YALE
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Yale ICF Working Paper No. 04-03
June 2004

AFFECTIVE DECISION MAKING IN INSURANCE MARKETS

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Affective Decision Making in Insurance Markets*

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Previous Version: December 2003
This Version: June 2004

*I thank Donald Brown and Leeat Yariv for immense discussions, feedback and encouragements. I also thank David Armor, Renzo Comolli, Dino Gerardi, Stephen Morris, Ben Polak, Sergio Turner, and seminar participants at Yale University for helpful comments.

Abstract: This paper suggests incorporating affective considerations into decision making theory and insurance decision in particular. I describe a decision maker with two internal "accounts" – the rational account and the mental account. The rational account decides on insurance to maximize expected (perceived) utility, while the mental account chooses risk perceptions which then affect the perceived expected utility. The two accounts interact to reach a decision which is composed of both risk perception and insurance level. The model is based on psychology research and shows interesting results for the insurance markets. Also, this framework helps to distinguish between report and choice tasks.

JEL classification numbers: D14, D80,D81,G22,Z00

Keywords: Insurance, risk perception, motivated reasoning and dual-processes.

Introduction

With von Neumann and Morgenstern's (1944) axiomatization of expected utility using objective probabilities, expected utility became the main framework for analyzing decision-making under conditions of risk and uncertainty. However, ample empirical and experimental studies (e.g., Allais,(1953); Grether and Plott,(1979)) question the descriptive adequacy of expected utility. That has engendered other decision theories, such as prospect theory (Kahneman and Tversky, (1979)), expected utility with rank dependent probability weight (for review see Camerer, (1995)), mixed fanning[1] and more. The new theories, trying to improve on expected utility and focusing on subsets of anomalies, depart from the expected utility paradigm by introducing either a weight function[2], a different shape of the utility function, or both. However, even in introducing a weight function, these theories assume that (1) assigned weights are a given function of the known true probability distribution and (2) the probability/weighting function is independent of payoffs[3]. This paper agrees that the decision maker indeed holds and work with systematically biased (subjective) probability distribution[4]. However, this paper suggests endogenizing the process of forming probability judgement (thereafter perceived probability) and argue, as suggested by Edwards (1955, 1961) and Irwin (1953) , that in the mind of the economic agent perceived probability and payoff are not independent. Following Georgescu-Roegen in 1958[5], this could explain anomalies that are difficult to explain by expected utility and its extensions. Therefore, this paper suggests a new decision theory that extend on expected utility theory in a new way, and does so by focusing on the insurance markets.

In insurance decision-making the relevant probability weight is personal risk – the likelihood of the agent to be in a bad state. As mentioned above, the claim is that the agent works with a perceived, rather than objective, risk; moreover the process of forming risk perception is endogenous and depends upon outcomes (for an excellent review of research on risk perception see Slovic, (2000)). To characterize the process of forming risk perception, section 1 reviews psychology research which suggests that the agent act as if she *chooses*[6] her risk perception. More specifically, the agent selects an optimal risk perception to balance two contradicting forces: (1) the desire to hold favorable personal risk perception (optimism) and (2) a taste for accuracy (reality constraint) which is a function of the agent's base rate[7]. This is obviously an abstraction of a much more complex process; however, I believe it captures the essence of it. This process is labeled the "mental account" and its outcome is the perception of risk for a given payoff structure and a base rate. This is one aspect of the decision process; using the perception of risk, the agent needs to decide on her optimal insurance purchase. The insurance decision is made under the process labeled "rational account". The rational account is assumed to take as inputs the perception of risk and payoff function, and then make a decision according to expected utility theory. Note, however, that the insurance decision does change the payoff structure and thus affects the perception of risk. Hence, the decision maker is described as an agent with two accounts – the mental account and the rational account – which play against each other in an intrapersonal game. Consistency of the two accounts is characterized by the pure strategy Nash equilibria of the intrapersonal game. In other words, the agent's decision is modeled as a decision of both insurance and risk perception, which is a Nash equilibrium in a simultaneous move game between her two selves - the mental and the rational.

The main conclusions arising from this model are that (1) affective considerations generally lead to multiple equilibria implying that people with the same information can have different beliefs on their risk and take different actions (2) choice set consists of two types of equilibria, one of which suggests that educating the public about its higher-than-perceived-risk can lead to a counter intuitive result of less insurance (3) observing greater insurance purchasing as income increases does not necessarily contradict the Decreasing Absolute Risk Aversion (DARA) hypothesis and (4) the insurance premium affects risk perceptions. Also, I show that this model is consistent with stylized fact in the insurance context; moreover its framework, once applied to other contexts, is consistent with stylized facts outside the insurance markets.

The remainder of this paper is organized as follows: section 1 summarizes the psychology and economic literature and argues that the process underlying the risk perception choice is as described above. Section 2 gives the setting of the rational and mental accounts. Section 3 proves the existence of a pure strategy Nash equilibrium for the intrapersonal game and provides comparative statics to understand the impact of changes in income, insurance premium and base rate on the individual's insurance decision. Section 4 discusses both general and specific implication of the model both in and outside the context of insurance. For example, it discusses the relationship between absolute risk aversion, income and insurance choice as well as the cautious optimism phenomenon. Section 5 concludes.

1 Related Literature

1.1 Risk Perception

In order to characterize the thinking process of forming risk perception, one has to turn to the psychology literature. The first distinction the psychology literature suggests is between perceived *personal* risk and perceived *societal* risk. These two are fundamentally different; one implication is, as Tyler and Cook's (1984) research indicates, that factors influencing perceived societal risk do not necessarily influence perceived personal risk. More specifically they show that mass media information about risk will influence risk judgment on societal level but not on a personal level. Since this paper is concerned with the choice behavior of a single decision maker, it is important to understand the forces underlying perceived personal risk. Casual observations show that most people exhibit the "this is not going to happen to me" phenomenon. More scientifically put, most people, according to psychological experiments, believe that they are less likely than the average to be injured in a car accident or be involved in other bad experiences, but more likely than others to experience a positive event such as living longer, having a healthy life, and being successfully employed (Weinstein, (1980); Armor and Taylor, (2002)). In other words, on average, people tend to be unrealistically optimistic. Psychologists explain optimistic predictions as a way to achieve emotional and motivational goals such as good mood and self-confidence. In fact, Taylor and Brown (1988) show that an overly positive view of the future is correlated with normal mental health.

The literature on motivated cognition (e.g., Kunda, (1990)) suggests that confirmatory bias, optimism, cognitive dissonance, self-esteem and many other well-recorded psychological phenomena are due to individuals extracting utility from beliefs per se. That is, people have preferences over beliefs: people *want* to believe they are better than average, less likely to be ill, unemployed or unhappy. Differently put, the psychology literature suggests that people act as if they *choose* their beliefs. It is interesting to note that optimism bias is not exclusive to cases where an outcome is correlated with personal abilities; experiments show that subjects' prediction of what will occur is highly correlated with what they would like to see happen, rather than with what is objectively likely to happen (Irwin, (1953)). This is true even in events that are totally random (Taylor and Brown, (1988) and references within).

Some of the biases that are said to be explained by motivated cognition have been recognized in economic studies. To mention a few, Akerlof and Dickens (1982) examine the consequences of cognitive dissonance for workers in a hazardous profession. Rabin and Schrag (1999) suggest confirmatory bias model and show that it leads to overconfidence. They then examine the impact of this bias on agents' belief. Economists have also recognized the value of beliefs per se. For example, Benaboù and Tirole (2001) study the value agents put on self-confidence – belief about the self. They argue that people get a positive utility from favorable beliefs for three main reasons: consumption value – people simply like to have favorable beliefs about the self – signaling value – if you perceive yourself as better than you really are, other people will tend to believe it as well – and motivation – people like to have a certain belief in order to motivate themselves to complete a task. Furthermore, Koszĕgi (2000) examines the impact of ego utility, i.e., an agent that derives utility from positive beliefs about herself, on behavior and information acquisition. Yariv (2001) presents an axiomatic foundation for belief dependent utility functionals and Brunnermeier and Parker (2002) allow the agent to choose beliefs (optimal beliefs) to maximize total well-being over time, recognizing the effect of beliefs on future decisions and thus utility.

In the context of risk perception, motivated reasoning means that individuals are choosing their personal risk perception and the desire of holding favorable beliefs guides them in that process. i.e., in choosing risk perceptions individuals are optimistically biased. However, if motivation were the only driving force in the process of forming beliefs in general, and risk perception in particular, then people would generally hold arbitrary beliefs, which is not the case. Kunda (1990) argues that motivated cognition is restricted by personal experience, prior belief, knowledge or, in general, reality. Differently put, individuals hold the most favorable risk perceptions that they can justify, i.e., are reasonable.

In experiments regarding probability judgment people are shown to use heuristics such as anchoring[8] and adjustment (Tversky and Kahneman, (1974); Shiller, (2000)), representativeness and availability to form their beliefs. These heuristics are related to motivated reasoning where agents are argued to balance motivation and taste for accuracy. To see that note that motivated reasoning can be rephrased in terms of anchoring and adjustment: we are anchored to motivated beliefs and adjust to reality as we perceive it (base rate)[9]. In fact, Kunda (1990) herself acknowledge that these two might not be distinguishable and further research is needed. Representativeness and availability could be seen as the cognitive strategy one employs in order to justify her motivated belief.

The interesting point that arises from this discussion is that having a taste for accuracy – choosing beliefs that are close to our base rate – is no guarantee for being accurate! This is especially interesting since there is mixed evidence of both over- and under-confidence (Mellers et al, (1998)) measured relative to the objective probability. Thus mixed evidence could be explained by balancing accuracy and motivation with the wrong base rate. In fact, Armor and Taylor (2002) argue this is indeed the reason to the lack of evidence for a punishing economic and psychological results of optimism bias. Related and possibly an alternative explanation for lack of punishing results is that people tend to have a higher taste for subjective accuracy as their prediction is more likely to be tested or challenged. This is important in the context of insurance decisions, as the prediction of risk could bear high personal consequences which leads one to become more accurate. Hence, this supports the hypothesis that in forming risk perception to make an insurance decision people will balance optimism with accuracy.

Indeed research on risk perception find optimism bias to be one of the sources in forming risk perception (Slovic et al, (1982)). Moreover, taste for accuracy can be viewed as the emotion of fear of being overly optimistic; the further away perception is from the agent's base rate the higher is her fear. Emotions such as fear then guide her in adjusting perception closer to her base rate. This is in accordance with the psychology literature where emotions are believed to influence decisions in general (Loewenstein et al, (2001)) and in particular negative emotions influence perceived risk (Johnson and Tversky, (1983); Shafir and LeBoeuf, (2002)), with fear increasing it (Lerner and

Ketler, (2000) ; Lerner et al, (2003)). Negative emotions such as regret and fear affect decisions because people will attempt to avoid or minimize those emotions in the future (Bell, (1982) ; Mellers et al, (1998); Shafir and LeBoeuf, (2002)).

In sum, psychology and economic research suggest that people have preference over, and extract utility from, beliefs. This motivates agents to choose favorable beliefs, which is balanced against accuracy goals or fear of excess optimism. Therefore, this is captured (see section 2.2) by introducing a "mental profit" from beliefs. Mental profit is composed of a gain that increases as beliefs tend more towards the best (most desirable) outcome, and cost that increases as beliefs departs from the agent's base rate. The agent is choosing perceived risk, for some payoffs (via insurance) that maximizes her mental profit.

1.2 Dual Processes

After reviewing the psychology literature with its implications about the personal perception judgement and, in particular, perceived personal risk, one needs to think about how the latter leads to choosing insurance. In other words, one should link together the probability judgement and choice behavior. This brings us to the distinction often made in psychology, and supported by neuropsychology, between two systems of reasoning.

The mapping of the human mind into multiple processes has its roots in Aristotle, but the modern, most influential formulation of such distinction was made by Freud. Freud suggests a distinction between two processes: the unconscious, labeled the primary process, and the conscious, labeled the secondary process. His hypothesis is that the primary process is symbolic and associative while the secondary process is a rational-thinking process. Throughout the years, many more scholars in various fields of psychology made similar distinctions between two processes, albeit distinguishing the two by different traits: a verbal and nonverbal process, logical and prototypical systems, explicit and implicit, analytical and intuitive information processing and more (Wilson and Dunn, (2004); Epstein, (1994) and references within). Generally speaking, then, the psychology literature agrees on the distinction Freud proposes between two processes and agrees that one is a more deliberate, rational reasoning process and the other more intuitive and emotionally based. In contrast to Freud, psychologists now believe that the intuitive mode is not the source that undermines people's attempt at rational thinking, but merely a different type of reasoning system. Epstein (1994), recognizing the similarities between the various dual process theories, developed a general theory of personality, the Cognitive-Experimental Self Theory (CEST), to encompass all such dual process models by making a distinction between the rational system – which is a deliberate, effortful, abstract system – and the experimental system – which is intuitive and emotionally driven. Recently, Kahneman (2003) introduced this distinction to economists, and argued that it should be incorporated in economic decision making.

Forming risk perception, as is clear from the discussion above, involves emotional motives such as feeling good about oneself or one's future. Therefore, it seems natural that the task of forming risk perception is generally an intuitive one performed by the experimental system in CEST and which I label the mental account, defined in section 2.2 below. Indeed, Wilson and Dunn (2004) argue that source of self-knowledge failure is due to inaccessibility of the mind to mental processes that involve perception, self esteem and alike. Thus, they agree that many self-perceptions are formed in implicit mental processes, which they argue, are generally unconscious. In contrast, choosing optimal action, such as insurance, is a deliberate task which demands logical effort, and it is therefore labeled the rational account. I follow the distinction psychologists often make between the rational and mental accounts and hypothesize that each is partially in charge of the insurance decision. The insurance decision, is modeled as an outcome of the two accounts working together, in accordance with the psychology literature. In contrast to the psychology literature, which often implies that the two systems are competing, this study views the two processes as complements. The reason for this difference is that one can decompose the decision- making into two main components, one is dominated by the rational account and the other is dominated by the mental account. In order to reach a decision, each process uses all available information including that supplied by the other process.

2 The Model

Consider an agent who is facing two possible future states of the world, $s \in \{s_1, s_2\}$ with an associated wealth level of $w \in \{w_1, w_2\}$. Without loss of generality assume $w_1 < w_2$ and denote the income shock as $z \equiv w_2 - w_1$. The agent has a strictly concave utility function, $U(w)$, which is assumed to be continuous and differentiable three times.

Unlike the standard insurance models, the agent is assumed *not* to know her risk level, where risk is defined as the likelihood of being in state s_1. Consequently, the agent forms a risk perception and works with that. As argued above, the agent has preferences over belief, which in the insurance context is risk perception. Acknowledging that the agent has a preference over risk perception, leads one to believe that the agent *chooses* her perception of risk. The forces underlying this choice and the selection process itself will be discussed in section 2.2. In general, I distinguish between two processes: the rational account is presented in section 2.1 and the mental account presented in 2.2.

In order to avoid her (perceived) risk the agent can purchase insurance $I \in (-\infty, \infty)$, to smooth her wealth across states of the world. The insurance premium rate is $\gamma > 0$ and it is fixed for all levels of insurance purchase.

2.1 Rational Account

The rational account chooses insurance for a *given perceived risk* β to maximize expected utility. Thus, the rational account is the standard expected utility model using probability judgement rather than objective probabilities. More specifically, the rational account maximizes the following objective function:

$$\underset{I}{Max}\ \beta U(w_1 + (1-\gamma)I) + (1-\beta)U(w_2 - \gamma I)$$

Where $U(\cdot)$ is the agent's utility of wealth and γ is the insurance premium. The optimal insurance level, I^*, satisfies the first order condition of this problem:

$$\beta U'(w_1 + (1-\gamma)I^*)(1-\gamma) - (1-\beta)U'(w_2 - \gamma I^*)\gamma = 0$$

Given a fixed wealth, shock size and insurance premium, I^* is only a function of risk perception β and will be denoted as $I^*(\beta)$.

Since perceived risk is determined by the mental account, as discussed in section 2.2 below, *one can think of $I^*(\beta)$ as the best response of the rational account for any given strategy of the mental account.*

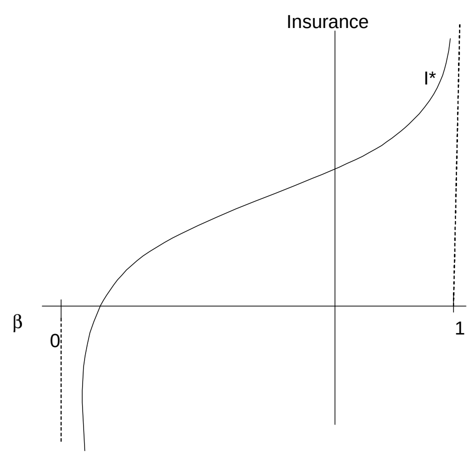
A straight-forward conclusion from the first order condition is that the best response $I^*(\beta)$ can be more than, less than or exactly equal to full insurance. For stating the condition on the model's parameters which determine that, recall the definition of z as the income shock size: $z \equiv w_2 - w_1$. The corollary below summarizes the result.

Corollary 1 *If* $\beta \gtreqless \gamma$ *then the optimal insurance level is* $I^*(\beta) \gtreqless z$.

Using the first order condition further more, and assuming that $U''(\cdot)$ is finite, one can determine the behavior of the optimal insurance $I^*(\beta)$ holding the insurance premium fixed.

Corollary 2 $I^*(\beta)$ *increases with perception of risk, and it is concave in* β *for* $\beta \to 0$ *and convex for* $\beta \to 1$. *Also, as* $\beta \to 0$, $I^*(\beta) \to -\infty$ *while as* $\beta \to 1$, $I^*(\beta) \to \infty$.

A possible illustration of $I^*(\beta)$ is:



2.2 Mental Account

The psychology literature dealing with motivated cognition (e.g., Kunda, (1990)) suggests that individuals extract utility from beliefs per se. That is, people have preferences over beliefs: people *want* to believe they are better than average, less likely to be ill, unemployed or unhappy. Acknowledging the effect of beliefs per se on well-being leads one to think that people *choose* their beliefs.

Thus, I allow the agent to choose her beliefs and derive the impact of that choice on perception and insurance choice.

Benaboù and Tirole (2001) argue that people get a positive utility from favorable beliefs for consumption value and signalling value among other reasons. I capture this by using expected utility formula. Note that the mental account takes the insurance level and therefore the utility level as given. Therefore, changing β, which is the weight assigned to the utility in state s_1, will change perceived expected utility. This also captures the dependency of perceived risk on payoffs. That is, the mental gain of changing β depends upon the utilities in both states of the world.

Notice that if expected utility were the only component in deciding on optimal perception of risk, then the agent would hold arbitrary values of β ($\beta \to 0$) for values of $I < z$, and ($\beta \to 1$) for values of $I > z$. However, as argued in section 1, people's beliefs are not arbitrary and are a result of balancing motivation with a taste for accuracy. As discussed, balancing motivation and accuracy can be viewed also as the phenomenon of anchoring and adjustment. I capture this by introducing a belief reference point, or base rate, β_0, which can be thought of as the agent's best assessment. Holding beliefs that are different from β_0 would bear a cost which captures the taste for accuracy.

According to the discussion above, the agent would generally prefer to hold beliefs such that $\beta \neq \beta_0$. However, due to her taste for accuracy, which can also be regarded as fear of being overly optimistic, there will be some optimal risk perception β^*, that balance these two forces. More specifically, β^* solves the following

$$\underset{\beta}{Max}\ \beta U(w_1 + (1-\gamma)I) + (1-\beta)U(w_2 - \gamma I) - cf(\beta, \beta_0)$$

where c is the importance the agent assigns to taste for accuracy relative to affective motivation. The first order condition is:

$$U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I) - c\frac{\partial f(\beta^*, \beta_0)}{\partial \beta} = 0$$

Given a fixed income, loss size, and c, β^* is only a function of insurance level I and β_0.

Notice that the marginal mental gain[10] of infinitesimally changing beliefs, $U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I)$[11], is influenced by the insurance level while the marginal cost of this, $c\frac{\partial f(\beta^*, \beta_0)}{\partial \beta}$, is determined by β_0. As a result, $\beta^*(I, \beta_0)$ is the probability judgement, or belief, that balances these two forces. Fixing a base rate β_0, β^* is a function of the insurance level only, and it is denoted $\beta^*(I)$. In other words, $\beta^*(I)$ is the perception that maximizes the mental well-being, for a given insurance level I, i.e., $\beta^*(I)$ *is the best mental response for a given strategy of the rational account.* It is interesting to note that the mental best response presents the risk perception one would expect the agent to report, for a given insurance level.

To analyze the behavior of $\beta^*(I)$ I assume the following:

Assumption 1 $f(\beta, \beta_0)$ *is a continuous, three times differentiable function of* β *and* β_0, it is *convex in* β and *reaches a minimum at* $\beta = \beta_0$.

In words, the further away β is from β_0, the greater are the psychological cost associated with it. This is, by definition, taste for accuracy.

I further assume that the cost function $f(\cdot)$ is submodular:

Assumption 2 *The mental cost function* $f(\beta, \beta_0)$ *is submodular, i.e.,* $\frac{\partial^2 f(\beta,\beta_0)}{\partial\beta\partial\beta_0} \leq 0$.

This assumption implies that the marginal cost of holding a risk perception β is nonincreasing in β_0. This assumption is used in later sections.

Lastly, experiments show that people attribute a special value for certainty, which is reflected by the extreme beliefs $\beta \in \{0, 1\}$. Thus, people would not generally choose to believe $\beta \in \{0, 1\}$ under conditions of risk and uncertainty. That can be captured by a mental cost function that is finite between two values $\{\underline{\beta}, \bar{\beta}\}$ with $0 < \underline{\beta} < \bar{\beta} < 1$ and is infinite at the limits. Note that $\{\underline{\beta}, \bar{\beta}\}$ can be arbitrarily close to $\{0, 1\}$. These arguments are summarized in Assumption 3 below:

Assumption 3 Mental costs are positive and infinite at the boundaries, $lim_{\beta\to\underline{\beta}} f(\beta, \beta_0) = lim_{\beta\to\bar{\beta}} f(\beta, \beta_0) \to +\infty$, which implies that the marginal mental cost at the limit are $lim_{\beta\to\underline{\beta}} \frac{\partial f(\beta,\beta_0)}{\partial\beta} \to -\infty$, $lim_{\beta\to\bar{\beta}} \frac{\partial f(\beta,\beta_0)}{\partial\beta} \to +\infty$.[12]

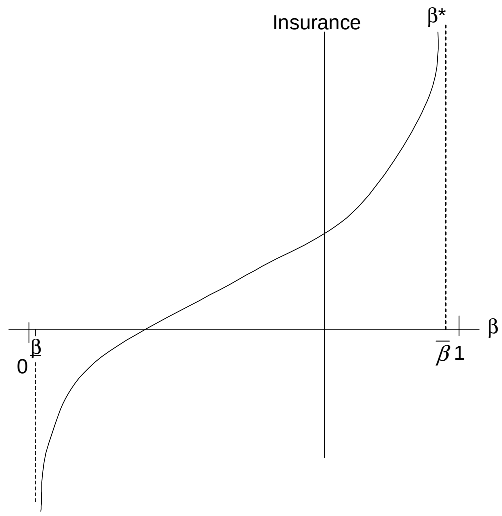
Using Assumption 1 and the first order condition, the following is a straight forward conclusion:

Corollary 3 *At full insurance,* $\beta^*(I = z) = \beta_0$.

With a little calculation, one can conclude:

Corollary 4 *$\beta^*(I)$ is strictly increasing with insurance I.*

For $c = 1$, the following is a possible illustration of $\beta^*(I)$:



Note that the above illustration assumes $c = 1$. Recall that c is the relative importance of the mental cost to gain as one marginally changes risk perception, β. Not surprisingly, as c changes, the mental best response changes. In particular, for any given insurance level, as c increases the probability judgment one selects is closer to the base rate β_0. This is summarized below:

Corollary 5 *Fix an insurance level I. For $I < z$ $(I > z)$ the mental best response $\beta^*(I)$ is increasing (decreasing) with c.*

In other words, the higher is c, the narrower is the domain of risk perception that the mental cost will choose. In fact, at the extreme, as $c \to \infty$, the mental account will always choose $\beta = \beta_0$; as $c \to 0$ the mental best response will resemble a step function: for less than full insurance $\beta^* \to 0$ and for more than full insurance $\beta^* \to 1$. Notice that, although different conceptually, Assumption 3 and a change in c have qualitatively the same impact – restricted domain of probability judgment. Therefore, I shall abuse notation and continue to label the belief domain $\left[\underline{\beta}, \overline{\beta}\right]$, regardless of the its source. Consequently, the illustration above is possible for any value of c.

Second, the illustration of the best response locus above assumes it follows a particular shape: β^* is initially convex and then becomes concave in insurance I. This is not necessarily the case; in fact, since $\beta^*(I)$ balances marginal mental gain and cost then its shape with respect to I depends on the effect of a change in I on the marginal gain, $\frac{\partial[U(w_1+(1-\gamma)I)-U(w_2-\gamma I)]}{\partial I}$, and the rate at which this happens, $\frac{\partial\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]}{\partial I}$, relative to the change in marginal cost as one changes belief β, $\frac{\partial f'(\beta^*,\beta_0)}{\partial\beta}$, its speed of change $\frac{\partial^2 f'(\beta^*,\beta_0)}{\partial\beta^2}$, and its relative importance c. More specifically the shape of $\beta^*(I)$ depends on the following condition:

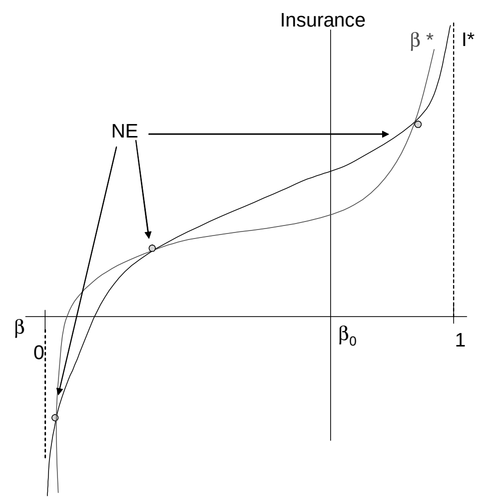
Corollary 6 *β^* is concave in I iff* $\frac{\frac{\partial\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right)}{\partial I}}{\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]^2} < \frac{\frac{\partial^2 f'(\beta^*,\beta_0)}{\partial\beta^2}}{c\left[\frac{\partial f'(\beta^*,\beta_0)}{\partial\beta}\right)^2}$ *and it is convex iff* $\frac{\frac{\partial\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right)}{\partial I}}{\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]^2} > \frac{\frac{\partial^2 f'(\beta^*,\beta_0)}{\partial\beta^2}}{c\left[\frac{\partial f'(\beta^*,\beta_0)}{\partial\beta}\right)^2}$

2.3 Both Accounts

Although presented separately, the rational and the mental account are not considered independent, but rather are thought, and assumed, to communicate in the process of reaching a decision. A decision in this context is a bundle of both risk perception and insurance level. As argued, the rational account produces an insurance best response to a certain risk perception β, while the mental account produces a belief best response to any strategy of the rational account, i.e., insurance decision I. The two accounts aim for consistency; otherwise the agent suffers a cost of cognitive dissonance. This situation is summarized in an intrapersonal static game of two accounts, defined below.

Definition 1 *An intrapersonal game is a simultaneous move game of two players – the rational and the mental account. The strategy of the rational account is insurance decision, $I \in (-\infty,\infty)$, and the strategy of the mental account is a choice of risk perception , $\beta \in [\underline{\beta},\overline{\beta}]$. The rational account's payoff function is $g(\beta,I) \equiv \beta U(w_1+(1-\gamma)I)+(1-\beta)U(w_2-\gamma I)$, $g:[\underline{\beta},\overline{\beta}]\times(-\infty,\infty)\rightarrow R$ and the mental account's payoff function is $\psi(\beta,\beta_0,c,I) \equiv g(\beta,I)-cf(\beta,\beta_0)$, $\psi:[\underline{\beta},\overline{\beta}]\times[\underline{\beta},\overline{\beta}]\times[0,\infty]\times(-\infty,\infty)\rightarrow R$, where β_0 is the agent's base rate, $f(\cdot)$ is the mental cost of holding beliefs $\beta \neq \beta_0$ and c is its importance relative to affect motivations.*

The pure strategy Nash equilibria of this game, if they exist, represent consistency between the two accounts. Thus, the set of Nash equilibria are the natural candidates of choice for the agent. Given the best responses of the two accounts one can illustrate graphically the set of Nash equilibria for this game. One possible illustration is presented in the figure below[13]:

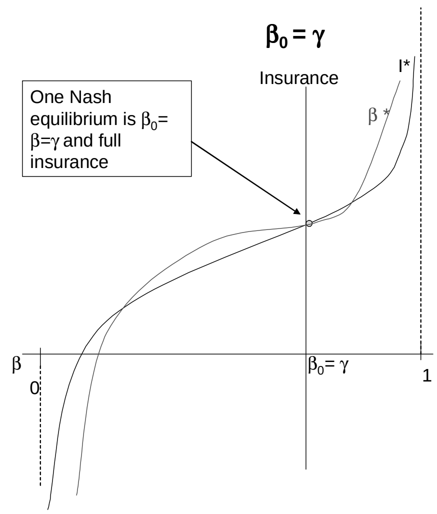


Notice that the implied risk perception at any Nash equilibrium is generally different from the self-reported risk perception one might record. This implies that the insurance choice need not be consistent with reported risk perception, a phenomenon recorded by Costa-Gomes and Weizsäcker (2003), albeit in a different context. This distinction is quite important, as will become clear in section 4.

Given the information on the best responses, one can argue that there always exists a Nash equilibrium and, moreover, to provide more details regarding one of the possible equilibria. The discussion below, split into three cases: $\gamma = \beta_0, \gamma > \beta_0$ and $\gamma < \beta_0$, is summarized in the following three propositions. Note that the graphical illustrations which accompany these propositions are assuming a specific shape of the β^* and I^* loci. However, the conclusion drawn are general and apply for all cases with all possible shapes of the mental and the rational best responses.

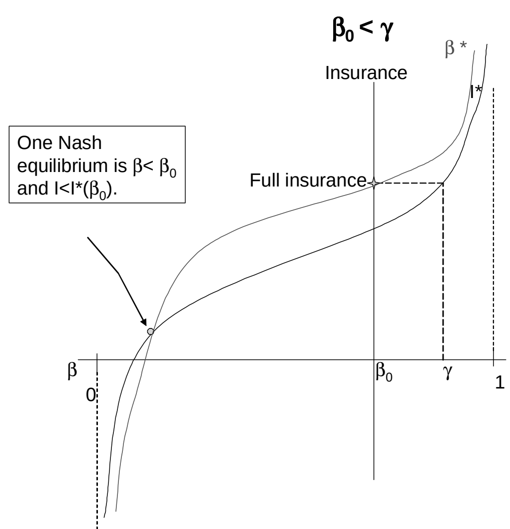
Proposition 1 *If $\gamma = \beta_0$, then there exists at least one Nash equilibrium with full insurance and*

$\beta = \beta_0 = \gamma$.



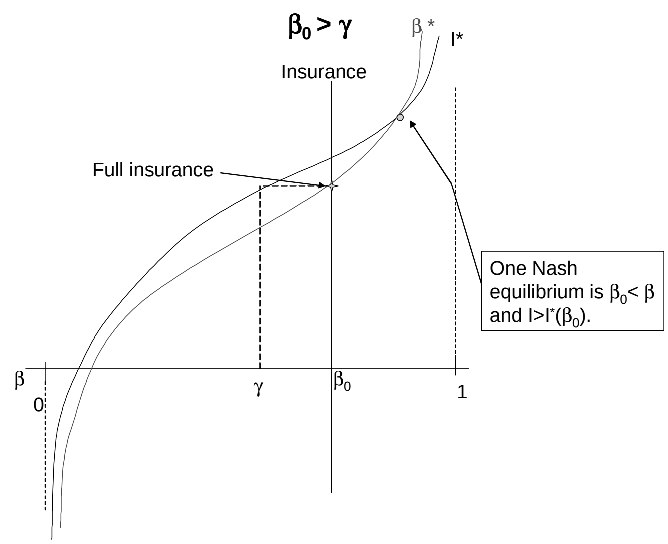
Note that this result is the same as the standard economic outcome. To prove its existence note that at full insurance there is no mental gain for holding beliefs $\beta \neq \beta_0$ but there exists mental cost. Therefore at full insurance the mental account's best response is $\beta = \beta_0$. Given that $\gamma = \beta_0 = \beta$, the rational account's best response is full insurance. Consequently, full insurance and $\beta = \beta_0$ is a Nash equilibrium of this case.

Proposition 2 *If $\gamma > \beta_0$, then there exists at least one Nash equilibrium, with $\beta < \beta_0$ and $I < I^*(\beta_0)$, i.e., less insurance relative to the standard economic model.*



To see this, note that because the insurance premium is higher than perceived risk then $I^*(\beta = \beta_0) < z$. Also, $\beta^* = \beta_0$ only at full insurance, where $I = z$. Therefore, at the point of $\beta = \beta_0$ the mental account's best response is above the rational account's best response. Since this relationship is reversed at the limit $\beta \to \underline{\beta}$ and both the mental and the rational best responses are increasing , the conclusion is that there exists a Nash equilibrium with $\beta < \beta_0$ and less insurance than the standard outcome.

Proposition 3 *If $\gamma < \beta_0$, then there exists at least one Nash equilibrium, with $\beta_0 < \beta$ and $I > I^*(\beta_0)$, i.e., more insurance relative to the standard economic model.*



To see this, notice that because the insurance premium is lower than β_0 then $I^*(\beta = \beta_0) > z$, while only full insurance, $I = z$ will make $\beta^* = \beta_0$. Therefore, at the point of $\beta = \beta_0$ the mental account's best response is below the rational account's best response. Since this relationship is reversed at the limit $\beta \to \overline{\beta}$ and both the mental and the rational best responses are increasing , one can conclude that there exists a Nash equilibrium with $\beta > \beta_0$ and more insurance than the standard outcome.

The above discussions and graphic illustrations suggest that there is at least one pure-strategy Nash equilibrium in our model and hint that there are multiple equilibria. In fact, the general result in the next section summarizes this intuition.

3 Results

This section proves existence of a pure strategy Nash equilibrium, provides sufficient conditions for a unique equilibrium and presents comparative statics.

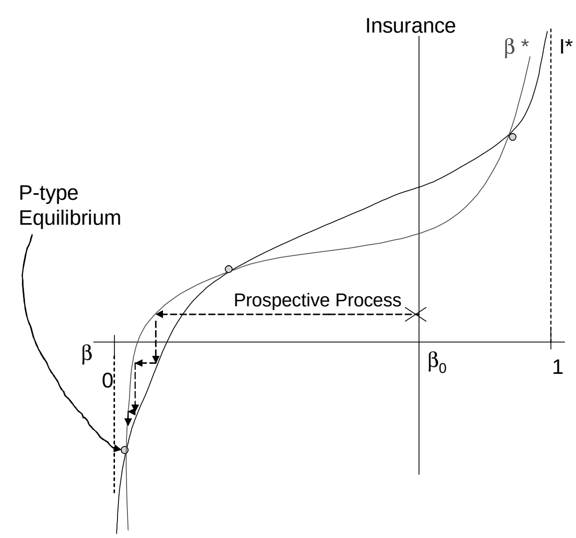
3.1 Nash Equilibria

Definition 2 *Define $\tilde{I}(\beta, \beta_0)$ as the insurance level such that $\beta^*(\tilde{I}, \beta_0) = \beta$ for some $\beta \in [\underline{\beta}, \overline{\beta}]$, i.e., $\tilde{I}(\beta, \beta_0) = \beta^{*-1}(I)$ is the inverse function of $\beta^*(I)$.*

Definition 3 *Let $I^* : [\underline{\beta}, \overline{\beta}] \to R$ be the best response of the rational account. Let $\beta^* : R \to [\underline{\beta}, \overline{\beta}]$ be the best response of the mental account strategy. Define a Prospective adjustment process (P) as a sequential play where the rational and the mental accounts play in turns, $h = I^* \circ \beta^*$, where $h : R \to R$.*

Definition 4 *A Nash equilibrium is of type P if an adjustment process P converges to it for all initial points in its neighborhood, i.e, fix an insurance level close enough to I^{NE}, $\dot{h}(t) > 0$ for $I < I^{NE}$ and $\dot{h}(t) < 0$ for $I > I^{NE}$[14].*

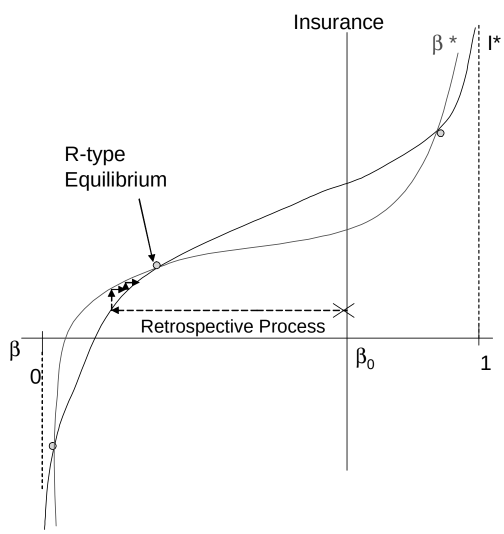
For example:



Definition 5 *Let* $I^{*-1}: R \rightarrow \left[\underline{\beta}, \overline{\beta}\right]$ *be the beliefs that makes* I^* *the rational account's best response. Let* $\beta^{*-1}: \left[\underline{\beta}, \overline{\beta}\right] \rightarrow R$ *be the insurance level that makes* β^* *the mental account's best response. Define a Retrospective adjustment process (R) as a sequential play where the rational and the mental accounts play in turns,* $h^{-1} = \beta^{*-1} \circ I^{*-1}$*, where* $h^{-1}: R \rightarrow R$.

Definition 6 *A Nash equilibrium is of type R if an adjustment process R converges to it for all initial points in its neighborhood, i.e, fix an insurance level close enough to* I^{NE}*,* $\dot{h}^{-1}(t) > 0$ *for* $I < I^{NE}$ *and* $\dot{h}^{-1}(t) < 0$ *for* $I > I^{NE}$[15].

For Example:



Theorem 1 *There exists a pure-strategy Nash equilibrium for the intrapersonal game. The Nash equilibria set contains an odd number of points with a lowest and a highest arguments and it forms a chain. Moreover, the Nash equilibria points alternate between being of type P to R. Under Assumption 3, the extreme equilibria points are of type* P[16].

Theorem 1, due to Milgrom and Roberts (1994), assures us that an agent can achieve internal consistency between her rational and mental accounts. Moreover, there is an odd number of such equilibria which implies that generally there are multiple equilibria. Having possibly multiple equilibria, a chain guarantees a unique order of equilibria from (low insurance, low risk perception) to (high insurance, high risk perception) and, excluding the case of a unique equilibrium, there is at least one equilibrium of type R. Since all equilibria are candidate for choice, the case of a unique equilibrium is of a special interest and will be discussed in the next section.

There are two remarks on the Nash equilibria that are worth noting. First, note that the agent's choice set captures the essence of the idea; the choice set is composed of Nash equilibria of the intrapersonal game which reflects the influence of insurance decision on both payoffs (thus utility) and probability judgement. In other words, the probability judgement is not independent of payoffs! This idea is similar to the moral hazard logic; in the moral hazard setting, agents are assumed to have two actions – insurance and , say, driving style. The argument is that agents will have incentives to change their driving style after buying insurance, leading to a change in their true risk. Thus insurance indirectly affect risk. In the current set-up, buying insurance *directly* affects the agent's *risk perception*. Therefore, the similarity is that insurance affects risk but the differences are that in the present model (1) insurance influences perceived risk, not true risk and (2) choice is predicted according to Nash equilibria implying that, in equilibrium, the insurance purchase will be consistent with risk perception. A second interesting point is that the Nash equilibria set, representing the choice candidates, contains both equilibria of type P and R . Although usually economists focus attention on type P equilibria, in fact the traditional result of full insurance and $\beta = \beta_0 = \gamma$ could be of type R. The exact conditions for this will be discussed in the following section. An additional support for looking at the type R equilibria is a stylized fact from empirical insurance study and a phenomenon that was recorded in experiments; both are consistent with a type R equilibrium and would not have been explained by this model otherwise.

3.2 Unique Equilibrium

The choice in an intrapersonal game with a unique equilibrium is clear. Thus, it is interesting to examine the conditions that guarantee this. The following proposition provides such a sufficient condition.

Proposition 4 *Define* $\Pi(\beta, \beta_0) = I^*(\beta) - \tilde{I}(\beta, \beta_0)$*. A monotone decreasing function* $\Pi(\beta, \beta_0)$ *in* β *is a sufficient condition for a unique Nash equilibrium of the intrapersonal game indexed by* β_0[17].

Recall that the relative importance of the mental cost, c, influences the optimal probabaility judgment of the mental account. Moreover, as c increases, the $[\underline{\beta}, \overline{\beta}]$ domain is reduced. Thus, a straight forward conclusion is:

Lemma 1 *For high enough values of c, Π will be monotonically decreasing, i.e., there exists a unique equilibrium.*

Theorem 1 guarantees that this equilibrium is of type P.

3.2.1 Full Insurance

Consider the case of $\beta_0 = \gamma$. Proposition 1 assures us that the unique equilibrium in this case is the standard economic outcome of $\gamma = \beta_0 = \beta$ and full insurance. As Proposition 2 and 3 hint this outcome will not be the outcome had the insurance premium or β_0 changed.

Changes in β_0 or insurance premium

Suppose the base rate β_0 changes, such that $\beta_0 < \gamma$. According to Proposition 2 the outcome in this case is less than full insurance and even less insurance relative to the expected utility model. If, on the other hand the base rate increases, such that $\beta_0 > \gamma$, then according to Proposition 3 the outcome in this case is more than full insurance and even more insurance relative to the standard economic prediction. The following Lemma summarizes:

Lemma 2 *As β_0 increases (decreases), the agent will buy more (less) insurance. This is in accordance with the standard economic model; however, the extent to which this happens is greater under the intrapersonal game relative to the expected utility model.*

This exercise makes the impact of affective considerations for type P equilibria quite clear: their presence leads the decision maker to enhance the extent of her rational account decision. For example, suppose the base rate decreases such that $\gamma > \beta_0$; the rational account (read: the rational effect) prescribes buying less than full insurance. The mental account, then, leads the decision maker to convince herself that she is at a lower risk, in order to feel better about her decision (motivational reasoning); this effect, which I label the mental effect, cause a further reduction in insurance purchase.

Clearly, a change in the insurance premium will yield similar results which are summarized in the following Lemma:

Lemma 3 *As the insurance premium increases (decreases), the agent will buy less (more) insurance. This is in accordance with the standard economic model; however, the extent to which this happens is greater under the intrapersonal game relative to the expected utility model.*

Changes in w_2 and shock size

Suppose income w_2 increases. For β_0, the rational account would prescribes full insurance, which is the same as before. The mental account, given full insurance, would again selects $\beta = \beta_0 = \gamma$. Therefore the full insurance unique equilibrium result sustains. When the shock size increases, the unique full insurance prevails as well, at its new level. Thus, the expected utility outcome, for the $\gamma = \beta_0$ case, is robust to mental considerations for all income and shock size. However, any disturbance to the fragile equality $\gamma = \beta_0$, such as a change in insurance premium or β_0, will result a different outcome than the one of expected utility.

Note that, as mentioned earlier, this equilibrium like any other unique equilibrium is of a prospective type. However, the standard outcome need not be of this type in the general case of multiple equilibria. This and the exact conditions that determine the standard outcome type will be discussed in the next section.

3.2.2 More or Less than Full Insurance

As Proposition 2 and 3 show in the cases of $\gamma > \beta_0$ and $\gamma < \beta_0$ the insurance purchase departs from the level prescribed by expected utility. This is due to the enhancement effect of the mental considerations, discussed previously. In contrast to the full insurance case, in these cases analyzing the changes in choice as one changes insurance premium, β_0, or income and shock size, can not be concluded from the discussion thus far; a more general analysis is required.

Note that analyzing the change in unique equilibrium is qualitatively the same as any other equilibrium of type P, regardless of uniqueness. To see that, note that one can define for any equilibrium a local neighborhood in which the equilibrium is unique. Therefore, the change in choice of the unique non-full-insurance equilibrium is presented as part of the analysis in the general case of multiple equilibria, next.

3.3 Multiple Equilibria

As section 3.1 indicates, the general case contains an odd number of multiple equilibria, some are of prospective (P) and some are of retrospective (R) type. Although we economists usually focus attention to equilibria which are stable under the P process, the intrapersonal model shows that the standard economic outcome of $\gamma = \beta_0 = \beta$ and full insurance can be stable under the R process. In fact, below are the exact conditions that determine whether or not the standard outcome is stable under the prospective process. These conditions depends on the relationship between the mental cost function and the utility function, as well as their relative importance, c, in the eyes of the agent.

Theorem 2 *Consider the case $\beta_0 = \gamma$. Define $\overline{w} \equiv \beta_0 w_1 + (1-\beta_0) w_2$ and let $r(\overline{w})$ be the absolute risk aversion measure. The full insurance standard outcome is of P type if and only if*

$$\frac{U'(\overline{w})}{r(\overline{w})} < c\frac{\partial^2 f(\beta = \beta_0, \beta_0)}{\partial \beta^2}\beta_0(1-\beta_0)$$

From these conditions it is clear that as the relative importance of the mental cost of holding some belief $\beta \neq \beta_0$ increases, the higher are the chances that the expected utility outcome will be of type P. Note that this resembles the conditions for uniqueness; indeed, for sufficiently large value of c there exists a unique equilibrium, and as Theorem 1 shows, the extreme (thus unique) equilibrium is of the prospective type. However, generally, there are cases with $\beta_0 = \gamma$, admitting the standard outcome, although it is of the retrospective type. An example is $c = 1$, a mental cost function $f(\beta, \beta_0) = \ln\left(\frac{1}{(\beta - \underline{\beta})(\overline{\beta} - \beta)}\right) - \beta\beta_0$ and a log utility function. Having this in mind, below are local comparative statics to help understand the change in equilibria, whether type P or R , as one changes the parameters of the model.

3.3.1 Comparative Statics

Changes in β_0 This part presents the changes in Nash equilibria, composed of (insurance, risk perception) as the agent's best assessment or base rate, β_0, changes. In order to achieve that, I use the following corollary which is a conclusion drawn from the mental best response and Assumption 2.

Corollary 7 *For $\beta^* \neq \beta_0$, $\beta^*(I, \beta_0)$ is non-decreasing in β_0. Assuming a strictly submodular mental cost function $f(\beta, \beta_0)$, i.e.$\frac{\partial^2 f(\beta,\beta_0)}{\partial\beta\partial\beta_0} < 0$, then $\beta^*(I, \beta_0)$ is increasing in β_0.*

The corollary above suggests that as the base rate β_0 increases, indicating a higher chance of being in state s_1, the optimal belief for any given insurance level is nondecreasing. To see that, note that as the base rate β_0 increases, the marginal gain from holding any belief β is unchanged while the marginal cost is nonincreasing. If marginal cost does not change, then the optimal belief for a given insurance level stays the same. If marginal cost does change, it declines and leads to higher perception of risk at every given insurance level.

Using Corollary 7, one can deduce the effect of a change in the base rate β_0 on both types of Nash equilibria. This is summarized in Theorem 3 below.

Theorem 3 *Consider a type P (R) equilibrium of the intrapersonal game. This equilibrium is nondecreasing (nonincreasing) in β_0. Moreover, if $\beta^*(I, \beta_0)$ is strictly increasing in β_0, then it is increasing (decreasing) in β_0.*

Theorem 3 suggests that as the base rate increases, the Nash equilibrium we consider might or might not increase, depending if it is equilibrium of type P or R. If it is a retrospective equilibrium then, as β_0 increases, it will consist of insurance level and risk perception which are less than or equal to their previous level. This result is counter intuitive, as one would expect an increase in the base rate to generate a higher Nash equilibrium with higher insurance and higher perception of risk just like the case for a type P equilibrium. However, this intuition in the retrospective case captures only one aspect: the change in β^*. Ceteris Paribus, the optimal belief is higher for a positive incremental change in β_0. However, the intrapersonal game is composed of both rational and mental considerations. If the rational and the mental account feed each other approaching the local Nash equilibrium, as is the case of type P equilibria, then one would maintain the intuitive result. By definition, in the neighborhood of the type R equilibrium, the insurance decision feeds beliefs in the opposite direction of the local Nash equilibrium. Therefore, for the retrospective equilibrium, an increase in the base rate results in a decrease in insurance choice *because* β^* increases with β_0. This result could have policy implications implying that manipulating the base rate upwards can cause the agent to choose less insurance and hold more favorable risk perception!

Changes in income and shock size This part studies the influence of a change in traditional insurance parameters, such as income and shock size, on the decision of an agent who is subject to both rational and mental considerations.

Theorem 4 *(i) Suppose that the income w_2 increases while the shock size, z, stays constant. Then, the change in choice is given by the following tables, distinguishing between choice due to type P or type R equilibrium as well as distinguishing between cases of a utility function with decreasing absolute risk aversion (DARA), constant absolute risk aversion (CARA) and increasing absolute risk aversion (IARA).*

Type P

	DARA	*CARA*	*IARA*
$I < z$	?	$NE \uparrow$	$NE \uparrow$
$I = z$	*unchanged*	*unchanged*	*unchanged*
$I > z$	?	$NE \downarrow$	$NE \downarrow$

Type R

	DARA	*CARA*	*IARA*
$I < z$	?	$NE \downarrow$	$NE \downarrow$
$I = z$	*unchanged*	*unchanged*	*unchanged*
$I > z$	?	$NE \uparrow$	$NE \uparrow$

(ii) Suppose that the shock size, z increases, while w_2 stays constant. Then, the impact of this change on the Nash equilibria is not clear.

Since choice is according to a Nash equilibrium and is composed of (insurance, risk perception), increase in equilibrium (or choice) means that both insurance and risk perception increases. Bearing this in mind, Theorem 4 states that if the utility function exhibits CARA, the initial choice consists of less than full insurance and follows a type-P equilibrium then as income increases choice will increase, leading to more insurance purchase and higher risk perception. However, in contrast to the expected utility model, such prediction in the current model is not constant; for instance if initially choice consists of more than full insurance, is due to a type-P equilibrium and we have

CARA utility function, then higher income leads to purchasing less insurance. Moreover, part (ii) of Theorem 4 implies that, ceteris paribus, an agent facing higher possible loss will not necessarily purchase more insurance. This is in contrast to the standard insurance model, which predicts more insurance purchase for such a change.

Changes in insurance premium This section examines the influence of a change in insurance premium on insurance decision. Generally speaking, I find results different from the standard outcome. A summary of the results is presented below.

Theorem 5 *Suppose the insurance premium increases. Then, the impact of this change on the Nash equilibria is as follows:*

		Type P	
	DARA	*CARA*	*IARA*
$I > z$	?	?	?
$I = z$	$NE \downarrow$	$NE \downarrow$	$NE \downarrow$
$0 < I < z$	$NE \downarrow$	$NE \downarrow$	?
$I < 0$	?	?	?

		Type R	
	DARA	*CARA*	*IARA*
$I > z$	?	?	?
$I = z$	$NE \uparrow$	$NE \uparrow$	$NE \uparrow$
$0 < I < z$	$NE \uparrow$	$NE \uparrow$	?
$I < 0$	?	?	?

Theorem 5 results are very interesting, suggesting that there are cases where insurance companies can increase insurance premium and not suffer reduced insurance purchases. This depends on the agent's initial insurance level, and the type of equilibrium her choice follows.

4 Discussion

This section generally discuss the implications of the model on risk perception and optimism as well as provides possible explanation for various stylized facts in the insurance markets. In addition, this section discusses experimental studies in psychology that this model can indirectly explain. Explaining phenomenon outside the insurance context suggests that the framework of dual processes in decision making is more general and can, with some adjustment, be applied to decision making in other contexts.

4.1 General Discussion

The current model shows that allowing motivational (affective) reasoning to interact with rational considerations give rise to possibly multiple equilibria with different probability judgment and actions. This is interesting as usually we think that decision making process leads to a unique outcome, i.e., unique choice. Multiple equilibria is consistent with the casual observations that different people holds different beliefs despite being exposed to the same events. Hence, this model suggests that affective motivations is one reason for this. Interestingly, often times, people explain such differences by 'different interpretation' of events. In this model, then, interpretation can be viewed as an equilibrium selection mechanism.

Note that the adjustment processes, either prospective or retrospective, discussed to define equilibrium type can be viewed as a way of selection mechanism. If the agent indeed uses either one of such selection mechanism then choice will be path dependent – the observed choice depends on whether the agent first chooses belief or insurance. This is in accordance with the framing effect and lab experiment where choices were demonstrated to depend upon the attribute subjects were induced to focus on. In the insurance context, a prospective process implies that manipulating subjects to report their risk perception first will, generally, lead to lower insurance purchase, relative to the case where subjects are manipulated to think about insurance first. This is of course for the case of multiple equilibria. In a context where the mental cost of distorting one's beliefs are very large (or their relative importance is very large) then there will be a unique equilibrium, which means behavior is not subject to framing effects.

Lastly, an insight that arises from this model is that it is possible for the experimenter to record both pessimism and optimism (relative to β_0). However, recording pessimism is possible only *because* the action taken (insurance) can change a state from being a 'bad' state to being a 'good' state. Thus, if the available action can not change the bad state into a good state, people will tend to be optimists and purchase insurance which is less than optimal. This might hold also in other fields similar to insurance, such as pharmaceutical drugs consumption. This discussion leads one to think about issues of control. A negative relationship between sense of control and perceived risk is a well recorded phenomenon. This seems to be in contrast to this model; unless we distinct between the sources of control sensation – available actions versus other factors such as familiarity (or ambiguity). Accepting such a distinction, this model suggests that the ability to take effective actions can lead an experimenter to record pessimism, while the other factors (which may or may not be familiarity) leads to record optimism.

4.2 Risk and Uncertainty

The reader might wonder at this point whether this model is a model of risk or uncertainty. Recall the distinction between risk and uncertainty; in both, the agent is faced with a future which outcome is not certain. Risk is the case where the agent knows the probability distribution over future event. Uncertainty, is the case where the agent is *uncertain* about this probability distribution; the uncertainty is captured by having a *set* of possible probability distributions over future events. Having these definitions in mind, it is clear that the *rational account* is a model of risk; the rational account takes the mental account's probability judgment and acts as if it is the true probability distribution. However, the agent's choice is a model of uncertainty since she does not know a priori the perceived probability: multiple equilibria means there is a set of perceived probabilities which the agent can believe. The linkage between risk and uncertainty is the mental account.

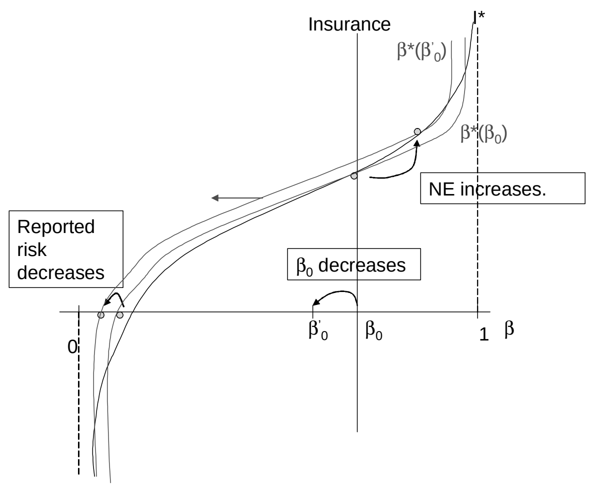
4.3 Insurance Market

4.3.1 Income, insurance choice and the DARA hypothesis

Eisenhauer (1997) find that life insurance purchases increase with wealth, a phenomenon consistent with the current model, as Theorem 4 shows. Eisenhauer (1997) bearing in mind the standard economic model, conclude that the findings reject the DARA hypothesis. However, this model suggests that is not necessarily the case. Greater insurance purchase as income increases can occur with a utility function exhibiting any of the absolute risk aversion properties: CARA, DARA or IARA. In fact, the current model implies that one can not conclude on the utility function absolute risk aversion property from observing changes in insurance choice as income changes.

4.3.2 Risk measure and insurance choice

Cawley and Philipson (1999) find that relatively risky individuals, under both self-reported and actual risk measures, are less likely to purchase life insurance, and once purchasing insurance the high-risk individuals, using self-reported risk measures, purchase less insurance. They find these results after controlling for wealth and proxis for bequest motives such as number of grandchildren, number of children, age of youngest child, average age of children, number of siblings, age of spouse and marital status. This model is consistent with such results. To see that, note that the model captures objective risk with β_0; by Theorem 3 we know that decreasing β_0, i.e., lower objective risk, the type-R equilibria increase, leading to higher insurance. As Corollary 7 shows, for a given insurance level, $\beta^*(I, \beta_0)$ moves always in the direction of β_0; Hence, for any given insurance level, lower reported risk will be associated with higher insurance. The figure below illustrates this point:



4.4 Other phenomena

4.4.1 Psychology Research: Cautious Optimism

Isen et al (1988) and Nygren et al (1996) examine the influence of positive affect on decision rule in risky situations. Both papers find a phenomenon labeled "cautious optimism". To illustrate this, consider the experiment in Nygren et al. In that experiment, participants are asked to make both numerical evaluations of verbal probabilities in three-outcome gambles and actual betting decisions for similar gambles. The participants who were induced with positive affect overestimated the probabilities of winning relative to losing, for the same phrases (optimism). However, when asked to gamble, these participants were much less likely to gamble relative to controls (cautious). The intrapersonal game is consistent with this phenomenon. Note that overestimated probabilities of winning in a report task is analogous to a shift in the mental best response, and therefore explaining cautious optimism is similar to explaining higher insurance purchase by low risk individuals (self reported measure). The key is that intrapersonal game framework makes the distinction between report and choice tasks. When engaged in a report task, participants will respond according to the mental best response. However, when engaged in a choice task; the choice is according to a Nash equilibrium which can be of type-R, leading lower perceived risk individual's choice to be more conservative, analogous to higher insurance.

Note that Nygren, Isen et al conclude that ..."these findings suggest that positive affect can promote an overt shift from a decision rule focusing primarily on probabilities to one focusing on utilities or outcome values, especially for losses." Translating this: positive affect promotes a shift from an adjustment process where the mental account moves first to one where the rational account moves first. Indeed, such a shift, in the presence of multiple Nash equilibria, means choosing more insurance or more cautiously. However, in this model, such a shift can not explain a higher reported win perception among the positive affect participants relative to controls.

5 Conclusions

Empirical evidence from the life insurance market as well as ample experimental evidence leads one to reevaluate economic decision theory. One of the main points arising from the psychology literature, as well as recent behavioral economic studies, is that people have preference over, and extract utility from, beliefs. Taking this point seriously and following the dual processes theory, I define a game – the intrapersonal game – in which the two accounts within the self, the rational and mental accounts, play simultaneously one against the other. The choice we observe is one of the Nash equilibria of this game.

Adding the mental account to the rational account can be viewed as adding a layer to, or enriching, the standard expected utility model. If the agent is not subject to mental considerations, then we are reduced to the classic model. However, if the agent decision involves some mental gain and cost then this model departs from the standard one, which suggests that the standard model will not provide a sufficient explanation. In other words, this model suggests that the failure of the traditional expected utility model to explain the data is in part due to systematic mental biases.

Taking the intrapersonal framework to the insurance context allows us to examine the insurance market and conduct comparative statics, given that people prefer to think the best outcome is more likely, or in other words, are optimistically biased. Many interesting dynamics arise from this framework; in particular it suggests a possible explanation for Cawley and Philipson's (1999) finding in the life insurance market. The suggested model can explain this since negative correlation between risk (actual and reported) and insurance decision is possible. In fact, this has interesting policy implications: educating the public to realize its higher-than-perceived risk can lead to an opposite reaction - lower insurance purchase!

Furthermore, this model is consistent (see theorem 4) with empirical results showing that risk aversion, as deduced from insurance decision, increases with wealth (Eisenhauer (1997)). Note that according to expected utility theory this rejects the hypothesis of DARA utility function, however, the present model does not. The current model suggests that the observed data is due to the interaction between the mental and rational account. Thus, one can not conclude the utility function risk aversion characteristics by observing behavior. In fact, we can have a DARA utility function and still have insurance choice increasing with wealth. Like the effect of an increase in income, an increase in insurance premium can lead either to higher or lower insurance purchase. This will depend on the utility function, the equilibrium type of the initial choice, as well as the ratio between initial insurance level and the shock size. Thus, insurance companies might actually have an incentive to decrease their insurance premium.

6 Appendix

Proof of corollary 2. Recall that I^* satisfies the following condition:

$$G \equiv \beta U^{'}(w_1 + (1-\gamma)I^*)(1-\gamma) - (1-\beta)U^{'}(w_2 - \gamma I^*)\gamma = 0$$

$\Rightarrow$

$$\frac{\partial I^*}{\partial \beta} = -\frac{\partial G/\partial \beta}{\partial G/\partial I^*} = -\frac{U^{'}(w_1 + (1-\gamma)I^*)(1-\gamma) + U^{'}(w_2 - \gamma I^*)\gamma}{\beta U''(w_1 + (1-\gamma)I^*)(1-\gamma)^2 + (1-\beta)U''(w_2 - \gamma I^*)\gamma^2} > 0$$

Note that $U^{'}(\cdot) \geq 0, U^{''}(\cdot) \leq 0$ for all $I \in (-\infty, \infty)$. As $\beta \to 0$, $I^* \to -\infty \Rightarrow U^{'}(w_1 + (1-\gamma)I^*) \longrightarrow \infty, U^{'}(w_2 - \gamma I^*) \longrightarrow 0$

assume that as $I^* \to \pm\infty$, $-\infty < U^{''}(\cdot) < 0 \Rightarrow$

$$lim_{\beta \to 0} \frac{\partial I^*}{\partial \beta} \to \infty,\ lim_{\beta \to 1} \frac{\partial I^*}{\partial \beta} \to \infty$$

Therefore, for at least one value of $\beta \in [0,1]$, $\frac{\partial^2 I^*}{\partial \beta^2} = 0$. Otherwise, it means that $\frac{\partial I^*}{\partial \beta}$ increases or decreases from ∞ to ∞ — contradiction. Also, one can conclude that for $\beta \to 0$, $\frac{\partial^2 I^*}{\partial \beta^2} < 0$ and $\beta \to 1$, $\frac{\partial^2 I^*}{\partial \beta^2} > 0$. Thus, for $\beta \to 0$, $I^*(\beta)$ is concave in β and as $\beta \to 1$, $I^*(\beta)$ is convex in β. ■

Proof of corollary 4. Recall that β^* satisfies the following condition:

$$G \equiv U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I) - c\frac{\partial f(\beta^*, \beta_0)}{\partial \beta} = 0$$

.

$f(\beta, \beta_0)$ is convex in β by Assumption 1 and suppose $\beta \neq \beta_0$:

$$\frac{\partial \beta^*}{\partial I} = -\frac{\partial G/\partial I}{\partial G/\partial \beta^*} = -\frac{U^{'}(w_1 + (1-\gamma)I)(1-\gamma) + U^{'}(w_2 - \gamma I)\gamma}{-c\frac{\partial^2 f(\beta^*, \beta_0)}{\partial \beta^2}} > 0$$

■

Proof of Corollary 5. Recall that β^* satisfies the following condition:

$$G \equiv U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I) - c\frac{\partial f(\beta^*, \beta_0)}{\partial \beta} = 0$$

.

$$\frac{\partial \beta^*}{\partial c} = -\frac{\partial G/\partial c}{\partial G/\partial \beta^*} = -\frac{-c\frac{\partial f(\beta^*,\beta_0)}{\partial \beta}}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}} = -\frac{\frac{\partial f(\beta^*,\beta_0)}{\partial \beta}}{\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}}$$

Recall that $f(\beta, \beta_0)$ is convex in β and reaches a minimum at $\beta = \beta_0$, by Assumption 1. Thus the sign of $\frac{\partial \beta^*}{\partial c}$ will be determined by the sign of $\frac{\partial f(\beta^*,\beta_0)}{\partial \beta}$. which is negative for values of $\beta^* < \beta_0$ where $I < z$, and positive for values $\beta^* > \beta_0$ where $I > z$. Consequently, $\frac{\partial \beta^*}{\partial c} \gtreqless 0 \Leftrightarrow \frac{\partial f(\beta^*,\beta_0)}{\partial \beta} \lesseqgtr 0 \Leftrightarrow I \lesseqgtr z$. ■

Proof of corollary 6. From the proof of Corollary 4:

$$\frac{\partial \beta^*}{\partial I} = -\frac{\partial G/\partial I}{\partial G/\partial \beta^*} = -\frac{U'(w_1 + (1-\gamma)I)(1-\gamma) + U'(w_2 - \gamma I)\gamma}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}}$$

$\Rightarrow$

$$\frac{\partial^2 \beta^*}{\partial I^2} = \frac{\partial \left[\frac{U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma}{c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}}\right)}{\partial I}$$

Taking the derivatives and rearranging implies that $\frac{\partial^2 \beta^*}{\partial I^2} \gtreqless 0 \Leftrightarrow$

$$\frac{\frac{\partial\left[U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma\right]}{\partial I}}{\left[U'(w_1 + (1-\gamma)I)(1-\gamma) + U'(w_2 - \gamma I)\gamma\right]^2} \gtreqless \frac{\frac{\partial^2 f'(\beta^*,\beta_0)}{\partial \beta^2}}{c\left[\frac{\partial f'(\beta^*,\beta_0)}{\partial \beta}\right]^2}$$

■

Proof of Theorem 1. **Existence** — Let $\Pi(\beta, \beta_0) = I^*(\beta) - \tilde{I}(\beta, \beta_0)$ where $\tilde{I}(\beta, \beta_0)$ is the insurance level such that $\beta^*(\tilde{I}, \beta_0) = \beta$ for some $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$, i.e., $\tilde{I}(\beta, \beta_0) = \beta^{*-1}(I)$ is the inverse function of $\beta^*(I)$ for values $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$. Assumption 3 guarantees that $0 < \underline{\beta} < \overline{\beta} < 1$. Thus, $\tilde{I}(\underline{\beta}, \beta_0) < I^*(\underline{\beta}) < I^*(\overline{\beta}) < \tilde{I}(\overline{\beta}, \beta_0)$, since $\tilde{I}(\underline{\beta}, \beta_0) \to -\infty$ and $\tilde{I}(\overline{\beta}, \beta_0) \to \infty$, while $I^*(\underline{\beta})$, $I^*(\overline{\beta})$ are finite. Consequently $\Pi(\underline{\beta}, \beta_0) > 0$ and $\Pi(\overline{\beta}, \beta_0) < 0$. Since both $\tilde{I}(\beta, \beta_0)$ and $I^*(\beta)$ are continuous in β, then $\Pi(\beta, \beta_0)$ is continuous in β $\forall \beta_0 \in [0, 1]$. Fix some $\beta_0 \in (0, 1)$. Having a continuous function Π which is $\Pi(\underline{\beta}, \beta_0) > 0$ and $\Pi(\overline{\beta}, \beta_0) < 0$ guarantees a solution $\Pi(\beta, \beta_0) = 0$ in the $\left[\underline{\beta}, \overline{\beta}\right]$ interval, which is a pure strategy Nash equilibrium of the intrapersonal game as it is a point of intersection of the two best responses.

Lowest and highest argument — By the boundary conditions, $\beta_L(\beta_0) \equiv \inf\{\beta | \Pi(\beta, \beta_0) \leq 0\}$, $\beta_H(\beta_0) \equiv \sup\{\beta | \Pi(\beta, \beta_0) \geq 0\}$ exist. One needs to show that $\beta_L(\beta_0), \beta_H(\beta_0)$ are solutions. Consider $\beta_L(\beta_0)$. By definition of β_L, the lim sup of $\Pi(\beta, \beta_0)$ as $\beta \uparrow \beta_L$ is nonnegative. Thus, $\Pi(\beta_L(\beta_0), \beta_0) \geq 0$. If $\beta_L(\beta_0) = \overline{\beta}$, then $\Pi(\overline{\beta}, \beta_0) < 0$ — contradiction. If $\beta_L(\beta_0) < \overline{\beta}$, and $\Pi(\beta_L(\beta_0), \beta_0) > 0$ then continuity implies that there is some $\varepsilon > 0$ such that $\Pi(\beta_L(\beta_0) + \varepsilon, \beta_0) > 0$ $\forall \beta \in [\beta_L(\beta_0), \beta_L(\beta_0) + \varepsilon]$ which is a contradiction to the definition of $\beta_L(\beta_0)$. Therefore, the conclusion is that $\Pi(\beta_L(\beta_0), \beta_0) = 0$. The case of $\beta_H(\beta_0)$ can be proved to be a solution by similar arguments.

Odd number of equilibria — Suppose there are two equilibria points $\beta_1 < \beta_2$. By the boundaries conditions, in the neighborhood of $\beta_1(\beta_0)$ $\Pi(\beta_1 - \varepsilon, \beta_0) > 0$ and $\Pi(\beta_1 + \varepsilon, \beta_0) < 0$. However, since there is only one more equilibrium point and Π is continuous then it must be that in the neighborhood of $\beta_2(\beta_0)$, $\Pi(\beta_2 - \varepsilon, \beta_0) < 0$ and $\Pi(\beta_2 + \varepsilon, \beta_0) > 0$ which is a contradiction to the boundaries conditions. This argument can be repeated for any case of even number of equilibrium points.

A chain — Note that the set of equilibrium points is defined as $\beta^{NE} \equiv \{\beta | \Pi(\beta, \beta_0) = 0\}$. Thus, the equilibrium points are beliefs $\beta \in \left[\underline{\beta}, \overline{\beta}\right]$ which forms a chain by definition. Since $I^*(\beta)$ is an increasing function, then it follows that the Nash equilibria of intrapersonal game which are vectors (I, β) form a chain.

Equilibrium Type — Recall the definition of the prospective adjustment process: $h = I^* \circ \beta^*$, where β^* is the mental BR and I^* is the rational BR. Given an insurance point I, β^* moves first and I^* second such that $h : R \to R$. A Nash equilibrium is of type P iff the prospective adjustment process converge to it. In other words, take a NE $\left(I^{NE}, \beta^{NE}\right)$. In the neighborhood of this equilibrium point it must be the case that for $I < I^{NE}$, $\dot{h} > 0$ and $I > I^{NE}$, $\dot{h} < 0$. This is equivalent to requiring that the slope of $\frac{\partial \tilde{I}(\beta^{NE}, \beta_0)}{\partial \beta} > \frac{\partial I^*(\beta^{NE})}{\partial \beta}$ which is equivalent to $\Pi(\beta^{NE} - \varepsilon, \beta_0) > 0$ and $\Pi(\beta^{NE} + \varepsilon, \beta_0) < 0$. By definition, then, β_L and β_H are of type P. A Nash equilibrium is of type R iff the retrospective adjustment process converge to it. In other words, take a NE $\left(I^{NE}, \beta^{NE}\right)$. In the neighborhood of this equilibrium point it must be the case that for $I < I^{NE}$, $\dot{h}^{-1} > 0$ and $I > I^{NE}$, $\dot{h}^{-1} < 0$. This is equivalent to requiring that the slope of $\frac{\partial \tilde{I}(\beta^{NE},\beta_0)}{\partial \beta} < \frac{\partial I^*(\beta^{NE})}{\partial \beta}$ which is equivalent to $\Pi(\beta^{NE}-\varepsilon,\beta_0) < 0$ and $\Pi(\beta^{NE}+\varepsilon,\beta_0) > 0$.By continuity of Π, then, the NE alternate from being of type P to type R.

Note that the existence, lowest and highest Nash equilibria and the chain results can be proved by defining a restricted intrapersonal game where the insurance strategy space is restricted between $\left[\underline{I},\overline{I}\right]$ such that the equilibria points of the intrapersonal game are not altered. The restricted game can be shown to be a supermodular game and thus these results follow from the properties of this class of games [see Topkis, 1998]. ■

Proof of Proposition 4. Define $\tilde{I}(\beta,\beta_0)$ to be the insurance level such that $\beta^*(\tilde{I},\beta_0)=\beta$ for some $\beta\in\left[\underline{\beta},\overline{\beta}\right]$, i.e., $\tilde{I}(\beta,\beta_0)=\beta^{*-1}(I)$ is the inverse function of $\beta^*(I)$ for values $\beta\in\left[\underline{\beta},\overline{\beta}\right]$. Assumption 3 guarantees that $\tilde{I}(\underline{\beta},\beta_0) < I^*(\underline{\beta}) < I^*(\overline{\beta}) < \tilde{I}(\overline{\beta},\beta_0)$ which implies $\Pi(\underline{\beta},\beta_0)>0$ and $\Pi(\overline{\beta},\beta_0)<0$. If $\Pi(\beta,\beta_0)$ is monotone decreasing in β, then there is a unique β where $\Pi=0$. This is the unique pure strategy Nash equilibrium. ■

Proof of Theorem 2. As in the proof of Theorem 1 a type-P equilibrium is where at (β^{NE},I^{NE}) $\frac{\partial \tilde{I}(\beta,\beta_0)}{\partial \beta} > \frac{\partial I^*(\beta)}{\partial \beta}$ or $\frac{\partial I^*(\beta)}{\partial \beta}\frac{\partial \beta^*(I)}{\partial I} < 1$.

$$\frac{\partial I^*(\beta)}{\partial \beta}\frac{\partial \beta^*(I)}{\partial I} = \frac{U'(w_1+(1-\gamma)I^*)(1-\gamma)+U'(w_2-\gamma I^*)\gamma}{\beta U''(w_1+(1-\gamma)I^*)(1-\gamma)^2+(1-\beta)U''(w_2-\gamma I^*)\gamma^2}\frac{U'(w_1+(1-\gamma)I)(1-\gamma)+U'(w_2-\gamma I)\gamma}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta^2}}$$

At $I^{NE}=z$ and $\beta^{NE}=\beta_0=\gamma$, define $\overline{w}\equiv\beta_0 w_1+(1-\beta_0)\,w_2$:

$$\begin{aligned}\frac{\partial I^*(\beta)}{\partial \beta}\frac{\partial \beta^*(I)}{\partial I} &= -\frac{U'(\overline{w})^2}{\beta_0(1-\beta_0)U''(\overline{w})c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta^2}}\\ &= \frac{U'(\overline{w})}{\beta_0(1-\beta_0)r_A(\overline{w})c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta^2}} < 1\end{aligned}$$

Rearranging:

$$\frac{U'(\overline{w})}{r_A(\overline{w})} < c\frac{\partial^2 f(\beta^*=\beta_0,\beta_0)}{\partial\beta^2}\beta_0(1-\beta_0)$$

■

Proof of Corollary 7. Recall that the choice of risk perception is determined by β^* which solves the following first order condition:

$$G\equiv U(w_1+(1-\gamma)I)-U(w_2-\gamma I)-c\frac{\partial f(\beta^*,\beta_0)}{\partial\beta}=0$$

Therefore, if the mental cost function $f(\beta,\beta_0)$ is submodular, i.e., $\frac{\partial^2 f(\beta,\beta_0)}{\partial\beta\partial\beta_0} \leq 0$, convex $\frac{\partial^2 f(\beta,\beta_0)}{\partial\beta^2} \geq 0$ and $\beta^* \neq \beta_0$:

$$\frac{\partial\beta^*}{\partial\beta_0} = -\frac{\frac{\partial G}{\partial\beta_0}}{\frac{\partial G}{\partial\beta^*}} = -\frac{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta\partial\beta_0}}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta^2}} = \frac{-\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta\partial\beta_0}}{\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta^2}} \geq 0$$

Furthermore, if $\frac{\partial^2 f(\beta,\beta_0)}{\partial\beta\partial\beta_0} < 0, and\ \beta \neq \beta_0$ then

$$\frac{\partial\beta^*}{\partial\beta_0} = -\frac{\frac{\partial G}{\partial\beta_0}}{\frac{\partial G}{\partial\beta^*}} = -\frac{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta\partial\beta_0}}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta^2}} = \frac{-\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta\partial\beta_0}}{\frac{\partial^2 f(\beta^*,\beta_0)}{\partial\beta^2}} > 0$$

■

Proof of Theorem 3. From Corollary 7, $\beta^*(I,\beta_0)$ is nondecreasing in β_0. Therefore $\tilde{I}(\beta,\beta_0), the$ inverse function, is nonincreasing in β_0, leading $\Pi(\beta,\beta_0) = I^*(\beta) - \tilde{I}(\beta,\beta_0)$ to be nondecreasing in β_0. Using Lemma 1 in Milgrom and Roberts [1994], provided below, one can conclude that the extreme Nash equilibria are nondecreasing in β_0, that is $\beta_L(\beta_0) = \inf\{\beta|\Pi(\beta) \leq 0\}$ and $\beta_H(\beta_0) = \sup\{\beta|\Pi(\beta) \geq 0\}$ are nondecreasing in β_0. Suppose $\beta^*(I,\beta_0)$ is strictly increasing in β_0, then $\tilde{I}(\beta,\beta_0)$ is strictly decreasing in β_0. Then $\Pi(\beta,\beta_0) = I^*(\beta) - \tilde{I}(\beta,\beta_0)$ is strictly increasing in β_0 meaning that there are no β such that $\Pi(\beta,\beta_0) = \Pi(\beta,\beta_0') = 0$. Using Lemma1, that means that the extreme Nash equilibria are strictly increasing in β_0. This argument can be applied for all equilibria of type P, as one can always find a local game that admits the same boundary conditions as the entire game and the equilibria we consider is one of the game's extreme points. Note that for an equilibrium of type R one can define a local game where it is one of the extreme equilibria. However, such a local game have the opposite boundary conditions and therefore the results are exactly the opposite.

Lemma 1 [Milgrom and Roberts, (1994)]: *Let $X \subset R$ and let $f,g : X \to R$. Suppose that for all $x \in X$, $g(x) \leq f(x)$. Then* $\inf\{x|g(x) \leq 0\} \leq \inf\{x|f(x) \leq 0\}$ *and* $\sup\{x|g(x) \geq 0\} \leq \sup\{x|f(x) \geq 0\}$ ■

Proof of Theorem 4. Recall that I^* solves the following equation:

$$G \equiv \frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = \frac{U'(w_2-z+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = 0$$

$$\frac{\partial I^*}{\partial w_2} = -\frac{\partial G/\partial w_2}{\partial G/\partial I^*}$$

$$= -\frac{\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*) - U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\right]\left[U'(w_2-\gamma I^*)\right]^2}{\left[U'(w_2-\gamma I^*)\right]^2\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)(1-\gamma) + U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\gamma\right]}$$

$\frac{\partial I^*}{\partial w_2} \gtreqless 0 \Leftrightarrow \frac{U''(w_2-z+(1-\gamma)I^*)}{U'(w_2-z+(1-\gamma)I^*)} \gtreqless \frac{U''(w_2-\gamma I^*)}{U'(w_2-\gamma I^*)}$

Using definition of absolute risk aversion $r(x) = -\frac{U''(x)}{U'(x)}$

$\frac{\partial I^*}{\partial w_2} \gtreqless 0 \Leftrightarrow r_A\left(w_2 - z + (1-\gamma)I^*\right) \lesseqgtr r_A\left(w_2 - \gamma I^*\right)$

Recall that β^* solves the following equation: $G \equiv U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I) - c\frac{\partial f(\beta^*,\beta_0)}{\partial \beta}$

$= U(w_2 - z + (1-\gamma)I) - U(w_2 - \gamma I) - c\frac{\partial f(\beta^*,\beta_0)}{\partial \beta} = 0$

If $\beta^* \neq \beta_0$:

$\frac{\partial \beta^*}{\partial w_2} = -\frac{\partial G/\partial w_2}{\partial G/\partial \beta^*} = -\frac{U'(w_2-z+(1-\gamma)I)-U'(w_2-\gamma I)}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}} \gtreqless 0$

$\Rightarrow \frac{\partial \beta^*}{\partial w_2} \gtreqless 0 \Leftrightarrow w_2 - z + (1-\gamma)I \lesseqgtr w_2 - \gamma I \Leftrightarrow I \lesseqgtr z$

Thus:

$$\begin{array}{llll} & DARA & CARA & IARA \\ I<z & \frac{\partial I^*}{\partial w_2}<0, \frac{\partial \beta^*}{\partial w_2}>0 & \frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}>0 & \frac{\partial I^*}{\partial w_2}>0, \frac{\partial \beta^*}{\partial w_2}>0 \\ I=z & \frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}=0 & \frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}=0 & \frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}=0 \\ I>z & \frac{\partial I^*}{\partial w_2}>0, \frac{\partial \beta^*}{\partial w_2}<0 & \frac{\partial I^*}{\partial w_2}=0, \frac{\partial \beta^*}{\partial w_2}<0 & \frac{\partial I^*}{\partial w_2}<0, \frac{\partial \beta^*}{\partial w_2}<0 \end{array}$$

Define $\Pi(\beta,\beta_0) = I^*(\beta) - \tilde{I}(\beta,\beta_0)$. $\Pi(\underline{\beta},\beta_0) > 0$ and $\Pi(\overline{\beta},\beta_0) < 0$ and equilibria of this game is where $\Pi(\beta,\beta_0) = 0$.

$$\begin{array}{llll} & DARA & CARA & IARA \\ I<z & \Delta\Pi\ ? & \Delta\Pi \uparrow & \Delta\Pi \uparrow \\ I=z & \Delta\Pi=0 & \Delta\Pi=0 & \Delta\Pi=0 \\ I>z & \Delta\Pi\ ? & \Delta\Pi \downarrow & \Delta\Pi \downarrow \end{array}$$

Using Lemma 1 in Milgrom and Roberts (1994) one can conclude the following for any equilibria of type P (see proof of Theorem 3 above for Lemma 1 and an argument why this holds for any type-P equilibrium) :

$$\begin{array}{llll} & DARA & CARA & IARA \\ I<z & ? & NE \uparrow & NE \uparrow \\ I=z & unchanged & unchanged & unchanged \\ I>z & ? & NE \downarrow & NE \downarrow \end{array}$$

Note that if the NE is of type R then the result is exactly the opposite.

For the second part of the Theorem:

An increase in the shock size z will increase I^* and will decrease β^* as is shown below:

Recall that I^* solves the following equation:

$$G \equiv \frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = \frac{U'(w_2-z+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = 0$$

$$\frac{\partial I^*}{\partial z} = -\frac{\partial G/\partial z}{\partial G/\partial I^*} =$$

$$-\frac{\left[-U''(w_2-z+(1-\gamma)I^*)\right]\left[U'(w_2-\gamma I^*)\right]^2}{\left[U'(w_2-\gamma I^*)\right]\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)(1-\gamma)+U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\gamma\right]}$$

$$\Rightarrow \frac{\partial I^*}{\partial z} > 0$$

Recall β^* solves the following equation

$$G \equiv U(w_1 + (1-\gamma)I) - U(w_2 - \gamma I) - c\frac{\partial f(\beta^*,\beta_0)}{\partial \beta} = 0$$

$$= U(w_2 - z + (1-\gamma)I) - U(w_2 - \gamma I) - c\frac{\partial f(\beta^*,\beta_0)}{\partial \beta}$$

$$\frac{\partial \beta^*}{\partial z} = -\frac{\partial G/\partial z}{\partial G/\partial \beta^*} = -\frac{-U'(w_2-z+(1-\gamma)I)}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}}$$

$$\Rightarrow \frac{\partial \beta^*}{\partial z} < 0$$

Consequently, as z increases both $\tilde{I}(\beta,\beta_0)$ and $I^*(\beta)$ increase and the change in $\Pi(\beta,\beta_0)$ is unclear. Therefore, it is not clear how NE changes with z. ■

Proof of Theorem 5. Recall that I^* solves the following equation: $G \equiv \frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = \frac{U'(w_2-z+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} - \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} = 0$

$$\frac{\partial I^*}{\partial \gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial I^*}$$

$$= \frac{I^*\left[U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)-U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)\right] - \frac{1}{(1-\gamma)^2}\frac{(1-\beta)}{\beta}\left[U'(w_2-\gamma I^*)\right]^2}{-\left[U''(w_2-z+(1-\gamma)I^*)U'(w_2-\gamma I^*)(1-\gamma)+U'(w_2-z+(1-\gamma)I^*)U''(w_2-\gamma I^*)\gamma\right]}$$

Rearranging and using $\frac{U'(w_1+(1-\gamma)I^*)}{U'(w_2-\gamma I^*)} = \frac{\gamma}{(1-\gamma)}\frac{(1-\beta)}{\beta} \Rightarrow$

$$\frac{\partial I^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow I^* \times \gamma(1-\gamma) \times \left[\frac{1}{r_A(w_2-z+(1-\gamma)I^*)} - \frac{1}{r_A(w_2-\gamma I^*)}\right] \gtreqless 1$$

Recall that β^* solves the following equation: $G \equiv U(w_2-z+(1-\gamma)I) - U(w_2-\gamma I) - c\frac{\partial f(\beta^*,\beta_0)}{\partial \beta} = 0$

$$\frac{\partial \beta^*}{\partial \gamma} = -\frac{\partial G/\partial \gamma}{\partial G/\partial \beta^*} = -\frac{U'(w_2-z+(1-\gamma)I)(-I)+U'(w_2-\gamma I)I}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}} = -\frac{\left[U'(w_2-\gamma I)-U'(w_2-z+(1-\gamma)I)\right]I}{-c\frac{\partial^2 f(\beta^*,\beta_0)}{\partial \beta^2}}$$

$$\Rightarrow \frac{\partial \beta^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow \left[U'(w_2-\gamma I) - U'(w_2 - z + (1-\gamma)I)\right] I \gtreqless 0$$

$$\Leftrightarrow \begin{cases} if\ I > 0 & \frac{\partial \beta^*}{\partial \gamma} \gtreqless 0 \Leftrightarrow I \gtreqless z \\ if\ I < 0 & \frac{\partial \beta^*}{\partial \gamma} > 0 \end{cases}$$

Thus:

	$DARA$	$CARA$	$IARA$
$I < 0$	$\frac{\partial I^*}{\partial \gamma}?, \frac{\partial \beta^*}{\partial \gamma} > 0$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} > 0$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} > 0$
$0 < I < z$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} < 0$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} < 0$	$\frac{\partial I^*}{\partial \gamma}?, \frac{\partial \beta^*}{\partial \gamma} < 0$
$I = z$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} = 0$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} = 0$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} = 0$
$I > z$	$\frac{\partial I^*}{\partial \gamma}?, \frac{\partial \beta^*}{\partial \gamma} > 0$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} > 0$	$\frac{\partial I^*}{\partial \gamma} < 0, \frac{\partial \beta^*}{\partial \gamma} > 0$

Define $\Pi(\beta,\beta_0) = I^*(\beta) - \tilde{I}(\beta,\beta_0)$. $\Pi(\underline{\beta},\beta_0) > 0$ and $\Pi(\overline{\beta},\beta_0) < 0$ and equilibria of this game is where $\Pi(\beta,\beta_0) = 0$.

	$DARA$	$CARA$	$IARA$
$I<0$	$\Delta\Pi$?	$\Delta\Pi$?	$\Delta\Pi$?
$0<I<z$	$\Delta\Pi\downarrow$	$\Delta\Pi\downarrow$	$\Delta\Pi$?
$I=z$	$\Delta\Pi\downarrow$	$\Delta\Pi\downarrow$	$\Delta\Pi\downarrow$
$I>z$	$\Delta\Pi$?	$\Delta\Pi$?	$\Delta\Pi$?

Using Lemma 1 in Milgrom and Roberts (1994) one can conclude the following for any equilibria of type P (see proof of Theorem 3 above for Lemma 1 and an argument why this holds for any type-P equilibrium) :

	$DARA$	$CARA$	$IARA$
$I<0$	?	?	?
$0<I<z$	$NE\downarrow$	$NE\downarrow$	?
$I=z$	$NE\downarrow$	$NE\downarrow$	$NE\downarrow$
$I>z$	?	?	?

Note that if the NE is of type R, then the result is exactly the opposite . ■

Notes

[1]Mixed fanning hypothesis argues that indifference curves fan out for less favorable lotteries, while they fan in for more favorable ones.

[2]Weight assigned to a given state of the world according to a given function of the true probability.

[3]Indeed, the weight of an event in the rank dependent expected utility (RDEU) is determined by its ranking in the distribution of possible outcomes (see Lopes (1995) and Camerer(1995)). In that sense RDEU allows outcomes to affect the weighting function. However, as long as the ranking is kept, the weight function according to RDEU is unchanged.

[4]Note that I will refer to subjective probability as probability judgment to distinguish from Savage's notion of subjective beliefs.

[5]"....[Many] controversies involving the nature of expectation could be avoided by recognizing at the outset that man's conscious actions are the reflection of his beliefs and of nothing else."

[6]Choice stands for a result of personal motives or goals and it is not to mean a deliberate, fully conscious act.

[7]Base rate can be thought of as the agent's probability reference point, or best assessment. This is the reality as the agent perceive it.

[8]Anchoring is the tendency of people to stick to an anchor rate in an ambiguous situation.

[9]Although experiments indicate that subjects are anchored to motivated beliefs and then adjust to the provided anchor rate, in reality the anchor rate is endogenous. Hence, one could argue that agents are anchored to the base rate and then adjust towards their motivational goals.

[10]Gain can be either positive or negative.

[11]For a low level of insurance, the difference in utilities in the two future states of the world is relatively large. Thus, changing beliefs will have a relatively large mental gain. As insurance increases, the same change in beliefs will have lower impact on the mental gain since the differences in the state contingent utilities is smaller.

[12]Note that the function

$$f(\beta,\beta_0)=\ln\left(\frac{1}{(\beta-\underline{\beta})(\overline{\beta}-\beta)}\right)-\beta\beta_0$$

satisfies all of the above assumptions.

[13]The illustration of the mental best response is for all strictly concave utility functions such that $\beta^*(I)$ is, for some ranges, convex in I and then becomes concave (see Corollary 6 for exact conditions).

[14]Note that this is equivalent to the function $\Pi = I^*(\beta) - \tilde{I}\ (\beta)$ being monotone decreasing $\left(\frac{\partial\Pi}{\partial\beta}<0\right)$ in the neighborhood of the equilibrium β^{NE}.

[15]Note that this is equivalent to the function $\Pi = I^*(\beta) - \tilde{I}\ (\beta)$ being monotone decreasing $\left(\frac{\partial\Pi}{\partial\beta}<0\right)$ in the neighborhood of the equilibrium β^{NE}.

[16]Note that if, alternatively, the mental cost function would be finite for $\forall\beta\in[0,1]$ and c is sufficiently small, then again the intrapersonal game would have odd numbers of equilibria with alternating types, but the extreme equilibria points are of type R.

[17]Note that this is equivalent to requiring $\frac{\partial\dot{h}(I)}{\partial I}=\frac{\partial h(I)}{\partial I}-1<0$ for all possible equilibrium values of I.

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