking at ϕ's extreme values. Setting it to zero implies consumers never switch firms. The current market shares are thus forever frozen in place. At the other end, as ϕ goes to infinity, customers instantly switch vendors and do so en masse at the drop of a coupon.

[3] Section I.C.1.b goes into greater detail and contains examples highlighting the half life statistic's implications while Section I.C.1.a contains estimates of its value. If one dislikes the model's behavior when u_i equals zero adding a fixed constant to Equation (1)'s denominator will eliminate it. However, this comes at the cost of having a half life without further assumptions.

[4] In general for a starting value of π(0) in Equation (2), multiply each equality by that amount. Since this has no impact on the model's equilibrium conditions the π(0) are suppressed to reduce the notational burden.

[5] We focus on pure strategies here. However, given the intuition that mixed strategies might allow firms to collude to limit wasteful spending, we have examined a two-firm version of the model with this property. In it, one firm receives an unanticipated (privately observed) positive shock to its profitability. In that setting, there exists an equilibrium in which spending on market share acquisition does not increase to reflect the positive shock. In this equilibrium, both firms earn more than they would in a full information environment.

[6] There are, of course, boundary conditions under which the solutions given here will not hold.

[7] Notice that as n goes to infinity the industry in this model becomes perfectly competitive. For the inequality to hold in the limit, one needs $\alpha_i s_i \overline{z} > 1$ for each firm i. But this can only be true if every "below average" firm is driven out, leaving a set of equally strong competitors. This, of course, conforms to the usual microeconomic view of what should happen in such industries.

[8] It is easy to generate examples where there are firms in the industry for which the comparative static goes both ways. Consider an industry with four firms. Three have values of $\alpha_i s_i$=1 and one that has a value equal to 2. In this case z=3.5. For the low $\alpha_i s_i$ firms $\alpha_i s_i z$=3.5>3 so they compete for market share, but since $\alpha_i s_i z$=3.5<6 for them $\partial u_i / \partial s_i < 0$. For the high $\alpha_i s_i$ firm $\alpha_i s_i z$=7>6 so it has the opposite reaction to its weaker rivals $\partial u_i / \partial s_i > 0$.

[9] A more extensive discussion of this general property relating when the deterministic and stochastic problems have identical solutions can be found in section 8.2 of Dockner et al. (2000).

[10] The main model assumes no exit; however, due to changing product mix and SIC code re-classification for some firms, we adjust for "exit" in the data by assuming that each firm in the industry gains market from the exiting firm, in proportion to its current market share. We do not need to adjust for entry because data filtering requires that all firms are in the sample at the beginning of rolling interval t.

[11] Data for estimation are from 1980 to 2009. Estimates end in 2004, given the data requirement of greater than 5 annual observations in the estimation interval t through $t+9$.

[12]The variables ϕ and α_{it} are constrained to be greater than or equal to zero, consistent with the model.

[13] The current mean and median market shares of sample firms are 19.7 and 12.1%, respectively, which suggests modest future growth among larger firms in the industry.

[14] The δ parameter is assumed to be positive in the model, but estimated δ's are sometimes negative. This can occur especially during high growth periods (for which the model is less appropriate). We exclude industry-years in which we observe negative values of δ. This reduces the sample size to 8,943 firm-year observations when industry-level δ_{It} is used, and to 11,231 observations when the market-wide definition of δ_{Mt} is used.

[15] While not reported in the tables, this explanation is bolstered by the fact that while the model-implied and actual market valuations are similar in magnitude across firms, the actual market values are less volatile. The mean (log) actual value equals 6.92 (i.e., value of approximately $1 billion) while the mean (log) model-implied value is 6.79 (i.e., value of approximately $890 million), a difference of only about 2%. In comparison, the actual market standard deviation is 1.83 while the model predicts a value of 2.23 a difference of about 18%.

[16] *R*-squares are shown in Table 8 for both OLS and GLS regressions. In the case of GLS, the coefficients and *t*-statistics are most relevant since *r*-squares are not bound between zero and one with GLS estimation.

[17] We thank the editor for encouraging this line of discussion.

[18] The value 1.41 is $\sqrt{2}$ rounded to two decimal points.

[19] One can see the importance of limiting the model's use to industries where its assumptions are likely to hold by examining horizontal mergers in industries that are likely to be poorly described by an oligopoly model like the one developed here. Using the same methodology to create Table XI but using data restricted to industries with 30 or more firms (rather than 20 or less, as in the current analysis), we find that the model displays almost no predictive power. This shows the importance of applying a structural model to conditions where the mathematical assumptions correspond reasonably well with the industry the researcher wishes to examine. While this limits a structural model's use in the cross section, it hopefully makes up for this shortcoming with improved insights into those industries to which it is applicable. To save space, the results of the analysis are tabulated in the Internet Appendix (Table IA.II).

[20] For example, in the market adjusted case this sets the constant to 1.0144, and the $1/n$ parameter to -1.2572.

Internet Appendix for “Dynamic Competition, Valuation and Merger Activity”[*]

Internet Appendix Table IA.I: Predicted Dynamics under Alternative Industry Definition (6 Digit NAICS)
This table supplements Table XI in the text. The dependent variable is the (log) market value of the firm, defined as equity market capitalization plus the book value of debt, in 2007 dollars, at the end of year *t*. The explanatory variables are (log) model-implied firm value, industry growth and the firm’s share of industry sales. The parameters $\phi,\ \alpha_i s_i z,\ \alpha_i,\ and\ \ f_i$ used to calculate model-implied V(m) are estimated via non-linear least squares estimation of Equation (19), which is based on the law of motion for market share *dm*, and OLS estimation of the firm profitability equation, Equation 2. Industry δ_{It} is defined as the average (unlevered) cost of capital, minus the average 5-year sales growth rate for all firms in the 6-digit NAICS code. The market-wide δ_{Mt} is defined as the historical market risk premium plus the risk-free rate, minus the long-run GDP growth rate and is identical for all firms. Panel A contains results of estimating the model for the sample of stable industries (industries with $r<|g|$ are excluded). Panel B contains results of the sample selection validation exercise, in which we use all industries for which we are able to obtain estimates and introduce a *lowgrowth* dummy variable equal to 1 if $r<|g|$ and interact it with *V(m)*. We test the hypothesis that the model does a better job estimating actual values for these industries. All firm-year observations of actual and predicted market values (V_{it} and $V(m_{it})$) are pooled. The model is estimated via OLS, with standard errors clustered at the industry level.

Panel A: Model Implied *V*(*m*) and Actual Firm Values

	V(m) calculated using Industry δ_{It}			V(m) calculated using market-wide δ_{Mt}		
Intercept	6.252	3.456	3.382	6.407	4.040	3.955
t-value	*38.94*	*11.41*	*11.48*	*44.10*	*12.61*	*12.85*
Market Share	2.978		1.733	2.951		1.650
t-value	*10.29*		*6.14*	*11.46*		*6.32*
Model-Implied V(m)		0.467	0.417		0.459	0.407
t-value		*12.49*	*9.54*		*10.74*	*8.52*
R-Sq (Adj)	0.149	0.380	0.426	0.144	0.354	0.395
N	3,432	3,432	3,432	4,648	4,648	4,648

Panel B: Sample Selection Check

Intercept	5.643	3.195	2.966	5.840	3.640	3.411
t-value	*36.37*	*11.84*	*11.51*	*40.13*	*13.61*	*13.29*
Market Share	2.17		1.380	2.053		1.284
t-value	*7.41*		*5.90*	*7.64*		*5.90*
Model-Implied V(m)		0.460	0.438		0.461	0.441
t-value		*12.40*	*10.89*		*11.67*	*10.57*
Lowgrowth	0.609	0.262	0.280	0.567	0.400	0.544
t-value	*3.96*	*1.29*	*1.35*	*4.27*	*1.95*	*2.77*
Lowgrowth* Share	0.799		0.008	0.898		0.367
t-value	*2.46*		*0.03*	*3.29*		*1.53*
Lowgrowth*V(m)		0.006	-0.021		-0.05	-0.134
t-value		*0.23*	*-.067*		*0.56*	*-1.09*
R-Sq (Adj)	0.158	0.409	0.445	0.150	0.389	0.421
N	6,224	6,224	6,224	8,230	8,230	8,230

[*] Spiegel, Matthew and Heather Tookes, 2011, Internet Appendix to “Dynamic Competition, Valuation and Merger Activity,” Journal of Finance, forthcoming, http://www.afajof.org/IA/[year].asp. Please note: Wiley-Blackwell is not responsible for the content or functionality of any supporting information supplied by the authors. Any queries (other than missing material) should be directed to the authors of the article.

Internet Appendix Table IA.II: Mergers and Acquisitions (Low Growth Industries with 30+ Firms)

This table presents the actual and predicted announcement returns of 209 industry rivals (119 unique rivals) near 65 M&A events. Panel A shows summary statistics, where Market-Adjusted returns are mean rivals' announcement returns minus the CRSP value-weighted returns, unlevered to reflect differences in capital structure. Beta-Adjusted returns are based on a market model where beta is calculated using monthly return data during the past 60 months (also unlevered). All CARs are calculated over a three day window from day -1 to +1 relative to announcement date 0. Predicted returns are calculated as the model-implied value changes, resulting from the change in the number of competitors from n_{pm} to $n_{pre}-1$ and the estimated change in the industry's competitive intensity, *z*. Panel A displays several summary statistics describing the M&A data. The estimated parameters in the Panel B come from the model Equations (21) and (22). In Panel C the realized returns are regressed against the predicted returns. Mean parameter estimates are in black (variable name row). Values in dark shaded cells (top row) are bootstrapped percentiles where data is drawn with replacement on a firm-by-firm basis. Values in light shaded cells (bottom row) are bootstrapped percentiles where industries are drawn with replacement. Firm specific parameters for year *t* are estimated using Equation (19) and 10 years of historical sales data, from year *t-9* through year *t*.

	Panel A: Summary Statistics									
	Market Adjusted Returns					Beta-Adjusted Returns				
Mean	0.159%					0.249%				
Median	-0.528%					-0.210%				
S.D.	4.193%					4.032%				
Obs.	209					209				
	Panel B: Nonlinear Least Squares Model Estimates									
Const.	0.9993					0.9984				
	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	0.9969	0.9975	0.9994	1.0044	1.0064	0.9954	0.9966	0.9984	1.0021	1.0039
	0.9971	0.9976	0.9995	1.0040	1.0059	0.9960	0.9969	0.9984	1.0018	1.0034
$1/n_{pre}$	-1.0012					-0.9421				
	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	-1.3363	-1.2410	-1.0117	-0.8735	-0.8252	-1.2109	-1.1284	-0.9475	-0.8192	-0.7540
	-1.3259	-1.2296	-1.0151	-0.8867	-0.8417	-1.2003	-1.1151	-0.9507	-0.8334	-0.7824
	Panel C: Bootstrapped Results from OLS Regression of Actual Against Model Returns									
Const.	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	-0.0058	-0.0038	0.0013	0.0070	0.0097	-0.0034	-0.0018	0.0022	0.0066	0.0087
	-0.0056	-0.0036	0.0012	0.0064	0.0089	-0.0040	-0.0022	0.0022	0.0068	0.0092
r_{model}	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	-3.8143	-0.5118	0.8500	1.3524	2.4013	-2.6207	-0.2930	0.9001	1.3317	2.2731
	-3.9158	-0.4863	0.9063	1.5963	2.5990	-2.1816	-0.1677	0.9170	1.5014	2.8352
R^2	1%	5%	50%	95%	99%	1%	5%	50%	95%	99%
	0.0000	0.0001	0.0051	0.0293	0.0463	0.0000	0.0001	0.0080	0.0393	0.0577
	0.0000	0.0001	0.0062	0.0285	0.0428	0.0000	0.0002	0.0087	0.0379	0.0566

Internet Appendix Dataset (Excel File)
This dataset supplements Table V in the text. It contains all of the estimated parameters shown in Table V, but includes all firms and years in the full sample (i.e., firms with market shares greater than 3% and that operate in low growth industries with fewer than 20 firms). The fields in the Excel file are as follows:

1. Year: Beginning of estimation window t.
2. SIC Code: 4-digit Compustat SIC Code
3. Permno: CRSP permanent number (Permno)
4. CoName: Company name, as listed in the CRSP/Compustat Database
5. Est. ϕ : Estimated customer disloyalty, ϕ , based on non-linear least squares estimation of Equation (19) in the text.
6. ϕ s.e.: Standard error of estimated ϕ .
7. Est. $\overline{m}_i$: Estimated steady state market share, $\overline{m}_i$, based on non-linear least squares estimation of Equation (19) in the text.
8. $\overline{m}_i$ s.e.: Standard error of estimated $\overline{m}_i$.
9. Est. α_i: Estimated firm profitability per unit market share (in millions of 2007 dollars per year). Estimated via OLS estimation of $\hat{\pi}_i(t) = e^{gt}(\alpha_i m_i(t) - f_i)$, where $\hat{\pi}_i(t)$=(Revenue – Cost of Goods Sold) and e^{gt} is the ratio of period t industry sales to industry sales in the first year of the sample.
10. α_i s.e.: Standard Error of estimated α_i are obtained for firms with market shares greater than 3%.

The estimation includes the following constraints: ϕ is between 0 and 25; $\overline{m}_i$ and α_i are non-negative. Standard errors equal to zero indicate corner solutions.