A Model of Stochastic Liquidity

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Abstract

This paper proposes a dynamic multi-security model in which liquidity reflects stochastic variation, persistence, and commonality of underlying information variance. Illiquidity, price-change variance, and trading volume all increase in the size of information. Using high frequency data, I perform structural estimation of the model by Bayesian Markov-Chain Monte-Carlo simulation, with the conditional volatility of underlying information modeled as stochastic volatility or realized volatility controlling for microstructure noise. I find that a Dow stock's liquidity decreases in the size of information about not only itself but also about other Dow stocks, demonstrating a significant cross-sectional effect of information on liquidity.

Introduction

The unobservability of information has been attracting, rather than repelling, researchers' interest in understanding how it disseminates within and across markets. Information has been long considered an important driving force behind the variation in measurable quantities in stock markets, such as return volatility and trading volume.[1] Recent research, however, has extended its attention to the next dimension of variability: that of liquidity. The stochastic nature of liquidity is now a presumption in many empirical studies, most prominently in the literature on liquidity risk.[2] In such applications, liquidity measures are typically constructed exogenously without imposing equilibrium restrictions. Some other applications, however, call for an econometric model that fully incorporates the mechanism through which prices and the terms of trade are determined in real markets. This paper attempts to fill such a request by explicitly modeling the dynamic relation between information and liquidity.

More specifically, this paper proposes a dynamic multi-security model in which liquidity reflects stochastic variation, persistence, and commonality of underlying information variance. Trading takes place over time in a centralized market a la Kyle (1985). Both informed traders and noise traders submit orders that are absorbed by a competitive market maker. As in Admati and Pfleiderer (1988), pieces of information about terminal payoffs are gradually revealed over time. Unlike these traditional models, however, the current model explicitly permits the underlying information process to be heteroskedastic. Important special cases include GARCH and stochastic volatility models, which are assumed to derive empirical implications and to

[1]This notion appears as early as Clark (1973). For subsequent work, see Tauchen and Pitts (1983), Karpoff (1987), Lamoureux and Lastrapes (1990), Gallant, Rossi, and Tauchen (1992), Andersen (1996), and references therein.

[2]Examination of liquidity-risk pricing includes work by Acharya and Pedersen (2005), Korajczyk and Sadka (2008), Pastor and Stambaugh (2003), and Watanabe and Watanabe (2008). Liquidity variation is studied in many other forms, such as unexpected illiquidity (Amihud (2002)), commonality in liquidity (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001)), and transmission of liquidity shocks across markets (Chordia, Sarkar, and Subrahmanyam (2005)). Also see related work by Chordia, Subrahmanyam, and Anshuman (2001), Domowitz, Hansch, and Wang (2005), and Goldreich, Hanke, and Nath (2005).

estimate the model. With persistence in the size of information, these models parsimoniously incorporate the feature that news tends to come in waves (Longin (1997, p.845)). This setting can be viewed as a variant of Longin (1997) to allow multi-security markets and imperfectly informed traders, or as a variant of Caballe and Krishnan (1994) to allow multiple trading sessions and possibly heteroskedastic information processes.

Using a multivariate GARCH information process, I first demonstrate that the conditional price-change variance, trading volume, and illiquidity (measured by Kyle's lambda) all become time-varying. The larger the size of the past information shock, the higher are the conditional price-change variance, expected trading volume, and illiquidity. Trading volume rises since a large information shock, whether positive or negative, provides an opportunity for informed traders to make a profit at the expense of noise traders. Prices become volatile due to a rise in information-based trade. The effect on liquidity is a little ambiguous *ex ante* because of two opposing effects. Given the persistence in information variance, the market maker will worsen the terms of trade due to adverse selection (which I call the volatility effect). At the same time, higher competition among more informed traders would allow him to improve the liquidity because each of them will make less profit (the competition effect). A numerical example shows that the former effect dominates the latter, decreasing liquidity as more information is produced. I also demonstrate that these effects can occur cross-sectionally, in that the generation of information about one firm increases another firm's illiquidity, return variance, and trading volume if there is flow of information from the former to the latter.

I provide empirical evidence consistent with the model's predictions. The theory provides a structural equation model that is amenable to Bayesian Markov-Chain Monte-Carlo (MCMC) estimation. Using high frequency data, I construct hourly return and signed and unsigned share turnover for 30 stocks in the Dow Jones Industrial Average (DJIA) Index. The conditional volatility of the information process is modeled as either stochastic volatility or realized volatility controlling for microstructure noise and infrequent sampling. The latter approach employs Zhang, Mykland, and Aït-Sahalia's (2005) two-scales realized variance, which is a bias-corrected average of subsampled realized variances. In both approaches, the specification nests various existing models of liquidity, such as Campbell, Grossman, and Wang (1993), Grossman and Miller (1988), Llorente et al. (2002), and Pastor and Stambaugh (2003). In these models, risk aversion and non-informational trade play a crucial role in forming market liquidity. Allowing for such a possibility lets us evaluate the relevance of an alternative mechanism of liquidity formation that clearly differs from our risk-neutral, asymmetric-information framework.

An advantage of the Bayesian MCMC framework is that it readily estimates unobservable state variables along with model parameters. This allows us to back out the information shock for each Dow stock, and therefore construct the "market" information shock as the average of the other 29 stocks in the DJIA Index. Using the estimated series, I find that a stock's price impact parameter is positively associated with lagged squared information shocks of both itself and the market for 28 out of the 30 Dow stocks. This provides evidence that a firm's illiquidity exacerbates in the size of information about not only itself, but also about other, presumably relevant firms, demonstrating a significant cross-sectional effect of information on liquidity. A caveat is that my result should be interpreted with caution as some of the model restrictions are violated. However, it is the virtue of structural estimation to be able to strictly impose theoretical restrictions on deep parameters, which are often ignored. The methodology employed in the current article measures information shocks as accurately as possible given the current state of the literature.

Kyle-Admati-Pfleiderer models with conditionally heteroskedastic information processes were first developed by Foster and Viswanathan (1993, 1995). The idea appears in Longin (1997) that the combination of GARCH information release and endogenous information acquisition can parsimoniously introduce a threshold effect in conditional price-change volatility.

Unlike these models, however, the current paper focuses on liquidity and its variation resulting from the generation of information. In addition, it closely examines cross-sectional implications that cannot be analyzed by these single-security models. In the informationally homoskedastic paradigm, there do exist multiple-security Kyle models that allow for a cross-sectional analysis. See, for example, Caballe and Krishnan (1994), Kumar and Seppi (1994), and Subrahmanyam (1991).[3] In these models, however, price-change variances, trading volume, and liquidity are all deterministic because both the underlying information and noise-trading processes have deterministic variances.[4] On the empirical side, the current paper proposes a new class of stochastic and realized volatility models in which the underlying information process affects the liquidity of assets. I demonstrate that such models can be readily estimated in a Bayesian MCMC framework.

The paper proceeds as follows. The next section sets out the model and solves for an equilibrium. It also examines the relation between information shocks and conditional price-change variance, trading volume, as well as liquidity. Section 2 provides the empirical evidence. The final section delivers concluding remarks. The appendix contains proofs. A technical appendix is available on the author's web site.

1 A Theory of Stochastic Liquidity

This section introduces a variant of Kyle-Admati-Pfleiderer model in which pieces of information about multiple securities are released with possible conditional heteroskedasticity and persistence. The number of informed traders is endogenously determined. I also examine the

[3]For related models with multiple securities or markets, see Chan (1993), Chowdhry and Nanda (1991), and Hagerty (1991).

[4]Outside the Kyle realm, Fernando (2003) investigates how liquidity shocks transmit across securities and risk-averse investors. The liquidity shock in his model is introduced as a change in investors' marginal valuation of risky assets without new information about the fundamental value of securities. In contrast, liquidity is endogenously determined in my model, depends critically on the asymmetric information between market participants, and is itself the key subject of investigation.

effects of information shocks on conditional variance, trading volume, and liquidity.

1.1 The Model

I modify Longin's (1997) setting to allow for a multi-security economy with imperfect information a la Caballe and Krishnan (1994). K risky assets are traded in a centralized market. Trading takes place over T periods. At the end of period T, the risky assets pay a vector of terminal dividends,

$$\widetilde{D}_T = D_0 + \sum_{t=1}^{T} \widetilde{\delta}_t, \qquad \widetilde{\delta}_t | \mathcal{F}_{t-1} \sim N(0, \Sigma_t), \tag{1}$$

where D_0 is a K vector of constants and Σ_t a $K \times K$ symmetric positive-definite matrix contained in the public information set at time $t-1$, $\mathcal{F}_{t-1} \equiv \{\Sigma_1, ..., \Sigma_t, \widetilde{\delta}_1, \ldots, \widetilde{\delta}_{t-1}\}$. As in Admati and Pfleiderer (1988), the random vector $\widetilde{\delta}_{t-1}$ is revealed to all market participants just prior to trading at time $t-1$ and therefore is in $\mathcal{F}_{t-1}$. Once revealed, it is considered a vector of pieces of public information about the terminal payoff. Specific formulation of Σ_t is not required for derivation of an equilibrium. At present, we note that it includes the class of GARCH and stochastic volatility models.

There are N_t informed traders at time t. Informed trader n receives a vector of noisy signals about the terminal payoffs,

$$\widetilde{y}_{t,n} = \widetilde{\delta}_{t+1} + \widetilde{\zeta}_t + \widetilde{\varepsilon}_{t,n}, \tag{2}$$

where $\widetilde{\zeta}_t \sim N(0, \Gamma_t)$ is the vector of common noise and $\widetilde{\varepsilon}_{t,n} \sim N(0, \Phi_t)$ is the vector of idiosyncratic noise, both of which are distributed independently, but possibly non-identically, over time. That is, the error-variance matrices, Γ_t and Φ_t, can change over time as long as they are deterministic and commonly known to all market participants at time t. These error-variance matrices can be zero; for example, if $\Gamma_t = \Phi_t \equiv 0$, traders acquire perfect private information, $\widetilde{y}_{t,n} = \widetilde{\delta}_{t+1}$. The specific forms of Γ_t and Φ_t are not required for the derivation of equilibrium.

Across informed investors, the idiosyncratic noise, $\widetilde{\varepsilon}_{t,n}$, is distributed independently and identically. N_t is a measurable function of $\mathcal{F}_t$ in general and can change over time as long as it is known to all market participants at time t. All informed traders are risk neutral and maximize their expected terminal profit in each period. Specifically, informed trader n solves

$$\max_{x_{t,n}} \mathbf{E}[(\widetilde{D}_T - \widetilde{P}_t)' x_{t,n} | \mathcal{F}_{t,n}], \tag{3}$$

where $\widetilde{P}_t$ is the K vector of prices, $x_{t,n}$ is the K vector of his demand in period t, $\mathcal{F}_{t,n} = \mathcal{F}_t \cap \{\widetilde{y}_{t,n},\ \widetilde{w}_1, \ldots, \widetilde{w}_{t-1}\}\ \forall n$ is his information set at time t, and $\widetilde{w}_{t-1}$ is the net order flow of the informed traders and noise traders in the previous trading session. Prior to trading at time t, informed traders observe neither the contemporaneous net order flow, $\widetilde{w}_t$, nor the current prices, $\widetilde{P}_t$, that will soon be announced by the market maker. Noise traders submit a vector of exogenous random orders, $\widetilde{z}_t \sim N(0, \Psi_t)$, where Ψ_t is a K dimensional symmetric positive-definite matrix and is a deterministic function of time.

A competitive market maker sets prices in this batch-trading market. After observing the vector of net order flows, $\widetilde{w}_t = \sum_{n=1}^{N_t} x_{t,n} + \widetilde{z}_t$, but not the individual trades separately. He sets prices according to the market-efficiency condition that prices equal the expected value of terminal payoffs given his information set,

$$\widetilde{P}_t = \mathbf{E}[\widetilde{D}_T | \mathcal{F}_{t,m}], \tag{4}$$

where $\mathcal{F}_{t,m} = \mathcal{F}_t \cap \{\widetilde{w}_1, \ldots, \widetilde{w}_t\}$. As usual, this can be viewed as a competitive zero-profit condition under risk neutrality. Clearly, the information sets of informed traders and the market maker do not nest each other, creating information asymmetry.

1.2 Equilibrium

Following the standard solution technique, we restrict our attention to a linear equilibrium. We begin by assuming that the market maker uses a pricing rule in which prices are a linear function of net order flow. Given the price function, we next solve the profit-maximization problem of informed traders in Equation (3). Finally, the resulting demand is substituted into the market-efficiency condition in Equation (4) to determine the coefficients in the original price conjecture. The result is summarized in the following theorem:

Theorem 1 *(Equilibrium) There exists a linear symmetric equilibrium in which the market maker sets prices, $\widetilde{P}_t$, according to the following pricing rule and informed trader n submits the following market order, $x_{t,n}$:*

$$\widetilde{P}_t = \widetilde{D}_t + A_t \widetilde{w}_t, \tag{5}$$

$$x_{t,n} = B_t \widetilde{y}_{t,n}, \tag{6}$$

where $\widetilde{D}_t \equiv \sum_{s=1}^{t} \widetilde{\delta}_s$ is the cumulative payoffs revealed up to time t, $\widetilde{w}_t = B_t \sum_{n=1}^{N_t} \widetilde{y}_{t,n} + \widetilde{z}_t$ is the net order flow, and

$$A_t \equiv \sqrt{N_t}\Psi_t^{-\frac{1}{2}} M_t^{\frac{1}{2}} \Psi_t^{-\frac{1}{2}}, \quad B_t \equiv A_t^{-1} J_t^{-1} \Sigma_{t,\xi} \Sigma_{t+1}^{-1},$$

$$M_t \equiv \Psi_t^{\frac{1}{2}} J_t^{-1} \Sigma_{t,\xi} J_t'^{-1} \Psi_t^{\frac{1}{2}}, \quad \Sigma_{t,\xi} \equiv \Sigma_{t+1}(\Sigma_{t+1} + \Gamma_t + \Phi_t)^{-1}\Sigma_{t+1},$$

$$J_t \equiv 2I + (N_t - 1)\Sigma_{t+1}(\Sigma_{t+1} + \Gamma_t + \Phi_t)^{-1}(\Sigma_{t+1} + \Gamma_t)\Sigma_{t+1}^{-1}.$$

Furthermore, this is the unique equilibrium in which A_t is symmetric.

Clearly, if Σ_{t+1} and Ψ_t are symmetric positive definite (and Γ_t and Φ_t are symmetric positive definite or zero matrices), so are M_t and A_t. Therefore, a stock's price increases with its own order flow. The price sensitivity to the order flow is represented by the corresponding diagonal element of A_t, a multivariate version of Kyle's lambda. This is an inverse measure of liquidity. The B_t matrix represents trade intensity; the larger the B_t matrix, the larger trades the informed traders will place based on their private signals, $\widetilde{y}_{t,n}$.

1.3 Information Acquisition

This subsection solves traders' information-gathering problem. Assume that traders can acquire the vector of private signals at a fixed cost of c_t, a positive scalar known at time t. Let π_t be the expected total profit of all informed traders when there are N_t of them. As in Admati and Pfleiderer (1988), since the market maker earns zero expected profit, the total profit of all informed traders equals the losses of the noise traders.

Lemma 1 *(Expected profit of informed traders) The expected total profit of all informed traders is given by*

$$\pi_t = \sqrt{N_t} tr(M_t^{\frac{1}{2}}),$$

where $tr(\cdot)$ is the trace operator and M_t is defined in Theorem 1.

If Ψ_t, Σ_{t+1}, Γ_t, and Φ_t are diagonal, so is M_t and it is easy to see that π_t is increasing in the noise-trading variance of any stock. This is so because, as the noise trading increases and gives informed traders more room for camouflage, they will trade more aggressively and thus can make a larger profit. Clearly, the profit per informed trader, π_t/N_t, decreases with N_t and traders decide to become informed as long as the per-trader profit covers the cost of information acquisition.

Corollary 1 *(Number of informed traders) When traders can acquire information at a cost of*

c_t, the number of informed traders is given by the maximum integer N_t such that

$$\frac{1}{\sqrt{N_t}} tr(M_t^{\frac{1}{2}}) \geq c_t. \tag{7}$$

We assume that $tr(M_t^{\frac{1}{2}})/c_t \geq 1 \ \forall t$ so that there is always at least one informed trader. An information shock, whether negative or positive, tends to increase Σ_{t+1} and hence π_t and N_t. If the conditional variance of information pieces persists, so does the number of informed traders.

To facilitate exposition, define two types of firms, informationally *active* and *passive*. While the prospect of the informationally active firm is affected by news about itself only, the prospect of the informationally passive firm is affected by news about both itself and other firms. The defining feature of the latter is that it receives a cross-sectional inflow of information, which could be earnings announcement of other firms in the same industry; we will come back to this point in Subsection 2.7. More specifically, the following numerical example is used for illustrative purposes throughout the rest of the paper.

Example 1 *(Two securities with asymmetrically dependent information variances) Suppose two securities, A and P, are traded. The first security, A, is informationally active with autonomous, persistent information variance. The second security, P, is informationally passive and has less persistent information variance that depends on Security A's. The information and noise-trading processes of the two securities are mutually and cross-sectionally uncorrelated. Specific parameter values are*

$$\Sigma_t = \begin{pmatrix} \sigma_{t,A}^2 & 0 \\ 0 & \sigma_{t,P}^2 \end{pmatrix}, \quad \begin{aligned} \sigma_{t,A}^2 &= .5 + .7\sigma_{t-1,A}^2 + .25\widetilde{\delta}_{t-1,A}^2, \\ \sigma_{t,P}^2 &= 2 + .7\sigma_{t-1,P}^2 + .1\widetilde{\delta}_{t-1,P}^2 + .1\widetilde{\delta}_{t-1,A}^2, \end{aligned} \tag{8}$$

$$\Psi_t \equiv \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \quad \Gamma_t = \Phi_t \equiv 0, \quad c_t \equiv 0.1, \ \forall t. \quad \blacksquare$$

Equation (8) implies that the unconditional information variances of Securities A and P are 10 and 15, respectively. It is set higher for Security P to capture the idea that its information is less precise than Security A's. $\widetilde{\delta}_{t-1,A}^2$ may be considered a common squared shock that affects the information variances of both securities, but Security P's information variance contains an additional squared noise, $\widetilde{\delta}_{t-1,P}^2$. Also, it has a smaller noise-trading variance. Security A may be interpreted as a large blue-chip firm with massive trading volume whose news abundantly flows through the media. In contrast, Security P can be thought of as a small, economically related firm on its supply chain with relatively little analyst coverage. The assumed characteristics of the two securities seem to gain some empirical support; for example, using a sequential trade model, Easley, Kiefer, O'Hara, and Paperman (1996) find that high volume stocks tend to have a higher probability of information events and higher arrival rates of both informed and uninformed traders. The example assumes that $\Gamma_t = \Phi_t \equiv 0$, i.e., traders acquire perfect information. This is perfectly admissible and is made here to aid intuition. However, in our empirical analysis, we will allow both Γ_t and Φ_t to be free parameters to be estimated.

Clearly, all covariance matrices are diagonal for any t and the two information processes are stationary with finite positive variances. In all the numerical analyses and formulae to follow, think of ourselves as an econometrician standing at time $t-1$ with the public information set $\mathcal{F}_{t-1}$ ($\ni \widetilde{\delta}_{t-1}$). That is, all graphs and formulae for time t quantities represent expected values conditional on $\mathcal{F}_{t-1}$, denoted by $\mathbf{E}_{t-1}[\cdot]$. However, for convenience, I follow the convention to call time t "current" and time $t-1$ the "past."

Panel (a) of Figure 1 plots the number of informed traders against the past squared information shocks of the two stocks. It is a nondecreasing step function of the two shocks due to integer programming at time $t-1$ similar to the time t problem in Corollary 1. The number of informed traders is affected more by Security A's squared shock than Security P's, because the former produces more information than the latter as represented by a larger coefficient on

$\widetilde{\delta}^2_{t-1,A}$ in the $\sigma^2_{t,A}$ equation in (8).

Panel (b) shows the expected number of informed traders at time t, $\mathbf{E}_{t-1}[N_t]$. Note that there is no analytic expression for this conditional expectation, as the conditional distribution of N_t given $\mathcal{F}_{t-1}$ is unknown because of the integer programming in Corollary 1. It is therefore computed by a Monte-Carlo simulation. In this and all other simulations to follow, 10,000 experiments are run on each grid with variance reduction methods known as the antithetic variable technique and moment matching.[5] The surface of $\mathbf{E}_{t-1}[N_t]$ in the panel looks somewhat like a "smoothed" version of Panel (a). This implies that the number of informed traders is also persistent; the larger N_{t-1}, the larger is N_t on average. Because of this feature, like the number of informed traders at $t-1$, its expected value at time t is affected more by the past squared shock to Security A than that to Security P.

1.4 Conditional Price-Change Variance

From Equation (5), the price-change vector can be written as

$$\Delta\widetilde{P}_t \equiv \widetilde{P}_t - \widetilde{P}_{t-1} = \widetilde{\delta}_t + A_t\widetilde{w}_t - A_{t-1}\widetilde{w}_{t-1}. \tag{9}$$

The following corollary computes the conditional price-change variance.

Corollary 2 *(Price-change variance) The price-change variance conditional on the public information set at time $t-1$, $\mathcal{F}_{t-1}$, is given by*

$$H_t \equiv Var_{t-1}(\Delta\widetilde{P}_t) = \mathbf{E}_{t-1}[\Sigma_{Aw,t+1}] + \Sigma_t - \Sigma_{Aw,t}, \tag{10}$$

$$\Sigma_{Aw,t+1} \equiv N_t\Sigma_{t+1}[(N_t+1)(\Sigma_{t+1}+\Gamma_t)+2\Phi_t]^{-1}\Sigma_{t+1}.$$

[5] Specifically, the first and second moments of $N(0,1)$-random draws are matched. Since the antithetic variable technique automatically matches all odd moments, it follows that up to the third moments and all the higher odd-order moments are matched.

When $\Gamma_t = \Phi_t \equiv 0$ for all t, $\Sigma_{Aw,t+1} = \frac{N_t}{N_t+1}\Sigma_{t+1}$ and Equation (10) simplifies as

$$H_t = \mathbf{E}_{t-1}\left[\frac{N_t}{N_t+1}\Sigma_{t+1}\right] + \frac{1}{N_{t-1}+1}\Sigma_t. \tag{11}$$

This is best understood if information is exogenously acquired, i.e., N_t is fixed at a constant, N, for all t. Then, H_t would be a weighted average of (the expected value of) Σ_{t+1} and Σ_t, with the weights given by $N/(N+1)$ and $1/(N+1)$, respectively. When information acquisition and hence the number of informed traders are endogenized, the two "weights" no longer add up to 1 in general and change over time.

Figure 2 plots the conditional price-change variance of Securities A and P in Example 1. Kroner and Ng (1998) call such plots the News Impact Surfaces (NIS). The figure indicates that Security A's conditional variance is almost solely determined by its own past squared shock, while Security P's conditional variance increases with both shocks.[6] Thus, $\widetilde{\delta}^2_{t-1,A}$ induces commonality in the conditional price-change variances.

1.5 Trading Volume

Following Admati and Pfleiderer (1988) and Foster and Viswanathan (1995), define trading volume as half the sum of absolute trades by the three groups of market participants:

$$V_t \equiv \frac{1}{2}\left(\left|\sum_{n=1}^{N_t} x_{t,n}\right| + |\widetilde{z}_t| + |\widetilde{w}_t|\right), \tag{12}$$

[6]Using the setting with $\Gamma_t = \Phi_t \equiv 0$, Longin (1997) demonstrates the existence of a threshold effect in conditional price-change variance. A similar effect obtains in our multi-security model. As we increase the past squared shocks, both $\mathbf{E}_{t-1}[N_t]$ and N_{t-1} increase (Figure 1). Since H_t is increasing in N_t and decreasing in N_{t-1} (see Equation (11)), there will be a discontinuous drop, or a kink, in the graph of H_t where N_{t-1} increases by a unit. The slope of the surface will be lower beyond each kink, roughly reflecting the concavity of the graph of $\mathbf{E}_{t-1}[N_t]$. This effect, however, is small and not visible in Figure 2.

where $|\cdot|$ denotes the elementwise absolute-value operator.[7] Using equilibrium characterization obtained in Theorem 1, we can easily compute this quantity as summarized in the following corollary:

Corollary 3 *(Expected trading volume) Expected trading volume at time t, conditional on the public information set at time $t-1$, $\mathcal{F}_{t-1}$, is given by*

$$\mathbf{E}_{t-1}[V_t] = \frac{1}{\sqrt{2\pi}}\mathbf{E}_{t-1}\left[\begin{array}{c} \sqrt{diag(N_t B_t[N_t(\Sigma_{t+1}+\Gamma_t)+\Phi_t]B_t')} + \sqrt{diag(\Psi_t)} \\ +\sqrt{diag(N_t B_t[N_t(\Sigma_{t+1}+\Gamma_t)+\Phi_t]B_t' + \Psi_t)} \end{array}\right].$$

where $\sqrt{\cdot}$ is the elementwise square-root operator and $diag(\cdot)$ gives a vector containing the principal diagonals of the argument matrix. When $\Phi_t \equiv 0$, this simplifies to

$$\mathbf{E}_{t-1}[V_t] = \frac{1}{\sqrt{2\pi}}\mathbf{E}_{t-1}[\sqrt{N_t}+1+\sqrt{N_t+1}]diag(\sqrt{\Psi_t}). \tag{13}$$

Equation (13) says that higher noise trading results in higher trading volume, since it allows informed traders to camouflage their trades more easily. This is analogous to the univariate results of Admati and Pfleiderer (1988) and Foster and Viswanathan (1995). More of our interest is the effect of information. Trading volume is linked to information variance through N_t, which increases in M_t and hence in Σ_{t+1} (see Corollary 1). Thus, trading volume increases with information variance since, with persistence in the size of information, a high profit opportunity introduced by a large absolute information shock induces more traders to become informed, thereby increasing N_t. This implies that even if the noise-trading variance is constant (as in Example 1), the volume becomes time-varying in equilibrium. Since N_t is persistent as shown

[7] The three terms in the bracket represent orders from informed and noise traders and the order imbalance absorbed by the market maker. The multiplier 1/2 corrects for double-counting of buys and sells. This definition implies that trades among informed traders, which can occur when signals are heterogeneous (i.e., $\Phi_t \neq 0$), are crossed. If one counts such trades separately, the volume will be larger than the one defined here. A similar remark is made by Admati and Pfleiderer (1988, p.24).

earlier, so is volume, which is a well-known empirical regularity.

Figure 3 plots the expected trading volume of the informationally active (A) and passive (P) securities, respectively, in Example 1. Like the expected number of informed traders at time t in Panel (b) of Figure 1, the expected trading volume of both securities is determined almost solely by Security A's past squared shocks.

1.6 Illiquidity

The Kyle's (1985) lambda matrix, A_t, is an illiquidity measure. A larger diagonal element of A_t means that the corresponding price is more sensitive to order flow and hence that the stock is more illiquid. When $\Gamma_t = \Phi_t \equiv 0$ for all t and additionally both Σ_{t+1} and Ψ_t are diagonal (as in our example) or more generally they commute, expected illiquidity conditional on the public information set at time $t-1$, $\mathcal{F}_{t-1}$, can be written as

$$\mathbf{E}_{t-1}[A_t] = \mathbf{E}_{t-1}\left[\frac{\sqrt{N_t}}{N_t+1}\Sigma_{t+1}^{\frac{1}{2}}\right]\Psi_t^{-\frac{1}{2}}. \tag{14}$$

As Longin (1997) notes, when the size of information persists, a large absolute information shock at time $t-1$ has two opposing effects on illiquidity through the terms inside the square bracket. The first effect increases illiquidity by raising the future information variance (Σ_{t+1}) due to assumed persistence. The second effect decreases illiquidity via an increased number of informed traders (N_t) because competition reduces their profits (see Lemma 1 and note that $M_t^{\frac{1}{2}}$ is proportional to $1/(N_t+1)$ when $\Gamma_t = \Phi_t \equiv 0$). Let us call these the volatility effect and the competition effect, respectively. The overall effect of information depends on which one dominates.

Figure 4 shows expected illiquidity of the two securities in Example 1. Both panels demonstrate that a security's expected illiquidity increases in its own past squared shock. It follows that, in the current setting, the volatility effect of a security's own information shock dominates its competition effect. In Panel (b), in addition, Security P's expected illiquidity also increases with the other security's past squared shock. Thus, there is commonality in illiquidity that is driven by the common information shock, $\widetilde{\delta}_{t-1,A}$.[8]

2 Empirical Analysis

This section performs a structural estimation of the proposed model via a Bayesian MCMC approach. The conditional volatility of the information process is modeled as either stochastic volatility or realized volatility controlling for microstructure noise. Using the estimates, the model's implications are tested.

2.1 Methodology: A Stochastic Volatility-Liquidity Model

To estimate the model, I first rewrite the equations in Theorem 1 by empirically observable quantities. The vector of price changes, $\Delta\widetilde{P}_t$, and order flows, $\widetilde{w}_t$, are closely related to the return and the signed share turnover which an econometrician observes with error. Equation (9) suggests a regression of price changes on the last two order flows, which is characterized by the following two conditional moments:

$$\mathbf{E}[\Delta\widetilde{P}_t|\widetilde{w}_t,\widetilde{w}_{t-1}] = A_t\widetilde{w}_t, \tag{15}$$

$$Var(\Delta\widetilde{P}_t|\widetilde{w}_t,\widetilde{w}_{t-1}) = \Sigma_t - N_{t-1}\Sigma_t[(N_{t-1}+1)(\Sigma_t+\Gamma_{t-1})+2\Phi_{t-1}]^{-1}\Sigma_t \equiv \Sigma_{\delta|w,t}. \tag{16}$$

[8]Interestingly, Panel (a) of Figure 4 indicates that for very small $\widetilde{\delta}_{t-1,A}$, Security A's illiquidity decreases in Security P's lagged squared information shock. This is a cross-sectional competition effect; having information about the passive stock induces traders to become informed, which increases competition, and yet the passive security's information shock does not affect the active security by construction (no cross-sectional volatility effect from Security P to A).

The first equation is a familiar price-impact regression. This relation easily follows by taking the conditional expectation of Equation (9) and using the fact that $\mathbf{E}[\widetilde{\delta}_t|\widetilde{w}_{t-1}] = A_{t-1}\widetilde{w}_{t-1}$, which is equivalent to the market efficiency condition in Equation (4).[9] The second equation says that the variance of the information shock, $\widetilde{\delta}_t$, in the price change is reduced from Σ_t because $\widetilde{w}_{t-1}$ contains information about it. It has the form of the conditional variance of a multivariate normal distribution.

It is difficult to estimate a multivariate system for computational reasons. Therefore, we focus on a univariate system for individual stocks. Alternatively, each univariate system can be interpreted as an entry of a multivariate system with cross-sectionally independent shocks. To fix notation, denote a generic entry of the following vectors and matrices by the lower case letters as shown in parentheses: $\widetilde{P}(p)$, $\widetilde{w}(w)$, $\Sigma(\sigma^2)$, $\Sigma_{\delta|w}(\sigma^2_{\delta|w})$, $\Gamma(\gamma)$, $\Phi(\phi)$, $\Psi(\psi)$, $A(a)$, and $B(b)$, where we have ignored the time subscript. Then, the quote midpoint return for an individual stock can be written as $r_t = (p_t - p_{t-1})/p_{t-1}$.[10] Dividing Equation (15) by the lagged price, I obtain $\mathbf{E}[r_t|w_t, w_{t-1}] = a_t w_t/p_{t-1}$. The right hand side of this equation is closely related to the signed share turnover, which can be written as

$$stov_t = w_t/s_{t-1} \sim N(\mu_{stov}, \sigma^2_{stov,t}), \tag{17}$$

where s_{t-1} is the number of shares outstanding at time $t-1$ and we have assumed that $stov_t$ is normally distributed with mean μ_{stov} and $\sigma^2_{stov,t}$, which are to be estimated.[11] Further controlling for additional variables to be explained shortly, I arrive at the following regression

[9] To see this, equate the right hand side of Equations (4) and (5) and rearrange to get $\mathbf{E}[\widetilde{\delta}_{t+1}|\mathcal{F}_{t,m}] = A_t\widetilde{w}_t$, and note that the relevant information in $\mathcal{F}_{t,m}$ is $\widetilde{w}_t$.

[10] Since the shocks are normally distributed, prices can be zero or negative, making the division by lagged price potentially problematic in theory. I abstract from this issue in the empirical analysis.

[11] p_t and s_t here are adjusted for stock splits. Specifically, if p_t^* and s_t^* are the unadjusted price and the unadjusted number of shares outstanding, respectively, $p_t = p_t^*(s_t^*/s_0^*)$ and $s_t = s_0^*$.

equation:

$$r_t = \beta_0 + \beta_1 \underline{a}_t stov_t + \beta_2 stov_{t-1} + \beta_3 r_{t-1} + \beta_4 tov_{t-1} r_{t-1} + e_t, \tag{18}$$

$$\underline{a}_t \equiv a_t s_{t-1}/p_{t-1}, \tag{19}$$

where e_t is a Gaussian white noise with variance $\sigma^2_{e,t}$, whose logarithm is assumed to follow an AR(1) process:

$$\ln \sigma^2_{e,t} = \alpha_0 + \alpha_2(\ln \sigma^2_{e,t-1} - \alpha_0) + \eta_t. \tag{20}$$

Here, the η_t shock to the log variance of the residual return, distributed normally with mean zero and variance α_1^2, is a measure of information generation (see, e.g., Andersen (1996)).[12] Dividing Equation (16) by the lagged squared price gives the residual return variance, $\sigma^2_{e,t}$. Rearranging that relation, we can connect the measurable residual-return variance to the unobservable conditional price-change variance, $\sigma^2_{\delta|w,t}$, and hence to σ^2_t:

$$\sigma^2_{\delta|w,t} = \sigma^2_t - \frac{N_{t-1}(\sigma^2_t)^2}{(N_{t-1}+1)(\sigma^2_t + \gamma_{t-1}) + 2\phi_{t-1}} = p^2_{t-1}\sigma^2_{e,t}. \tag{21}$$

Observe that the first equality of Equation (21), after rearranging, yields a quadratic equation for σ^2_t. Solving,

$$\sigma^2_t = \frac{1}{2}(f_{1,t-1} + \sqrt{f^2_{1,t-1} + 4f_{2,t-1}}) > 0, \tag{22}$$

$$f_{1,t-1} \equiv (N_{t-1}+1)(\sigma^2_{\delta|w,t} - \gamma_{t-1}) - 2\phi_{t-1}, \tag{23}$$

$$f_{2,t-1} \equiv [(N_{t-1}+1)\gamma_{t-1} + 2\phi_{t-1}]\sigma^2_{\delta|w,t} > 0. \tag{24}$$

From Equation (17), the variance of $stov_t$, $\sigma^2_{stov,t}$, can be translated into the order-flow variance

[12] The η_t shocks should not be confused with information shocks, δ_t, which can be readily backed out using estimated unobservable state variables in the Bayesian MCMC framework. See the discussion preceding Equation (28). An alternative GARCH specification would restrict $\eta_t \equiv \delta_t$.

via $\sigma^2_{w,t} = \sigma^2_{stov,t} s^2_{t-1}$, where $\sigma^2_{w,t}$ is a generic diagonal entry of $\Sigma_{w,t} \equiv Var_t(\widetilde{w}_t)$ in Equation (A29) of the appendix. When all matrices are scalars, it is straightforward to confirm that the expression simplifies to $\Sigma_{w,t} = \left[\frac{N_{t-1}(\sigma^2_t+\gamma_{t-1})+\phi_{t-1}}{\sigma^2_t+\gamma_{t-1}+\phi_{t-1}} + 1\right]\psi_{t-1}$. Equating the simplified expression to $\sigma^2_{stov,t} s^2_{t-1}$ and solving for the noise trading variance, we obtain

$$\psi_{t-1} = \sigma^2_{stov,t} s^2_{t-1} \frac{\sigma^2_t + \gamma_{t-1} + \phi_{t-1}}{(N_{t-1}+1)(\sigma^2_t + \gamma_{t-1}) + 2\phi_{t-1}}. \tag{25}$$

The number of informed traders is determined by solving the traders' information acquisition problem. Substituting the expression for M_t in Theorem 1 into Equation (7), stepping back one period, and rearranging gives a cubic inequality for N_{t-1}:

$$N_{t-1}\left[\frac{(N_{t-1}+1)(\sigma^2_t + \gamma_{t-1}) + 2\phi_{t-1}}{\sigma^2_t}\right]^2 \leq \frac{\psi_{t-1}(\sigma^2_t + \gamma_{t-1} + \phi_{t-1})}{c^2_{t-1}}. \tag{26}$$

Given draws of other parameters, we can solve this inequality for N_{t-1} at each node in each iteration of the Bayesian MCMC procedure. Following the standard practice, we look for real-numbered (rather than natural-numbered) N_{t-1} assuming that the inequality is binding.[13] We next model the cost of acquiring one-period-ahead information as a truncated linear function of the residual return variance,

$$c_{t-1} = \max(h_0 + h_1 \sigma^2_{e,t}, 0.1). \tag{27}$$

The truncation ensures that the cost is positive. As we will see, the truncation point is low enough not to bind for most stocks. Finally, a scalar counterpart of the equations in Theorem 1 interconnects the parameters. I estimate the system restricting the variances of the two signal noises and the signed share turnover to be constant over time, i.e., $\gamma_{t-1} \equiv \gamma$, $\phi_{t-1} \equiv \phi$, and

[13] This cubic equation is very accurately solved; the difference between both sides of the equation is minimized to the order of 10^{-8} or smaller in relative magnitude to either side.

$\sigma^2_{stov,t} \equiv \sigma^2_{stov}$, to improve their identifiability.[14]

The above system of equations constitutes a stochastic volatility model in which innovations to the conditional residual return variance also affect the price-impact coefficient, $\underline{a}_t$ (an illiquidity measure). To see this, first note that a positive η_t shock increases the information variance, σ^2_t, by Equations (20) and (21). This, in turn, affects a_t and hence $\underline{a}_t$ via a scalar counterpart of equations in Theorem 1 (this effect is depicted in Figure 4). This provides the model's namesake, a stochastic volatility-liquidity model.

2.2 Testable Hypotheses

We are now ready to state testable hypotheses. Equation (15) imposes very tight restrictions on the slope coefficients in Equation (18), as summarized in the following statistical hypothesis:

Hypothesis 1 *(Slope coefficients in the mean equation)* $\beta_1 = 1$ *and* $\beta_2 = \beta_3 = \beta_4 = 0$.

The specification in Equation (18) is designed to examine alternative hypotheses. A potentially important feature of capital markets absent from our model is the risk aversion of agents. The specification controls for proxies of factors that affect liquidity under risk aversion, most likely inventory risk and supply pressure accommodated by risk-averse market makers and investors. The existing literature suggests the following sings for the coefficients in Equation (18):

$\beta_2 < 0$: This coefficient is similar to the return-reversal measure of Pastor and Stambaugh (2003, the gamma coefficient in their Equation (1)), who base their theoretical foundation on Campbell, Grossman, and Wang (1993).[15] A generalized implication of Campbell, Grossman,

[14] While the two signal noises are strictly exogenous processes, signed share turnover is not. Restricting its variance and hence the variance of w_t/s_{t-1} (see Equation (17)) to be constant imposes a restriction on a free parameter, ψ_{t-1}, through Equation (25).

[15] Note that Pastor and Stambaugh (2003) use volume signed by return in excess of the market as a measure of order flow. Their objective is to examine the pricing of liquidity risk using series long enough to bear asset pricing implications. We do not have to use such a proxy for order flow because signed share turnover is available at the intraday frequency.

and Wang's (1993) model (explained further in the next paragraph) is that price pressure caused by non-informational trades will be mitigated in subsequent periods, making the slope coefficient on the lagged signed share turnover negative.

$\beta_3 < 0$: Grossman and Miller (1988, Section I.C) predict a negative correlation between successive price changes. In their model of inventory risk, risk-averse market makers willing to offset the liquidity trades of "outside customers" command a price discount, which will subsequently be reversed when counter-trades arrive.[16] A counterpart of β_2 and β_3 also appears in Hasbrouck (1991), who finds that intraday price changes are related to previous price changes and signed trades in a VAR framework.

$\beta_4 \gtreqless 0$: Campbell, Grossman, and Wang (1993) present and test a model in which risk-averse market makers accommodate buying or selling pressure from liquidity traders. They show that the price concession required by such market makers will be reversed in later periods, leading to a negative return autocorrelation similar to Grossman and Miller (1988). They additionally show that the return autocorrelation decreases (becomes more negative) with volume, which measures the level of liquidity trades that the market makers are accommodating. This implies that $\beta_4 < 0$. However, Llorente et al. (2002) remark another possibility: when trades derive from long-lived asymmetric information, return autocorrelation will rise and can become even positive as information gets impounded into prices gradually. Hence, while returns generated by hedging trades tend to reverse themselves ($\beta_4 < 0$), returns generated by speculative trades continue themselves ($\beta_4 > 0$). Thus, the sign of β_4 is ambiguous *ex ante* and is an empirical question.

In all these models, risk aversion plays a key role in forming liquidity. This clearly represents a different mechanism from our risk-neutral framework, in which liquidity is formed through the

[16] A negative serial price-change covariance is also consistent with Roll's (1984) bid-ask spread measure. However, Grossman and Miller's (1988) model would be more relevant to the interpretation of our β_3 coefficient because we use returns computed from quote-midpoints rather than transaction prices.

process of capitalizing information asymmetry into price. Thus, including the control variables allows us to not only examine robustness, but also explore the reason *why* the null hypothesis is rejected, when it is indeed rejected.

Next, Equation (27) allows us to examine whether the value of private information varies over time. Recall that $\sigma^2_{e,t+1}$ is positively related to the information shock variance, σ^2_{t+1}, via Equation (21) at time $t+1$. Consider the value of private signals in Equation (2) when the variance of $\widetilde{\delta}_{t+1}$ (Σ_{t+1}) rises. Fixing the noise variances, the private signals should become more valuable as they become more informative and the signal-to-noise ratio rises. Said differently, the cost of information acquisition rises during periods of volatile fundamentals in which forecasting terminal payoffs is difficult. This is summarized this in the following empirical hypothesis:

Hypothesis 2 *The value of private information rises during times of volatile fundamentals:* $h_1 > 0$.

An advantage of the Bayesian MCMC framework is that it readily estimates unobservable state variables along with model parameters. This allows us to back out an individual stock's information shock, δ_t, via Equation (9). Thus, for a given stock, we can further construct the market information shock, $\delta_{t,M}$, as the equally weighted average of the information shocks of the other 29 stocks in DJIA. Then, to examine the effect of information shocks on liquidity, I regress the stock's estimated price impact parameter, $\underline{a}_t$, on the lagged squared individual shock and the market information shock:

$$\underline{a}_t = g_0 + g_1\delta^2_{t-1} + g_2\delta^2_{t-1,M} + \epsilon_t. \tag{28}$$

Note that $\underline{a}_t$ is closely related to the Kyle's lambda parameter, a_t, via Equation (19). As indicated in Figure 4, if the individual stock in question is an informationally active stock, its price impact parameter is almost solely determined by its own past squared information shock ($g_1 > 0$ and $g_2 \approx 0$). If the individual stock is an informationally passive stock, its price impact parameter is affected by both its own past squared information shock and the market's past squared information shock ($g_1 > 0$ and $g_2 > 0$). This leads to the following hypothesis, reflecting the lack of prior knowledge about whether the Dow stocks are informationally active or passive:

Hypothesis 3 *Information generation decreases liquidity:* $g_1 > 0$ *and* $g_2 \geq 0$*.*

2.3 Data

I estimate the model using the 30 Dow stocks because information generation and liquidity are most accurately measured for actively traded stocks. For all the 30 stocks in the DJIA index as of September 2004, I extract from the TAQ dataset quotes and trades in years 2002 and 2003, the latest available at the time of analysis. Here, I limit the sample period to 2002 and later to allow enough time after the decimalization of NYSE and NASDAQ. I divide each trading day into an opening 30 minute interval (9:30a.m.-10:00a.m.) and six consecutive hourly intervals (10:00a.m.-4:00p.m.). Intraday return, r_t, is computed using the last quote midpoint in each interval. Discarding the opening 30 minute interval to avoid the undue effect of overnight events, this produces six hourly returns per day. The duration of one hour is chosen to capture the intraday interaction between market makers and traders. I calculate signed and unsigned share turnover following Breen, Hodrick, and Korajczyk (2002). Trades and quotes are matched and then signed by the Lee and Ready (1991) algorithm. Signed share turnover, $stov_t$, for period t is buyer-initiated share volume less seller-initiated share volume, normalized by the number of shares outstanding ($SHROUT$) in CRSP. Since $SHROUT$ is recorded in units of thousands, $stov_t = 1$ means that 0.1% of the shares outstanding are bought in net. Unsigned share turnover, tov_t, is the sum of the buyer- and seller-initiated share trades with the same standardization.[17]

To save space, I provide brief summary statistics of the constructed series here without a table. Over the 30 Dow stocks, the mean hourly return is 26 bp and the mean signed share turnover is 0.047, which implies that more shares are bought than sold in the sample period. This is consistent with the fact that there are on average more buy orders (556) than sell orders (527) per stock each hour. The mean unsigned share turnover is 0.49, meaning that 0.049% of each stock's outstanding shares are traded hourly.

2.4 Estimation

A general feature of a stochastic volatility model, including our stochastic volatility-liquidity model, is that innovations to the conditional variance are independent of innovations to return, making maximum likelihood estimation difficult, if not impossible, unless one makes some approximations to the true unknown likelihood function (see, e.g., Aït-Sahalia and Kimmel (2007)). Fortunately, a Bayesian MCMC method allows us to estimate such a model. I draw $10,000$ samples at each node and discard the initial $5,000$ samples for the system to arrive at a steady state. Using the next $5,000$ samples, I compute the mean and the 95% confidence interval of each parameter, including the unobservable state variables such as $\sigma^2_{e,t}$ at each time.

Table 1 shows the prior distributions of parameters used in the Bayesian MCMC estimation. I choose relatively informative priors for parameters specifying conditional variances of return and signed share turnover, which are known to be estimated quite accurately. The relatively informative priors for α_1^2 and α_2 follow Kim, Shephard, and Chib (1998, Equations (4) and (5)). The prior distribution for α_1^2, the "variance of log variance," is inverse-gamma, which is the conjugate prior for the normal variance and has a positive support. I set $\alpha_2 = 2\alpha_2^* - 1$,

[17] Following the standard practice, I calculate r_t, $stov_t$, and tov_t without adjusting the raw variables for stock splits as in Footnote 11. These quantities are correctly computed as they are standardized and the opening overnight periods are excluded from my sample.

where α_2^* has a beta distribution shown in the table caption. Since its support is $\alpha_2^* \in [0, 1]$, the prior of $\alpha_2 \in [-1, 1]$. For other parameters, I use diffuse prior distributions containing the nulls in Hypotheses 1 and 2 to make those hypotheses rejectable.[18]

2.5 Result

Table 2 shows selected parameter estimates (means of the posterior distributions) for the 30 Dow stocks along with the 2.5 and 97.5 percentiles in parentheses. The first column shows the mean log variance parameter, α_0. The average of annualized volatility, $\sqrt{252 \times 6.5 \exp(\alpha_0)}$, over the 30 stocks is 0.19 (not shown), which is in the right ballpark. The next column shows the volatility of log variance, α_1, whose 2.5 percentile is well above zero for all stocks, indicating active information production. As expected, the persistence parameter, α_2, is slightly below 1, implying slow decay of information shocks. Panel A of Figure 5 shows the posterior mean of residual return variance ($\sigma_{e,t}^2$) along with the 95% confidence interval for a representative stock, IBM. I chose IBM because it is used in many existing studies including Foster and Viswanathan (1995), which facilitates comparison. The panel depicts a very persistent series, consistent with α_2 being close to 1. The tight confidence interval relative to other panels in the figure suggests that this unobservable state variable is well identified. Importantly, the conditional residual return variance changes substantially over time, representing active information generation as implied by the reliably positive α_1 estimate.

The next two columns of Table 2 show the estimated coefficients on the current and lagged signed share turnovers. The estimated β_1 is positive for all stocks, but only five of them have a 95% confidence interval that brackets the theoretical value of 1. This suggests that for most stocks, the current order flow plays a somewhat larger or smaller role than the model implies.

[18]The Bayesian MCMC estimation is performed using WinBUGS (Bayesian inference Using Gibbs Sampling), a freely available software package. The code is modified from Yu and Meyer's (2006) programs for various multivariate stochastic volatility models. The zero-profit condition in Equation (26) is solved using Robert Campbell's numerical methods library written in Component Pascal. The WinBUGS code and the Component Pascal routine are linked by Dave Lunn's WinBUGS Development Interface.

Also, the estimated β_2 is negative for all stocks, and significantly so for 28 out of the 30 stocks. As discussed above, this result is more consistent with risk-averse investors requiring price discount to accommodate noninformational trades (Campbell, Grossman, and Wang (1993) and Pastor and Stambaugh (2003)), rather than competitive market makers capitalizing information into prices as in our model. Next, β_3 is not significantly different from zero for 20 out of 30 stocks. Although this is consistent with our model's implications, the other ten stocks have a significantly negative estimate, as predicted by Grossman and Miller (1988, Section I.C). The last coefficient in the mean equation, β_4, is not significantly different from zero for any stock; this is signified by the fact that the 95% confidence intervals shown are essentially those of the prior distribution for all stocks. To summarize, the null in Hypothesis 1 is rejected for all parameters but β_4. Since the model is rejected, care is required for the interpretation of the results to follow.

While the first set of hypotheses is rejected, we do have some support for the second (Hypothesis 2). The last column in the first panel of Table 2 shows that the estimates of h_1 are significantly positive for 19 stocks, insignificant for five stocks, and significantly negative for only six stocks. Thus, for 63% of the stocks the cost of information acquisition rises during times of volatile fundamentals.

The second panel of Table 2 reports the estimates of additional parameters of interest. For a quantity with a time subscript, the table shows the time-series average of the posterior mean along with the time-series averages of the 2.5 and 97.5 percentiles in parentheses. The average trading cost, c_t, and the average number of informed traders, N_t, in the first two columns tell an intriguing story. For example, the estimates for IBM imply that there are on average 28 informed traders who each acquires private signals worth $820 and makes just the same amount of profit per hour. Unlike some existing estimates, the estimates here do not seem implausible; for example, using a 1988 data, Foster and Viswanathan (1995) estimate the number of informed traders and the cost of private information for IBM to be $2,404$ (Table 4) and 0.28 cents per half hour (Table 2), respectively. They note that "[t]hese numbers are, however, not entirely plausible given our existing knowledge of the markets." (p.390) My estimates imply that fewer traders acquire more accurate and expensive information about IBM than Foster and Viswanathan's (1995) do. A caveat, however, is that my table also shows several stocks with hundreds of informed traders who acquire information as inexpensive as $10 or less.

Panel B of Figure 5 shows the estimated time series of c_t for IBM. Since estimated h_1 for IBM is positive in Table 2, c_t is a positive, and by construction linear, function of $\sigma^2_{e,t+1}$, whose lag is shown in Panel A (consider Equation (27) at time t). Again, the higher the residual return variance and hence the information variance, the higher is the signal-to-noise ratio (see Equation (2)), and the more accurate and expensive is the private signal. The number of informed traders, N_t, in Panel C moves in the opposite direction from c_t, because these two quantities are negatively related via Equation (26); as the cost of private information becomes higher, the fewer traders will acquire it. The confidence interval in the panel suggests that our working assumption that $N_t \geq 1$ is not significantly violated for IBM at any point in the sample period. Moreover, the assumption appears to have held for most stocks; back in Table 2, the average 2.5 percentile for N_t is significantly larger than 1 for 26 out of the 30 stocks (87%). For the remaining four stocks, the 95% confidence interval still contains 1.

The relative magnitude of estimated σ^2_{t+1}, γ, and ϕ in Table 2 implies that the signal-to-noise ratio of private signals varies across stocks. For some stocks, γ and ϕ are very close to zero (note that $\gamma = 0$ and/or $\phi = 0$ is perfectly admissible). In some cases, however, the relatively large size of their confidence intervals indicates that these parameters may not have been reliably estimated. Nevertheless, it is illuminating to examine the correlation between the private signals of two informed traders. From Equation (2), this correlation is given by

$(\sigma_{t+1}^2 + \gamma)/(\sigma_{t+1}^2 + \gamma + \phi)$. According to the table, the correlation estimated at average σ_{t+1}^2 ($Corr$) ranges between 0.17 and 0.99 across the 30 stocks with an (unreported) cross-sectional average of 0.71. Thus, private signals appear highly correlated across traders for an average stock, suggesting the possibility that traders acquire private information from same sources.

Like Foster and Viswanathan (1995), we find that the noise trading variance, ψ_t, is a large number. Informed traders take advantage of this as their total profit is proportional to $M_t^{\frac{1}{2}}$ and hence to $\psi_t^{\frac{1}{2}}$ (Lemma 1). This shows up as large estimates of the informed-trading intensity parameter, b_t. Finally, the price impact parameter, $\underline{a}_t$, is reliably positive; along with positive estimates of β_1 in Equation (18), this implies that the order flow tends to move the price in the same direction. The estimated time series of $\underline{a}_t$ for IBM is shown in Panel D of Figure 5. We see that price impact increases as the conditional residual return variance in Panel A increases. Again, a high residual return variance, or a high information variance, represents a profit opportunity for informed traders and increases the price impact.

2.6 Addressing Microstructure Noise: A Semi-parametric Approach using Realized Volatility

While the stochastic volatility model presented above is a natural approach to estimating unobservable information shocks, it may be doomed to microstructure noise at an intraday frequency. Although our returns are calculated from quote midpoints and hence are much less prone to bid-ask bounce than transaction-price returns are, microstructure noise could arise from various other sources. In addition, infrequent sampling can cause a serious measurement problem if the true price-generating process is close to being continuous. To save space, I refer the reader to a thorough discussion on these points by Zhang, Mykland, and Aït-Sahalia (2005). In particular, Aït-Sahalia and Yu (2008) find that microstructure noise is positively correlated with volatility and illiquidity including price impact measures. Importantly to my purpose, this implies that the strong relation I will later find between information variance and (il)liquidity may partly be driven by microstructure noise. To address this issue, I employ Zhang, Mykland, and Aït-Sahalia's (2005) two-scales realized variance measure that explicitly controls for microstructure noise and infrequent sampling. Very briefly, this is a bias-corrected average of subsampled realized variances based on log price differences.[19] Assuming that the realized variance measure, denoted by $\sigma^2_{Re,t}$, represents the residual return variance, I substitute it for $\sigma^2_{e,t}$ in Equation (20),

$$\ln \sigma^2_{Re,t} = \alpha_0 + \alpha_2(\ln \sigma^2_{Re,t-1} - \alpha_0) + \eta_t, \tag{29}$$

and re-estimate the whole system. Here, the information shock η_t is computed as the residual from an autoregressive (AR) model for $\sigma^2_{Re,k}$. I call this system a realized volatility-liquidity model.[20]

Table 3, comparable to Table 2, presents the selected parameter estimates of the realized volatility-liquidity model. Generally, the estimates are remarkably similar to those of the stochastic volatility-liquidity model except for α_1 and α_2. For example, the estimates for our representative stock, IBM, are as follows, with the corresponding estimates from the stochastic volatility-liquidity model in parentheses: $\alpha_0 = -10.8$ (-11.2), $\alpha_1 = 0.67$ (0.09), $\alpha_2 = 0.56$ (0.99), $\beta_1 = 0.17$ (0.19), $\beta_2 = -6.2$ (-6.8), $\beta_3 = 0.003$ (0.017), $\beta_4 = 0.042$ (0.050), and $h_1 = 3.8$ (3.8). The first column of the table allows us to calculate the mean annualized volatility, $\sqrt{252 \times 6.5 \exp(\alpha_0)}$. Its average across the 30 stocks is 0.216 (not shown), which is close to the value estimated from the stochastic volatility-liquidity model in the preceding subsection. The next two columns show the estimates of α_1 and α_2. We find that, in general, α_1 is larger and α_2 is smaller than those from the stochastic volatility-liquidity model. This perhaps reflects the "unfiltered" nature of realized volatility, as opposed to volatility estimated as an unobservable

[19] This is their "first-best approach." I implement it by subsampling every twentieth observations.

[20] We note that, in the realized volatility-liquidity model, $\sigma^2_{Re,t}$ is no longer an unobservable state variable to be estimated.

state variable in the stochastic volatility framework.

The estimated beta parameters are generally in line with those of the stochastic volatility-liquidity model. While β_1 is positive for all stocks, its 95% confidence interval contains the null of 1 for only two stocks. β_2 is significantly negative for 26 out of the 30 stocks. However, β_3 is now more consistent with our model's implication. The estimates are not significantly different from zero for all but four stocks, which have significantly negative estimates. Again, β_4 is not significantly different from zero for all stocks, since the prior confidence interval shows up as the almost common posterior interval. The distribution of estimated h_1 is also similar to Table 2. 17 out of 30 stocks (57%) have a significantly positive h_1, suggesting that the value of private information rises during times of volatile fundamentals. It is also noted, however, that nine stocks (30%) now have a significantly negative h_1.

The second panel of Table 3 shows additional quantities of the realized volatility-liquidity model corresponding to that of Table 2. Again, for many stocks, the estimates are quite similar to those from the stochastic volatility-liquidity model. For example, the estimates for IBM are as follows, with the corresponding estimates from the stochastic volatility-liquidity model in Table 2 in parentheses: $c_t = 990$ (820), $N_t = 23.4$ (27.5), $\sigma^2_{t+1} = 3.67$ (3.20), $\gamma = 0.00$ (0.00), $\phi = 0.04$ (0.04), $\psi_t = 2.3 \times 10^9$ (2.1×10^9), $\underline{a}_t = 167.1$ (156.5), and $b_t = 6.6$ (6.6). The pair of estimated c_t and N_t tells us that on average 23 traders acquire private signals at a cost of \$990 per hour. This is comfortably close to the estimates in the preceding subsection, which gives us confidence about the reliability of the IBM estimates. From the second column of the table, we find that the average number of informed traders is significantly greater than 1 for 27 out of 30 stocks (90%), validating our working assumption that $N_t \geq 1$. The average of estimated σ^2_{t+1} is again reliably positive. Moreover, while ϕ continues to be mostly insignificant, for many stocks the common noise variance, γ, now appears to be bounded fairly above zero. Perhaps for this reason, the implied correlation between two private signals averages at 0.77 (calculated using the values in the table), which is higher than that of the stochastic volatility-liquidity model by 0.06. The remaining two parameters, $\underline{a}_t$ and b_t, exhibit some departure from the estimates of the stochastic volatility-liquidity model for several stocks.

2.7 Cross-sectional Information Flow and Liquidity

Finally, I examine the cross-sectional implication of the model. Table 4 presents the result of the regression in Equation (28) for both the stochastic volatility-liquidity model and the realized volatility-liquidity model. We see that g_1 and g_2 are strongly positive for all but a couple of stocks for both models, meaning that liquidity of a Dow stock typically decreases in the size of information about itself but also about other Dow stocks. Moreover, g_2 is larger than g_1 for most stocks. This result suggests the existence of a substantial information spillover, producing a significant cross-sectional effect of information on liquidity.

What is the source of such strong cross-sectional information flows? The accounting literature has long been aware that an earnings announcement by one firm provides information about other firms in the same industry; see Thomas and Zhang (2008) and references cited therein. Moreover, news can be produced much more broadly and frequently than accounting information, for example, in the form of industry or macroeconomic news through newswires. Also, being an average, the cross-sectional information flow may be less noisy than the individual information flow. The more frequent production and less noisiness of cross-sectional information flow will tend to make g_2 larger than g_1.

3 Conclusion

This paper examines the dynamic cross-sectional effect of information on liquidity, trading volume, and conditional return variance. In a centralized market similar to Kyle (1985) and Admati and Pfleiderer (1988), trading of multiple securities takes place over time. Unlike these traditional models, however, information is gradually revealed over time with possible conditional heteroskedasticity and persistence in size. With persistence, a higher level of information generation results in higher volume, volatility, and illiquidity in subsequent periods. Using high frequency data for the 30 Dow stocks, I perform a structural estimation of the model by Bayesian MCMC simulation. The conditional volatility of the information process is modeled as either stochastic volatility or realized volatility controlling for microstructure noise (Zhang, Mykland, and Aït-Sahalia (2005)). I provide evidence that a Dow stock's illiquidity exacerbates in the size of information about not only itself, but also about other Dow stocks, demonstrating a significant cross-sectional effect of information on liquidity.

An important question left unanswered in our risk-neutral framework is the equilibrium relation between liquidity risk and expected return under asymmetric, heteroskedastic information. A theoretical investigation into this issue has been limited perhaps due to the lack of analytic solutions in a fully dynamic setting. A workaround may be to introduce investor myopia or bounded-rationality (Kyle and Xiong (2001), and Llorente et al. (2002)) or even irrationality (Baker and Stein (2004)).

Finally, I note that there is an alternative empirical specification based on a GARCH model, a natural counterpart to our stochastic and realized volatility models. Such a model may prove useful in linking the recent microstructure literature and the more traditional, early literature on volatility and information spillover developed in a GARCH framework (Conrad, Gultekin, and Kaul (1991), Fleming, Kirby, and Ostdiek (1998), Hamao, Masulis, and Ng (1990), Kroner and Ng (1998), Lin, Engle, and Ito (1994)). This direction is left for future research.

A Appendix

A.1 Proof of Theorem 1

Let the vectors of prices and the demand of informed trader n be

$$\widetilde{P}_t = A_{0t} + A_{1t}\widetilde{w}_t, \tag{A1}$$

$$x_{t,n} = B_{0t} + B_{1t}\widetilde{\xi}_{t,n}, \tag{A2}$$

respectively, where A_{0t} and B_{0t} are $K \times 1$ vectors and A_{1t} and B_{1t} are $K \times K$ matrices to be determined. Here, using the relevant information $\widetilde{y}_{t,n} \in \mathcal{F}_{t,n}$,

$$\widetilde{\xi}_{t,n} \equiv \mathbf{E}[\widetilde{\delta}_{t+1}|\mathcal{F}_{t,n}] = Cov_t(\widetilde{\delta}_{t+1}, \widetilde{y}'_{t,n})Var_t^{-1}(\widetilde{y}_{t,n})\widetilde{y}_{t,n} = \Sigma_{t+1}(\Sigma_{t+1} + \Gamma_t + \Phi_t)^{-1}\widetilde{y}_{t,n}, \tag{A3}$$

where subscript t denotes conditioning on the public information set, $\mathcal{F}_t$. We first solve the optimization problem of an informed trader given the market maker's linear pricing rule, and then verify that the prices are consistent with the market efficiency condition. The net order flow is

$$\widetilde{w}_t = x_{t,n} + \sum_{i \neq n} x_{t,i} + \widetilde{z}_t. \tag{A4}$$

Substituting this into the price function in Equation (A1), we may write the profit maximization problem in Equation (3) as

$$\max_{x_{t,n}} \mathbf{E}[\{\widetilde{D}_T - A_{0t} - A_{1t}(x_{t,n} + \sum_{i \neq n} x_{t,i} + \widetilde{z}_t)\}' x_{t,n}|\mathcal{F}_{t,n}].$$

Noting that informed traders do not observe prices by assumption and that $\widetilde{z}_t$, $\widetilde{\delta}_s \notin \mathcal{F}_{t,n}$, $\forall s > t+1$ and assuming the symmetry of A_{1t}, we obtain the first order condition

$$0 = \widetilde{D}_t + \widetilde{\xi}_{t,n} - A_{0t} - A_{1t}(2x_{t,n} + \sum_{i \neq n} \mathbf{E}[x_{t,i}|\mathcal{F}_{t,n}]). \tag{A5}$$

The second order condition for maximization is met if A_{1t} is positive definite. Here, from (A2),

$$\mathbf{E}[x_{t,i}|\mathcal{F}_{t,n}] = B_{0t} + B_{1t}\mathbf{E}[\widetilde{\xi}_{t,i}|\mathcal{F}_{t,n}], \tag{A6}$$

where $\mathbf{E}[\widetilde{\xi}_{t,i}|\mathcal{F}_{t,n}]$ is informed trader n's estimate of another informed trader i's estimate of $\widetilde{\delta}_{t+1}$. To compute this, first derive the prior moments from Equation (A3),

$$\Sigma_{t,\xi} \equiv Var_t(\widetilde{\xi}_{t,n}) = \Sigma_{t+1}(\Sigma_{t+1} + \Gamma_t + \Phi_t)^{-1}\Sigma_{t+1}, \tag{A7}$$

$$\Sigma_{t,c} \equiv Cov_t(\widetilde{\xi}_{t,i}, \widetilde{\xi}'_{t,n}) = \Sigma_{t+1}(\Sigma_{t+1} + \Gamma_t + \Phi_t)^{-1}(\Sigma_{t+1} + \Gamma_t)(\Sigma_{t+1} + \Gamma_t + \Phi_t)^{-1}\Sigma_{t+1}, \tag{A8}$$

for any $i \neq n$. Then, since the prior mean $\mathbf{E}_t[\widetilde{\xi}_{t,i}] = 0$,

$$\mathbf{E}[\widetilde{\xi}_{t,i}|\mathcal{F}_{t,n}] = \Sigma_{t,c}\Sigma_{t,\xi}^{-1}\widetilde{\xi}_{t,n}. \tag{A9}$$

Substituting this for $\mathbf{E}[\widetilde{\xi}_{t,i}|\mathcal{F}_{t,n}]$ in Equation (A6) and using the conjectured demand function in Equation (A2), we can rewrite the first-order condition in Equation (A5) as

$$0 = \widetilde{D}_t + \widetilde{\xi}_{t,n} - A_{0t} - 2A_{1t}(B_{0t} + B_{1t}\widetilde{\xi}_{t,n}) - (N_t - 1)A_{1t}[B_{0t} + B_{1t}\Sigma_{t,c}\Sigma_{t,\xi}^{-1}\widetilde{\xi}_{t,n}]. \tag{A10}$$

For this equation to hold for any realization of $\widetilde{\xi}_{t,n}$, its coefficient must be zero:

$$0 = I - 2A_{1t}B_{1t} - (N_t - 1)A_{1t}B_{1t}\Sigma_{t,c}\Sigma_{t,\xi}^{-1}, \text{ or}$$
$$A_{1t}B_{1t} = J_t^{-1}, \text{ with } J_t \equiv 2I + (N_t - 1)\Sigma_{t,c}\Sigma_{t,\xi}^{-1}. \tag{A11}$$

Substituting Equations (A7) and (A8) for $\Sigma_{t,\xi}$ and $\Sigma_{t,c}$, respectively, produces the expression for J_t in Theorem 1. The constant term in Equation (A10) must also be zero, which, after rearranging, produces

$$A_{0t} = \widetilde{D}_t - (N_t + 1)A_{1t}B_{0t}. \tag{A12}$$

We next solve the market maker's problem. Recall that $\mathcal{F}_{t,m} = \mathcal{F}_t \cap \{\widetilde{w}_1, \ldots, \widetilde{w}_t\}$. Since the prior mean $\mathbf{E}_t[\widetilde{w}_t] = N_t B_{0t}$, we can rewrite the market efficiency condition in Equation (4) as

$$\widetilde{P}_t = \widetilde{D}_t + \mathbf{E}[\widetilde{\delta}_{t+1}|\mathcal{F}_{t,m}] = \widetilde{D}_t + Cov_t(\widetilde{\delta}_{t+1}, \widetilde{w}_t')Var_t^{-1}(\widetilde{w}_t)(\widetilde{w}_t - N_t B_{0t}). \tag{A13}$$

Here, from Equations (A4) and (A2),

$$Cov_t(\widetilde{\delta}_{t+1}, \widetilde{w}_t') = \sum_{i=1}^{N_t} Cov_t(\widetilde{\delta}_{t+1}, \widetilde{\xi}_{t,i}')B_{1t}' = N_t \Sigma_{t,\xi} B_{1t}', \tag{A14}$$

where we have used Equations (A3) and (A7) in the second equality. Also,

$$Var_t(\widetilde{w}_t) = B_{1t}Var_t(\sum_{n=1}^{N_t} \widetilde{\xi}_{t,n})B_{1t}' + \Psi_t = N_t B_{1t}\{\Sigma_{t,\xi} + (N_t - 1)\Sigma_{t,c}\}B_{1t}' + \Psi_t \equiv \Sigma_{w,t}. \tag{A15}$$

So, Equation (A13) becomes

$$
\begin{aligned}
\widetilde{P}_t &= \widetilde{D}_t + N_t \Sigma_{t,\xi} B'_{1t} [N_t B_{1t} \{\Sigma_{t,\xi} + (N_t - 1)\Sigma_{t,c}\} B'_{1t} + \Psi_t]^{-1} (\widetilde{w}_t - N_t B_{0t}) \\
&= \widetilde{D}_t + [B_{1t} \{I + (N_t - 1)\Sigma_{t,c}\Sigma_{t,\xi}^{-1}\} + N_t^{-1} \Psi_t B_{1t}'^{-1} \Sigma_{t,\xi}^{-1}]^{-1} (\widetilde{w}_t - N_t B_{0t}),
\end{aligned}
$$

assuming nonsingularity of B_{1t}. Comparing with the price conjecture in Equation (A1), we have

$$
\begin{aligned}
A_{1t} &= [B_{1t} \{I + (N_t - 1)\Sigma_{t,c}\Sigma_{t,\xi}^{-1}\} + N_t^{-1} \Psi_t B_{1t}'^{-1} \Sigma_{t,\xi}^{-1}]^{-1}, \text{ or} \\
A_{1t} B_{1t} &= [J_t - I + N_t^{-1} B_{1t}^{-1} \Psi_t B_{1t}'^{-1} \Sigma_{t,\xi}^{-1}]^{-1}, \text{ and} \qquad \text{(A16)} \\
A_{0t} &= \widetilde{D}_t - N_t A_{1t} B_{0t}, \qquad \text{(A17)}
\end{aligned}
$$

where we have eliminated $(N_t - 1)\Sigma_{t,c}\Sigma_{t,\xi}^{-1}$ from the first equation using Equation (A11).

We will solve four equations (A11), (A12), (A16), and (A17) for four unknowns, A_{0t}, A_{1t}, B_{0t}, and B_{1t}. First, we immediately have $A_{1t} B_{0t} = 0$ from (A12) and (A17). Again assuming the nonsingularity of A_{1t}, we find that $B_{0t} = 0$ and that $A_{0t} = \widetilde{D}_t$.

Next, equating Equations (A11) and (A16) and canceling J_t,

$$B_{1t}^{-1} \Psi_t B_{1t}'^{-1} = N_t \Sigma_{t,\xi}. \qquad \text{(A18)}$$

We shall replace B_{1t} in this equation with the (assumed) symmetric matrix A_{1t}. From (A11), we have

$$B_{1t} = A_{1t}^{-1} J_t^{-1}. \qquad \text{(A19)}$$

So, equation (A18) becomes

$$A_{1t} \Psi_t A_{1t} = N_t J_t^{-1} \Sigma_{t,\xi} J_t'^{-1}. \qquad \text{(A20)}$$

Since Ψ_t is symmetric positive definite, there exists a unique symmetric positive definite matrix $\Psi_t^{\frac{1}{2}}$ such that $(\Psi_t^{\frac{1}{2}})^2 \equiv \Psi_t^{\frac{1}{2}}\Psi_t^{\frac{1}{2}} = \Psi_t$. Now pre- and post-multiply $\Psi_t^{\frac{1}{2}}$ to both sides of the above equation to write

$$(\Psi_t^{\frac{1}{2}} A_{1t} \Psi_t^{\frac{1}{2}})^2 = N_t \Psi_t^{\frac{1}{2}} J_t^{-1} \Sigma_{t,\xi} J_t'^{-1} \Psi_t^{\frac{1}{2}} \equiv N_t M_t, \tag{A21}$$

which gives the expression for M_t in Theorem 1. Since M_t is clearly symmetric positive definite, there exists a unique symmetric positive-definite matrix $M_t^{\frac{1}{2}}$ such that $(M_t^{\frac{1}{2}})^2 = M_t$. Take the square root of both sides of Equation (A21) and retain the positive sign for A_{1t} to be positive definite. Solving for A_{1t} yields the expression for A_t in Theorem 1 (i.e., $A_{1t} = A_t$). Then, using Equations (A19) and (A3) in Equation (A2),

$$x_{t,n} = B_{1t}\widetilde{\xi}_{t,n} = A_{1t}^{-1} J_t^{-1} \Sigma_{t+1} (\Sigma_{t+1} + \Gamma_t + \Phi_t)^{-1} \widetilde{y}_{t,n} = A_{1t}^{-1} J_t^{-1} \Sigma_{t,\xi} \Sigma_{t+1}^{-1} \widetilde{y}_{t,n}, \tag{A22}$$

where we have used Equation (A7). The coefficient on $\widetilde{y}_{t,n}$ gives the expression for B_t in Theorem 1. Clearly, $A_{1t} = A_t$ is indeed symmetric positive definite as has been assumed. Since all coefficients are unique, so is the equilibrium in which A_t is symmetric. ■

A.2 Proof of Lemma 1

The expected losses of noise traders are given by

$$\pi_t = \mathbf{E}_t[(\widetilde{P}_t - \widetilde{D}_T)'\widetilde{z}_t] = \mathbf{E}_t[\widetilde{w}_t' A_t \widetilde{z}_t] = \mathbf{E}_t[\widetilde{z}_t' A_t \widetilde{z}_t],$$

where we have substituted for $\widetilde{P}_t$ from Theorem 1 and retained only relevant terms. Here, $\widetilde{z}_t' A_t \widetilde{z}_t = tr(\widetilde{z}_t' A_t \widetilde{z}_t) = tr(A_t \widetilde{z}_t \widetilde{z}_t')$ by the property of the trace operator. Noting that $A_t \in \mathcal{F}_t$, we have

$$\pi_t = tr(A_t \mathbf{E}_t[\widetilde{z}_t \widetilde{z}_t']) = tr(A_t \Psi_t) = \sqrt{N_t}\, tr(\Psi_t^{-\frac{1}{2}} M_t^{\frac{1}{2}} \Psi_t^{\frac{1}{2}}),$$

where we have substituted the expression for A_t in Theorem 1. The two Ψ_t terms can circulate and cancel out, which proves Lemma 1. ■

A.3 Proof of Corollary 2

Noting the independence of terms in Equation (9) except between $\widetilde{\delta}_t$ and $\widetilde{w}_{t-1}$,

$$H_t \equiv Var_{t-1}(\Delta\widetilde{P}_t) = Var_{t-1}(A_t\widetilde{w}_t) + Var_{t-1}(\widetilde{\delta}_t - A_{t-1}\widetilde{w}_{t-1}). \tag{A23}$$

Here, since $A_t \in \mathcal{F}_t$ and $\mathbf{E}_{t-1}[\widetilde{w}_t] = \mathbf{E}_t[\widetilde{w}_t] = 0$, using the law of iterated expectations,

$$Var_{t-1}(A_t\widetilde{w}_t) = \mathbf{E}_{t-1}[A_t\widetilde{w}_t\widetilde{w}_t'A_t] = \mathbf{E}_{t-1}[A_t\mathbf{E}_t(\widetilde{w}_t\widetilde{w}_t')A_t] = \mathbf{E}_{t-1}[A_tVar_t(\widetilde{w}_t)A_t]. \tag{A24}$$

But equating the coefficients on $\widetilde{w}_t$ in Equations (A13) and (5) and rearranging, we have

$$Var_t(\widetilde{w}_t) = A_t^{-1}Cov_t(\widetilde{\delta}_{t+1}, \widetilde{w}_t') = N_tA_t^{-1}\Sigma_{t,\xi}B_{1t}' = N_tA_t^{-1}\Sigma_{t,\xi}J_t'^{-1}A_t^{-1}, \tag{A25}$$

where we have used Equations (A14) and (A19) in the second and third equalities, respectively. We conclude that

$$Var_{t-1}(A_t\widetilde{w}_t) = \mathbf{E}_{t-1}[N_t\Sigma_{t,\xi}J_t'^{-1}]. \tag{A26}$$

Note that this is symmetric since

$$\begin{aligned} N_t\Sigma_{t,\xi}J_t'^{-1} &= N_t(J_t'\Sigma_{t,\xi}^{-1})^{-1} = N_t[2\Sigma_{t,\xi}^{-1} + (N_t-1)\Sigma_{t,\xi}^{-1}\Sigma_{t,c}\Sigma_{t,\xi}^{-1}]^{-1} \text{ by (A11)} \\ &= N_t[2\Sigma_{t,\xi}^{-1} + (N_t-1)\Sigma_{t+1}^{-1}(\Sigma_{t+1}+\Gamma_t)\Sigma_{t+1}^{-1}]^{-1} \text{ by (A7) and (A8)} \\ &= N_t\Sigma_{t+1}[(N_t+1)(\Sigma_{t+1}+\Gamma_t) + 2\Phi_t]^{-1}\Sigma_{t+1} \equiv \Sigma_{Aw,t+1} \text{ by (A7)} \end{aligned} \tag{A27}$$

is. Following a similar procedure, compute the second variance in the rightmost side of Equation (A23) as

$$Var_{t-1}(\widetilde{\delta}_t - A_{t-1}\widetilde{w}_{t-1}) = \Sigma_t - Cov_{t-1}(\widetilde{\delta}_t, \widetilde{w}'_{t-1})A_{t-1} - [Cov_{t-1}(\widetilde{\delta}_t, \widetilde{w}'_{t-1})A_{t-1}]' + Var_{t-1}(A_{t-1}\widetilde{w}_{t-1})$$
$$= \Sigma_t - \Sigma_{Aw,t}. \tag{A28}$$

where we have used the fact that each of the last three terms in the first line resolves to $N_{t-1}\Sigma_{t-1,\xi}J'^{-1}_{t-1} = \Sigma_{Aw,t}$. To see this, write the first covariance term as $Cov_{t-1}(\widetilde{\delta}_t, \widetilde{w}'_{t-1})A_{t-1} = N_{t-1}\Sigma_{t-1,\xi}J'^{-1}_{t-1} = \Sigma_{Aw,t}$, which obtains by pre- and post-multiplying A_t to Equation (A25) and going back one period. Further, this is symmetric as Equation (A27) at time $t-1$ shows. Thus, the second covariance term equals the first. Finally, the last variance term can be written as $Var_{t-1}(A_{t-1}\widetilde{w}_{t-1}) = N_{t-1}\Sigma_{t-1,\xi}J'^{-1}_{t-1} = \Sigma_{Aw,t}$, which clearly results from lagging the time in the parentheses of Equations (A24) and (A26). Substituting Equations (A26) and (A28) back into Equation (A23) and using Equation (A27), we obtain Equation (10) in the corollary. ■

A.4 Proof of Corollary 3

We first state the following lemma that is necessary in computing the volume:

Lemma 2 *If $\widetilde{u} \sim MVN(0, \Sigma)$, then $\mathbf{E}|\widetilde{u}| = \sqrt{\frac{2}{\pi}diag(\Sigma)}$, where $\sqrt{\cdot}$ is the elementwise square root operator and $diag(\cdot)$ returns a vector containing the diagonal elements of the argument matrix.*

The proof is by direct computation. Now, compute the variance of the net order flow as

$$\Sigma_{w,t} = Var_t(\widetilde{w}_t) = Var_t\left(B_t\sum_{n=1}^{N_t}\widetilde{y}_{t,n}\right) + Var_t(\widetilde{z}_t) = N_tB_t[N_t(\Sigma_{t+1} + \Gamma_t) + \Phi_t]B'_t + \Psi_t. \tag{A29}$$

By the law of iterated expectations, the expected trading volume can be written as

$$\mathbf{E}_{t-1}[V_t] = \frac{1}{2}\mathbf{E}_{t-1}\left[\mathbf{E}_t\left|\sum_{n=1}^{N_t} x_{t,n}\right| + \mathbf{E}_t|\widetilde{z}_t| + \mathbf{E}_t|\widetilde{w}_t|\right].$$

Noting that the first term in Equation (A29) is the variance of all the informed traders' orders and applying the above lemma, we obtain the first expression in the corollary.

Next, compute the variance of a single informed trader's order. From Equation (6) and the results in Theorem 1, given the public information set $\mathcal{F}_t \ni \{B_t,\, N_t,\, \Sigma_{t+1}\}$ at time t,

$$\begin{aligned} Var_t(x_{t,n}) &= B_t(\Sigma_{t+1} + \Gamma_t + \Phi_t)B_t' \\ &= A_t^{-1}J_t^{-1}\Sigma_{t,\xi}\underbrace{\Sigma_{t+1}^{-1}(\Sigma_{t+1} + \Gamma_t + \Phi_t)\Sigma_{t+1}^{-1}}_{\Sigma_{t,\xi}^{-1}}\Sigma_{t,\xi}J_t'^{-1}A_t^{-1} \\ &= A_t^{-1}J_t^{-1}\Sigma_{t,\xi}J_t'^{-1}A_t^{-1} = \frac{1}{N_t}\Psi_t, \end{aligned} \tag{A30}$$

by Equation (A20) (recall that $A_{1t} = A_t$). When $\Phi_t = 0$, the right hand side of the first line implies that Equation (A29) simplifies to $\Sigma_{w,t} = (N_t + 1)\Psi_t$. This proves Equation (13) in the corollary. ■

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Table 1: **Prior distributions of parameters used in the Bayesian MCMC estimation**

	α_0	α_1^2	α_2	β_0	β_1
Distribution	$N(-10, 10)$	$IG(2.5, 0.05)$	See caption	$N(0, 0.01)$	$N(1, 1)$
Mean	-10	0.033	0.86	0	1
Stdev	3.2	0.047	0.11	0.1	1

	β_2	β_3	β_4	$10^{-6}h_0$	$10^{-7}h_1$
Distribution	$N(0, 100)$	$N(0, 1)$	$N(0, 1)$	$N(1, 1)I(10^{-6},)$	$N(0, 10)$
Mean	0	0	0	1.3	0
Stdev	10	1	1	0.79	3.2

	γ	ϕ	μ_{STOV}	$10^6\sigma^2_{STOV}$
Distribution	$N(0, 1)I(0,)$	$N(0, 1)I(0,)$	$N(0, 0.1)$	$IG(2.25, 0.0625)$
Mean	0.8	0.8	0	0.05
Stdev	0.60	0.60	0.32	0.10

This table shows the prior distributions of parameters used in the Bayesian MCMC estimation. 'Stdev' is the standard deviation. $N()$ is the normal distribution. $IG()$ is the inverse-gamma distribution. $I(b,)$ represents truncation below at b. γ and ϕ are truncated below at zero, which is perfectly admissible for these variance parameters. α_2 is set equal to $2\alpha_2^* - 1$, where $\alpha_2^* \sim beta(20, 1.5)$. To avoid numerical overflow or loss of precision, draws of h_0 are given by 10^6 times draws from the truncated normal distribution shown in the column labeled "$10^{-6}h_0$." h_1 and σ^2_{STOV} are drawn in a similar manner.

Table 2: **Selected parameter estimates of the stochastic volatility-liquidity model**

Ticker	α_0	α_1	α_2	β_1	β_2	β_3	β_4	h_1
MSFT	-10.4 (-10.6, -10.2)	0.16 (0.13, 0.21)	0.97 (0.94, 0.98)	2.51 (1.23, 4.09)	-0.4 (-0.9, 0.0)	-0.012 (-0.05, 0.02)	0.057 (-1.9, 2.0)	0.3 (-6.3, 6.4)
HON	-10.2 (-10.4, -9.9)	0.16 (0.13, 0.20)	0.97 (0.96, 0.99)	0.22 (0.12, 0.34)	-1.9 (-3.4, -0.4)	-0.022 (-0.06, 0.01)	0.066 (-1.9, 2.1)	2.3 (0.4, 6.0)
KO	-11.2 (-11.4, -10.9)	0.12 (0.10, 0.14)	0.98 (0.97, 0.99)	0.31 (0.21, 0.44)	-6.1 (-8.5, -3.7)	-0.074 (-0.11, -0.04)	0.013 (-2.0, 2.0)	4.9 (1.9, 9.5)
DD	-10.9 (-11.1, -10.7)	0.13 (0.10, 0.16)	0.97 (0.96, 0.99)	2.70 (1.81, 3.85)	-1.8 (-3.4, 0.0)	-0.028 (-0.06, 0.01)	-0.029 (-2.0, 1.9)	-3.6 (-8.2, -0.5)
XOM	-11.5 (-11.9, -11.1)	0.16 (0.13, 0.20)	0.98 (0.97, 0.99)	0.21 (0.12, 0.29)	-13.3 (-16.3, -10.2)	-0.037 (-0.07, 0.00)	0.013 (-2.0, 2.0)	4.5 (0.9, 10.4)
GE	-11.0 (-11.1, -10.9)	0.28 (0.22, 0.34)	0.87 (0.81, 0.92)	4.19 (3.09, 5.49)	-10.1 (-12.9, -7.2)	-0.079 (-0.11, -0.05)	0.013 (-1.9, 2.0)	-4.2 (-8.7, -0.9)
GM	-10.6 (-10.9, -10.3)	0.19 (0.14, 0.23)	0.97 (0.96, 0.99)	0.18 (0.11, 0.24)	-2.2 (-2.9, -1.5)	0.002 (-0.04, 0.04)	0.159 (-1.8, 2.1)	4.7 (1.2, 9.3)
IBM	-11.2 (-11.7, -10.8)	0.09 (0.08, 0.11)	0.99 (0.99, 1.00)	0.19 (0.13, 0.27)	-6.8 (-8.3, -5.4)	0.017 (-0.02, 0.05)	0.050 (-1.9, 2.0)	3.8 (1.2, 9.3)
MO	-11.6 (-11.7, -11.5)	0.74 (0.65, 0.82)	0.57 (0.47, 0.68)	0.63 (0.39, 0.93)	-1.5 (-2.6, -0.3)	-0.041 (-0.07, -0.01)	-0.167 (-2.1, 1.8)	-2.0 (-4.9, 0.3)
UTX	-10.8 (-11.2, -10.4)	0.14 (0.09, 0.18)	0.98 (0.97, 0.99)	0.21 (0.14, 0.28)	-2.9 (-4.2, -1.6)	-0.007 (-0.05, 0.03)	0.024 (-1.8, 1.9)	4.8 (1.7, 9.1)
PG	-11.6 (-11.9, -11.3)	0.14 (0.12, 0.17)	0.98 (0.97, 0.99)	0.16 (0.11, 0.21)	-2.8 (-4.6, -1.0)	-0.090 (-0.13, -0.05)	0.022 (-1.9, 2.0)	4.2 (1.6, 8.4)
CAT	-10.7 (-10.9, -10.4)	0.13 (0.10, 0.16)	0.98 (0.96, 0.99)	1.81 (0.67, 3.66)	-2.0 (-2.7, -1.3)	-0.007 (-0.04, 0.03)	-0.168 (-2.1, 1.8)	3.4 (-2.0, 8.4)
BA	-10.5 (-10.7, -10.4)	0.26 (0.23, 0.31)	0.93 (0.90, 0.95)	0.33 (0.16, 0.51)	-4.2 (-5.6, -2.7)	-0.035 (-0.07, 0.00)	0.016 (-1.9, 2.0)	2.9 (0.5, 7.0)
PFE	-11.0 (-11.2, -10.8)	0.19 (0.14, 0.27)	0.96 (0.93, 0.98)	0.20 (0.10, 0.35)	-7.4 (-9.7, -5.2)	-0.040 (-0.08, 0.00)	0.013 (-1.9, 2.0)	2.2 (0.3, 6.3)
JNJ	-11.3 (-11.5, -11.2)	0.20 (0.16, 0.24)	0.95 (0.92, 0.97)	2.70 (1.29, 4.27)	-3.1 (-5.0, -1.1)	-0.083 (-0.12, -0.05)	0.037 (-2.0, 2.0)	3.1 (-6.1, 7.8)
MMM	-11.3 (-11.6, -11.1)	0.12 (0.10, 0.15)	0.98 (0.97, 0.99)	0.14 (0.08, 0.24)	-2.4 (-3.5, -1.3)	-0.006 (-0.04, 0.03)	-0.030 (-2.0, 1.9)	2.3 (0.4, 8.5)
MRK	-10.9 (-11.2, -10.7)	0.16 (0.13, 0.19)	0.97 (0.96, 0.99)	1.02 (0.20, 2.60)	-4.7 (-6.7, -2.7)	-0.033 (-0.07, 0.00)	0.053 (-1.9, 2.1)	5.6 (2.5, 10.1)
AA	-10.4 (-10.6, -10.2)	0.15 (0.12, 0.18)	0.97 (0.95, 0.98)	1.02 (0.33, 2.68)	-1.6 (-2.8, -0.3)	-0.034 (-0.07, 0.00)	0.043 (-1.9, 2.1)	-0.1 (-5.0, 5.3)
DIS	-10.3 (-10.6, -10.1)	0.23 (0.17, 0.30)	0.96 (0.93, 0.98)	0.26 (0.09, 0.45)	-4.7 (-6.3, -3.1)	-0.039 (-0.07, 0.00)	0.037 (-1.9, 2.0)	2.6 (0.1, 7.4)
HPQ	-9.9 (-10.2, -9.6)	0.15 (0.13, 0.17)	0.98 (0.97, 0.99)	2.47 (1.28, 3.84)	-4.3 (-6.1, -2.6)	-0.055 (-0.09, -0.02)	-0.019 (-2.0, 1.9)	4.2 (1.9, 8.1)
MCD	-10.8 (-10.9, -10.7)	0.39 (0.34, 0.46)	0.80 (0.73, 0.86)	3.25 (2.35, 4.57)	-1.2 (-2.2, -0.1)	-0.063 (-0.10, -0.03)	0.059 (-1.9, 2.0)	-5.0 (-10.7, -1.4)
JPM	-10.5 (-11.0, -10.1)	0.10 (0.08, 0.12)	0.99 (0.99, 1.00)	2.88 (1.87, 4.23)	-6.6 (-8.6, -4.7)	0.006 (-0.03, 0.04)	-0.064 (-2.1, 1.9)	5.1 (2.0, 9.3)
WMT	-11.2 (-11.5, -11.0)	0.15 (0.13, 0.17)	0.98 (0.97, 0.99)	0.14 (0.09, 0.22)	-8.1 (-10.4, -5.9)	-0.078 (-0.11, -0.05)	0.025 (-2.0, 2.0)	0.6 (0.2, 2.0)
AXP	-10.6 (-10.9, -10.2)	0.14 (0.12, 0.16)	0.99 (0.98, 0.99)	0.34 (0.21, 0.52)	-5.5 (-7.4, -3.6)	-0.003 (-0.04, 0.03)	0.027 (-1.9, 2.0)	3.8 (1.0, 7.9)
INTC	-9.8 (-9.9, -9.7)	0.31 (0.24, 0.38)	0.91 (0.88, 0.94)	1.76 (0.41, 3.28)	-1.5 (-2.6, -0.5)	0.008 (-0.03, 0.04)	-0.026 (-2.0, 1.9)	-4.8 (-7.9, -2.0)
VZ	-10.6 (-10.9, -10.3)	0.19 (0.15, 0.24)	0.98 (0.96, 0.99)	0.21 (0.15, 0.28)	-5.8 (-8.4, -3.1)	-0.026 (-0.06, 0.01)	0.010 (-1.9, 2.0)	4.5 (1.7, 9.3)
SBC	-10.3 (-10.5, -10.1)	0.15 (0.14, 0.18)	0.97 (0.96, 0.98)	2.93 (2.11, 4.06)	-5.4 (-8.2, -2.6)	-0.034 (-0.07, 0.00)	0.018 (-2.0, 2.0)	-3.8 (-8.4, -0.5)
HD	-10.6 (-10.7, -10.5)	0.23 (0.18, 0.27)	0.93 (0.90, 0.96)	3.57 (2.54, 4.66)	-3.4 (-4.4, -2.3)	-0.052 (-0.09, -0.02)	-0.038 (-2.0, 1.9)	-6.9 (-11.3, -3.4)
AIG	-10.8 (-11.1, -10.5)	0.16 (0.12, 0.22)	0.98 (0.96, 0.99)	0.24 (0.18, 0.33)	-3.8 (-6.6, -0.9)	-0.028 (-0.06, 0.01)	0.040 (-2.0, 2.0)	4.1 (1.7, 8.7)
C	-10.8 (-11.2, -10.5)	0.18 (0.15, 0.25)	0.98 (0.96, 0.99)	0.25 (0.19, 0.31)	-5.3 (-8.1, -2.4)	-0.070 (-0.11, -0.03)	0.037 (-1.9, 2.0)	7.8 (3.5, 12.7)

Table 2: **Continued**

Ticker	$c_t \times 10^{-3}$	N_t	σ^2_{t+1}	γ	ϕ	$Corr$	$\psi_t \times 10^{-9}$	$\underline{a_t}$	$b_t \times 10^{-3}$
MSFT	203.70 (44.46, 604.17)	0.5 (0.1, 1.9)	0.12 (0.06, 0.26)	0.09 (0.00, 0.39)	0.23 (0.03, 0.61)	0.47	1017.4 (728.9, 1401.4)	7.8 (2.4, 20.1)	4601.2 (1028.3, 13568.5)
HON	1.77 (0.26, 5.26)	7.6 (2.2, 19.7)	0.34 (0.10, 0.88)	0.00 (0.00, 0.00)	0.01 (0.00, 0.03)	0.97	2.5 (0.8, 5.0)	105.3 (53.5, 185.3)	57.3 (8.6, 175.8)
KO	1.24 (0.37, 3.45)	8.4 (3.2, 16.1)	0.29 (0.11, 0.58)	0.00 (0.00, 0.01)	0.02 (0.00, 0.06)	0.91	2.8 (1.4, 5.5)	162.5 (95.7, 247.3)	48.3 (15.0, 136.4)
DD	0.00 (0.00, 0.02)	256.9 (121.0, 441.3)	0.04 (0.02, 0.07)	0.97 (0.46, 1.83)	0.70 (0.02, 2.13)	0.59	0.1 (0.1, 0.8)	10.2 (5.2, 18.3)	1.0 (0.3, 6.2)
XOM	0.83 (0.14, 2.19)	21.1 (8.7, 49.1)	0.35 (0.13, 0.85)	0.00 (0.00, 0.00)	0.00 (0.00, 0.02)	0.98	5.5 (2.0, 10.1)	328.3 (200.5, 544.3)	41.2 (8.1, 96.7)
GE	0.00 (0.00, 0.02)	817.6 (288.1, 1549.1)	0.02 (0.01, 0.04)	0.30 (0.18, 0.47)	0.82 (0.03, 2.30)	0.28	4.0 (0.9, 23.8)	17.3 (7.5, 34.5)	4.2 (0.8, 27.6)
GM	1.82 (0.43, 4.52)	10.9 (5.4, 24.3)	0.68 (0.29, 1.59)	0.00 (0.00, 0.00)	0.01 (0.00, 0.03)	0.98	3.3 (1.4, 5.5)	50.2 (32.4, 80.5)	30.3 (7.8, 61.2)
IBM	0.82 (0.23, 2.13)	27.5 (11.9, 51.2)	3.20 (1.32, 6.32)	0.00 (0.00, 0.01)	0.04 (0.00, 0.15)	0.99	2.1 (1.0, 4.0)	156.5 (100.7, 226.9)	6.6 (2.0, 16.2)
MO	7.42 (2.28, 17.56)	2.9 (0.7, 7.5)	0.13 (0.01, 0.56)	0.00 (0.00, 0.00)	0.00 (0.00, 0.01)	0.97	25.5 (10.1, 46.4)	44.3 (11.0, 114.9)	805.6 (76.7, 3222.9)
UTX	1.62 (0.48, 3.61)	7.7 (4.0, 15.0)	0.98 (0.46, 1.92)	0.00 (0.00, 0.01)	0.03 (0.00, 0.09)	0.97	0.7 (0.4, 1.1)	83.0 (56.1, 122.2)	11.8 (4.0, 22.8)
PG	0.54 (0.17, 1.26)	16.4 (8.5, 29.3)	1.29 (0.59, 2.51)	0.00 (0.00, 0.01)	0.03 (0.00, 0.10)	0.97	0.6 (0.3, 0.9)	166.6 (111.9, 239.8)	6.2 (2.3, 13.0)
CAT	13.50 (0.59, 87.89)	1.7 (0.0, 6.2)	0.09 (0.05, 0.19)	0.13 (0.00, 0.71)	0.34 (0.00, 1.58)	0.39	3.7 (2.0, 6.4)	9.2 (2.7, 22.8)	242.4 (19.6, 1466.8)
BA	3.29 (0.43, 8.89)	4.1 (1.3, 13.8)	0.25 (0.06, 0.76)	0.00 (0.00, 0.01)	0.00 (0.00, 0.01)	0.98	3.5 (1.0, 6.5)	69.8 (32.1, 140.9)	113.2 (13.4, 344.1)
PFE	0.50 (0.05, 1.63)	47.3 (13.9, 113.6)	0.72 (0.11, 2.52)	0.00 (0.00, 0.01)	0.12 (0.02, 0.31)	0.80	11.2 (2.3, 28.6)	266.3 (100.3, 572.6)	32.4 (3.7, 92.9)
JNJ	0.63 (0.00, 1.59)	92.1 (2.8, 669.0)	0.05 (0.02, 0.10)	0.28 (0.01, 0.79)	0.82 (0.04, 2.00)	0.29	14.3 (0.2, 24.2)	17.9 (7.1, 41.3)	43.4 (0.5, 98.1)
MMM	0.40 (0.06, 1.56)	29.4 (7.7, 66.9)	6.58 (1.78, 15.82)	0.00 (0.00, 0.02)	0.17 (0.00, 0.73)	0.97	0.2 (0.1, 0.5)	120.0 (62.1, 197.3)	1.6 (0.3, 5.7)
MRK	1.49 (0.58, 3.57)	8.1 (3.1, 16.6)	0.31 (0.04, 0.96)	0.03 (0.00, 0.18)	0.52 (0.00, 1.61)	0.37	8.1 (2.3, 14.9)	90.3 (12.3, 212.2)	42.3 (15.2, 89.9)
AA	22.77 (1.96, 91.33)	0.8 (0.0, 3.9)	0.05 (0.02, 0.15)	0.00 (0.00, 0.01)	0.00 (0.00, 0.01)	0.94	11.8 (4.3, 17.7)	26.2 (6.7, 68.1)	1252.7 (126.0, 4822.2)
DIS	2.29 (0.07, 8.30)	10.5 (1.7, 56.8)	0.16 (0.03, 0.78)	0.00 (0.00, 0.00)	0.01 (0.00, 0.04)	0.94	11.5 (1.4, 26.9)	122.8 (48.1, 319.7)	218.9 (7.3, 830.6)
HPQ	2.89 (1.01, 6.54)	3.0 (1.0, 7.6)	0.04 (0.02, 0.08)	0.13 (0.03, 0.36)	0.64 (0.21, 1.36)	0.21	34.5 (27.9, 40.4)	13.6 (5.7, 29.2)	158.5 (71.1, 295.6)
MCD	0.04 (0.00, 0.27)	493.9 (55.4, 1124.6)	0.01 (0.00, 0.03)	0.22 (0.12, 0.38)	1.09 (0.10, 2.56)	0.17	4.9 (0.3, 20.3)	6.9 (1.7, 17.0)	25.8 (0.5, 164.3)
JPM	2.42 (0.84, 4.97)	3.4 (1.6, 7.8)	0.04 (0.02, 0.06)	0.15 (0.03, 0.35)	0.75 (0.25, 1.57)	0.20	36.0 (30.2, 41.3)	10.2 (5.4, 17.5)	138.4 (61.0, 237.5)
WMT	0.12 (0.03, 0.39)	79.5 (28.6, 151.7)	2.89 (0.86, 6.36)	0.01 (0.00, 0.01)	0.09 (0.01, 0.26)	0.96	1.1 (0.5, 2.4)	507.3 (277.3, 788.5)	3.2 (0.8, 10.4)
AXP	3.46 (0.76, 10.54)	4.5 (1.4, 10.6)	0.24 (0.09, 0.56)	0.00 (0.00, 0.01)	0.01 (0.00, 0.04)	0.96	4.2 (1.8, 8.4)	104.4 (55.8, 172.8)	95.4 (20.6, 303.1)
INTC	16.92 (8.47, 29.05)	21.4 (5.3, 66.4)	2.82 (0.08, 11.30)	0.16 (0.00, 0.32)	0.50 (0.05, 1.24)	0.31	1033.4 (515.7, 1364.1)	21.8 (2.7, 87.0)	1663.4 (499.6, 4042.6)
VZ	1.84 (0.54, 4.60)	10.4 (5.2, 18.6)	0.48 (0.21, 0.96)	0.00 (0.00, 0.00)	0.01 (0.00, 0.02)	0.98	4.1 (2.2, 6.9)	221.7 (146.5, 324.3)	37.2 (14.2, 77.3)
SBC	0.00 (0.00, 0.01)	488.2 (242.1, 869.1)	0.03 (0.02, 0.06)	0.50 (0.27, 0.91)	0.72 (0.03, 2.18)	0.42	0.6 (0.2, 2.6)	21.2 (10.5, 39.0)	1.7 (0.5, 9.0)
HD	0.01 (0.00, 0.05)	731.1 (296.7, 1265.2)	0.04 (0.02, 0.07)	0.59 (0.31, 1.02)	1.90 (0.71, 3.32)	0.25	1.9 (0.4, 10.6)	10.1 (4.3, 20.2)	2.8 (0.4, 18.6)
AIG	1.45 (0.45, 3.58)	13.6 (6.6, 23.9)	1.22 (0.48, 2.50)	0.01 (0.00, 0.02)	0.06 (0.00, 0.17)	0.94	2.9 (1.6, 5.1)	221.5 (133.3, 332.2)	17.7 (6.5, 41.2)
C	2.77 (0.88, 6.07)	14.0 (8.4, 23.8)	0.55 (0.28, 1.01)	0.00 (0.00, 0.00)	0.00 (0.00, 0.01)	0.99	14.5 (8.3, 21.6)	206.1 (148.4, 287.9)	52.7 (23.4, 86.5)

This table shows the estimates of selected parameters and quantities of the stochastic volatility-liquidity model. The model is described by the mean return equation in (18) with the price impact coefficient (19), conditional variance in (20), conditional information variance in (21)-(24), noise trading variance in (25), and the cost of private information in (27). The zero-profit condition (26) is numerically solved with equality at each node in each iteration. The variances of the two signal noises and the signed share turnover are restricted as $\gamma_{t-1} \equiv \gamma$, $\phi_{t-1} \equiv \phi$, and $\sigma^2_{stov,t} \equiv \sigma^2_{stov}$. b_t is computed as a generic entry of B_t using the relations in Theorem 1. For parameters without a time subscript, the mean of the Bayesian MCMC sampling distribution is shown along with the 2.5 and 97.5 percentiles in parentheses. For quantities with a time subscript, the time-series average of the sampling mean is shown along with the time-series averages of the sampling 2.5 and 97.5 percentiles in parentheses. $Corr$ is the correlation between the private signals of two informed traders, calculated as $(\sigma^2_{t+1} + \gamma)/(\sigma^2_{t+1} + \gamma + \phi)$ at mean parameter estimates.

Table 3: **Selected parameter estimates of the realized volatility-liquidity model**

Ticker	α_0	α_1	α_2	β_1	β_2	β_3	β_4	h_1
MSFT	-10.4 (-10.5, -10.4)	0.49 (0.48, 0.50)	0.76 (0.74, 0.79)	2.00 (0.42, 3.75)	-0.2 (-0.9, 0.5)	-0.017 (-0.05, 0.01)	0.058 (-1.9, 2.0)	0.5 (-5.3, 6.5)
HON	-10.1 (-10.1, -10.0)	0.75 (0.73, 0.77)	0.57 (0.55, 0.60)	0.35 (0.17, 0.57)	-1.4 (-3.0, 0.3)	-0.008 (-0.05, 0.03)	0.040 (-1.9, 2.0)	3.6 (0.5, 8.3)
KO	-10.9 (-11.0, -10.9)	0.66 (0.65, 0.68)	0.59 (0.57, 0.61)	3.18 (2.56, 3.89)	-5.3 (-8.3, -2.3)	-0.049 (-0.09, -0.01)	0.007 (-1.9, 1.9)	-3.9 (-8.2, -0.7)
DD	-10.7 (-10.7, -10.6)	0.69 (0.68, 0.71)	0.62 (0.60, 0.64)	2.37 (1.70, 3.08)	-2.4 (-4.2, -0.5)	-0.013 (-0.05, 0.03)	-0.018 (-2.0, 1.9)	-3.6 (-8.0, -0.5)
XOM	-11.0 (-11.1, -10.9)	0.69 (0.68, 0.71)	0.71 (0.69, 0.73)	0.19 (0.15, 0.22)	-13.6 (-17.9, -9.4)	-0.007 (-0.05, 0.04)	0.010 (-2.0, 2.0)	11.6 (7.1, 16.4)
GE	-10.5 (-10.6, -10.5)	0.71 (0.69, 0.73)	0.56 (0.54, 0.58)	3.74 (3.01, 4.56)	-10.8 (-14.7, -6.9)	-0.031 (-0.07, 0.01)	0.006 (-2.0, 2.0)	-3.8 (-8.5, -0.5)
GM	-10.5 (-10.5, -10.4)	0.82 (0.80, 0.84)	0.61 (0.59, 0.63)	0.15 (0.10, 0.22)	-1.9 (-2.4, -1.3)	0.021 (-0.01, 0.06)	0.071 (-1.9, 2.0)	3.4 (1.0, 8.0)
IBM	-10.8 (-10.9, -10.8)	0.67 (0.65, 0.69)	0.56 (0.54, 0.58)	0.17 (0.11, 0.23)	-6.2 (-8.1, -4.4)	0.003 (-0.04, 0.04)	0.042 (-2.0, 2.0)	3.8 (1.2, 8.9)
MO	-10.9 (-11.0, -10.9)	0.77 (0.75, 0.79)	0.54 (0.52, 0.56)	2.91 (2.13, 3.81)	-0.3 (-1.9, 1.3)	-0.019 (-0.06, 0.02)	-0.210 (-2.2, 1.8)	4.2 (0.7, 8.9)
UTX	-10.6 (-10.6, -10.5)	0.75 (0.73, 0.77)	0.59 (0.57, 0.62)	0.24 (0.09, 0.37)	-3.7 (-5.1, -2.4)	0.018 (-0.02, 0.06)	-0.009 (-1.9, 1.9)	3.2 (0.2, 7.8)
PG	-11.4 (-11.4, -11.3)	0.68 (0.67, 0.70)	0.60 (0.58, 0.62)	0.13 (0.09, 0.18)	-5.0 (-7.1, -2.9)	-0.065 (-0.11, -0.02)	-0.016 (-2.0, 1.9)	4.0 (1.2, 7.9)
CAT	-10.5 (-10.6, -10.5)	0.73 (0.71, 0.74)	0.47 (0.45, 0.49)	0.19 (0.15, 0.24)	-3.0 (-3.8, -2.1)	0.025 (-0.01, 0.06)	-0.141 (-2.1, 1.8)	-1.3 (-2.3, -0.6)
BA	-10.2 (-10.2, -10.1)	0.75 (0.73, 0.77)	0.46 (0.44, 0.48)	2.40 (1.90, 2.93)	-3.3 (-5.0, -1.6)	-0.017 (-0.06, 0.02)	0.010 (-1.9, 2.0)	-3.9 (-8.4, -0.6)
PFE	-10.6 (-10.7, -10.6)	0.66 (0.65, 0.68)	0.61 (0.59, 0.63)	2.82 (1.91, 3.81)	-6.3 (-9.3, -3.3)	-0.011 (-0.05, 0.03)	0.012 (-2.0, 2.0)	5.2 (-2.6, 9.8)
JNJ	-10.9 (-11.0, -10.9)	0.74 (0.73, 0.76)	0.60 (0.58, 0.62)	0.20 (0.14, 0.26)	-4.7 (-7.2, -2.2)	-0.053 (-0.09, -0.01)	0.027 (-1.9, 2.0)	4.8 (1.6, 9.7)
MMM	-11.1 (-11.2, -11.1)	1.20 (1.17, 1.23)	0.36 (0.33, 0.38)	0.44 (0.31, 0.60)	-2.9 (-4.1, -1.7)	-0.011 (-0.05, 0.03)	-0.083 (-2.1, 1.9)	6.5 (3.4, 9.8)
MRK	-10.8 (-10.8, -10.7)	0.78 (0.76, 0.80)	0.57 (0.55, 0.59)	0.22 (0.13, 0.31)	-4.9 (-7.2, -2.5)	-0.014 (-0.05, 0.03)	0.013 (-1.9, 1.9)	4.3 (0.9, 8.9)
AA	-10.3 (-10.3, -10.2)	0.80 (0.78, 0.82)	0.48 (0.45, 0.50)	2.58 (1.79, 3.51)	-1.8 (-3.2, -0.4)	-0.009 (-0.05, 0.03)	0.060 (-1.9, 2.0)	-3.3 (-8.1, -0.4)
DIS	-10.1 (-10.1, -10.0)	0.73 (0.72, 0.75)	0.61 (0.59, 0.63)	2.91 (2.03, 4.05)	-6.5 (-8.5, -4.6)	-0.011 (-0.05, 0.03)	0.003 (-1.9, 1.9)	-3.8 (-8.4, 0.4)
HPQ	-9.8 (-9.8, -9.7)	1.16 (1.13, 1.19)	0.41 (0.38, 0.43)	4.86 (3.96, 5.75)	-3.2 (-5.2, -1.2)	-0.026 (-0.06, 0.01)	0.012 (-2.0, 2.0)	13.1 (11.2, 15.5)
MCD	-10.4 (-10.4, -10.4)	0.75 (0.73, 0.77)	0.45 (0.43, 0.47)	3.15 (2.31, 4.12)	-2.2 (-3.5, -0.9)	-0.020 (-0.06, 0.02)	0.029 (-1.9, 2.0)	-3.9 (-8.9, -0.7)
JPM	-10.1 (-10.2, -10.0)	0.72 (0.70, 0.74)	0.64 (0.62, 0.66)	0.24 (0.18, 0.31)	-6.8 (-8.9, -4.8)	0.008 (-0.03, 0.05)	-0.011 (-2.0, 2.0)	5.5 (2.6, 10.0)
WMT	-10.8 (-10.9, -10.8)	0.76 (0.74, 0.78)	0.53 (0.51, 0.54)	0.25 (0.17, 0.33)	-8.3 (-11.7, -4.7)	-0.062 (-0.10, -0.02)	0.012 (-2.0, 1.9)	5.1 (1.9, 10.1)
AXP	-10.4 (-10.4, -10.3)	0.75 (0.73, 0.77)	0.61 (0.59, 0.63)	2.54 (1.69, 3.67)	-6.8 (-8.9, -4.7)	0.025 (-0.01, 0.06)	0.008 (-2.0, 1.9)	-1.4 (-7.9, 6.0)
INTC	-9.7 (-9.7, -9.7)	0.50 (0.49, 0.52)	0.75 (0.75, 0.76)	1.38 (0.59, 2.21)	-1.0 (-2.1, 0.0)	0.017 (-0.01, 0.05)	0.003 (-2.0, 2.0)	-0.7 (-0.7, -0.7)
VZ	-10.3 (-10.4, -10.3)	0.72 (0.71, 0.74)	0.65 (0.63, 0.68)	0.21 (0.12, 0.34)	-6.7 (-9.8, -3.6)	-0.007 (-0.05, 0.03)	0.020 (-1.8, 2.0)	3.5 (0.6, 7.8)
SBC	-10.1 (-10.2, -10.1)	0.76 (0.74, 0.78)	0.61 (0.60, 0.63)	3.00 (2.17, 4.01)	-5.2 (-8.3, -2.0)	-0.035 (-0.08, 0.00)	0.012 (-1.9, 2.0)	-3.7 (-8.5, -0.6)
HD	-10.3 (-10.3, -10.2)	0.73 (0.71, 0.75)	0.52 (0.50, 0.54)	0.37 (0.28, 0.48)	-2.7 (-4.6, -0.8)	-0.033 (-0.07, 0.01)	0.019 (-1.9, 2.0)	6.5 (2.9, 10.1)
AIG	-10.6 (-10.6, -10.5)	0.75 (0.73, 0.77)	0.65 (0.63, 0.67)	0.17 (0.10, 0.24)	-4.5 (-7.6, -1.2)	-0.022 (-0.06, 0.02)	0.012 (-1.9, 2.0)	3.5 (0.8, 8.1)
C	-10.5 (-10.6, -10.4)	0.66 (0.65, 0.68)	0.76 (0.74, 0.78)	0.22 (0.17, 0.25)	-4.7 (-8.1, -1.3)	-0.043 (-0.09, 0.00)	0.032 (-1.9, 1.9)	12.6 (6.7, 18.0)

Table 3: **Continued**

Ticker	$c_t \times 10^{-3}$	N_t	σ^2_{t+1}	γ	ϕ	$Corr$	$\psi_t \times 10^{-9}$	$\underline{a}_t$	$b_t \times 10^{-3}$
MSFT	192.03 (16.30, 590.52)	0.8 (0.1, 5.3)	0.26 (0.09, 0.24)	0.08 (0.00, 0.40)	0.39 (0.10, 1.28)	0.31	963.7 (619.7, 1247.7)	6.1 (2.3, 20.5)	5031.3 (499.6, 14529.8)
HON	7.58 (1.11, 20.07)	2.2 (0.5, 6.9)	0.14 (0.07, 0.33)	0.00 (0.00, 0.02)	0.01 (0.00, 0.02)	0.94	6.1 (2.2, 10.2)	55.6 (30.8, 104.6)	260.1 (41.1, 657.1)
KO	0.00 (0.00, 0.02)	244.8 (177.1, 325.9)	0.05 (0.05, 0.05)	2.07 (1.36, 2.96)	0.72 (0.02, 2.07)	0.75	0.3 (0.1, 1.0)	12.5 (10.2, 15.2)	1.2 (0.4, 4.8)
DD	0.00 (0.00, 0.02)	236.8 (161.8, 347.6)	0.05 (0.05, 0.05)	1.48 (0.80, 2.34)	0.72 (0.03, 2.10)	0.68	0.1 (0.1, 0.5)	10.0 (7.5, 13.4)	1.0 (0.3, 4.4)
XOM	3.07 (1.87, 4.37)	7.2 (5.3, 10.0)	0.23 (0.18, 0.31)	0.00 (0.00, 0.00)	0.00 (0.00, 0.01)	0.99	12.0 (8.8, 15.1)	268.4 (230.9, 319.0)	96.3 (57.6, 141.0)
GE	0.00 (0.00, 0.02)	427.3 (313.8, 573.4)	0.03 (0.03, 0.03)	1.82 (1.20, 2.65)	0.73 (0.03, 2.10)	0.72	3.1 (1.2, 10.4)	11.5 (9.3, 14.0)	3.7 (1.0, 13.0)
GM	1.14 (0.35, 2.72)	13.6 (5.9, 25.7)	0.76 (0.37, 1.38)	0.00 (0.00, 0.00)	0.01 (0.00, 0.05)	0.97	2.8 (1.3, 5.0)	54.0 (36.3, 76.1)	45.7 (8.0, 154.1)
IBM	0.99 (0.32, 2.34)	23.4 (11.1, 44.4)	3.67 (1.82, 6.79)	0.00 (0.00, 0.01)	0.04 (0.00, 0.15)	0.99	2.3 (1.1, 4.2)	167.1 (116.5, 233.9)	6.6 (2.1, 15.8)
MO	0.96 (0.17, 2.00)	2.7 (1.3, 6.5)	0.04 (0.04, 0.04)	1.86 (1.00, 2.94)	0.39 (0.01, 1.40)	0.83	25.6 (13.3, 35.0)	4.1 (3.2, 5.5)	74.6 (26.2, 124.1)
UTX	1.57 (0.11, 4.38)	8.4 (2.4, 35.9)	0.92 (0.37, 3.80)	0.02 (0.00, 0.05)	0.10 (0.00, 0.39)	0.87	0.8 (0.2, 1.6)	74.5 (42.9, 171.8)	17.1 (1.3, 45.9)
PG	0.66 (0.20, 1.34)	14.6 (7.5, 29.3)	1.41 (0.75, 2.72)	0.01 (0.00, 0.02)	0.08 (0.00, 0.27)	0.93	0.7 (0.3, 1.1)	173.4 (123.1, 251.3)	6.8 (2.0, 14.1)
CAT	1.28 (0.64, 2.33)	6.8 (4.0, 10.5)	0.64 (0.40, 0.97)	0.00 (0.00, 0.01)	0.01 (0.00, 0.03)	0.98	1.0 (0.6, 1.4)	55.1 (42.3, 69.6)	19.9 (9.8, 35.7)
BA	0.00 (0.00, 0.03)	225.0 (167.7, 290.6)	0.06 (0.06, 0.06)	2.60 (1.71, 3.70)	0.75 (0.03, 2.11)	0.78	0.2 (0.1, 0.7)	6.6 (5.4, 8.0)	1.0 (0.3, 4.3)
PFE	1.72 (0.00, 3.16)	17.3 (1.5, 376.1)	0.04 (0.04, 0.04)	1.14 (0.47, 2.18)	1.11 (0.12, 2.43)	0.51	76.0 (1.6, 96.3)	9.6 (7.0, 13.8)	119.9 (1.9, 192.1)
JNJ	1.30 (0.45, 2.68)	13.6 (7.0, 26.1)	0.71 (0.40, 1.30)	0.01 (0.00, 0.01)	0.08 (0.00, 0.22)	0.85	4.8 (2.4, 8.0)	180.5 (130.4, 255.0)	28.0 (9.4, 58.3)
MMM	1.25 (0.60, 2.06)	7.8 (4.7, 12.9)	0.87 (0.57, 1.35)	0.03 (0.00, 0.09)	1.01 (0.47, 1.59)	0.44	0.8 (0.5, 1.1)	36.2 (25.5, 49.8)	6729.0 (4.1, 22635.8)
MRK	2.52 (0.49, 5.95)	7.6 (3.1, 19.7)	0.54 (0.28, 1.28)	0.01 (0.00, 0.01)	0.02 (0.00, 0.08)	0.94	4.7 (1.7, 8.2)	150.1 (102.6, 247.8)	54.9 (10.7, 128.3)
AA	0.05 (0.00, 0.29)	197.3 (63.0, 366.4)	0.03 (0.03, 0.03)	1.20 (0.59, 2.12)	0.64 (0.02, 2.09)	0.66	1.1 (0.1, 4.5)	6.1 (4.3, 8.6)	8.3 (0.4, 39.7)
DIS	0.02 (0.00, 0.29)	292.7 (4.1, 491.1)	0.02 (0.02, 0.02)	0.91 (0.44, 1.65)	0.70 (0.02, 2.08)	0.57	1.5 (0.3, 16.9)	8.3 (5.8, 11.5)	5.9 (0.7, 71.7)
HPQ	7.87 (6.68, 9.46)	0.3 (0.2, 0.4)	0.04 (0.04, 0.04)	1.00 (0.75, 1.34)	0.88 (0.71, 1.06)	0.54	48.8 (45.1, 53.3)	3.2 (2.8, 3.9)	30961.9 (294.9, 82843.8)
MCD	0.00 (0.00, 0.02)	222.9 (146.7, 333.0)	0.02 (0.02, 0.02)	1.19 (0.63, 1.96)	0.68 (0.02, 2.00)	0.64	0.8 (0.3, 2.7)	3.7 (2.8, 5.0)	2.8 (0.8, 10.9)
JPM	3.05 (1.40, 5.58)	8.3 (4.8, 13.4)	0.36 (0.23, 0.55)	0.00 (0.00, 0.00)	0.00 (0.00, 0.02)	0.98	10.7 (6.6, 15.9)	121.1 (93.3, 155.5)	77.0 (35.2, 141.5)
WMT	1.33 (0.52, 2.69)	14.8 (7.7, 26.3)	0.59 (0.33, 1.07)	0.00 (0.00, 0.02)	0.22 (0.03, 0.44)	0.67	6.5 (3.4, 10.3)	216.8 (156.3, 309.0)	28.4 (10.6, 57.4)
AXP	0.45 (0.00, 2.54)	182.2 (1.1, 391.5)	0.06 (0.05, 0.06)	1.44 (0.46, 2.57)	0.59 (0.01, 2.03)	0.71	2.7 (0.1, 9.4)	11.0 (7.4, 15.8)	20.6 (0.3, 96.3)
INTC	16.32 (16.03, 16.64)	2.1 (1.2, 4.3)	0.75 (0.21, 2.75)	0.51 (0.12, 1.03)	0.08 (0.00, 0.37)	0.86	1176.6 (909.1, 1373.6)	8.7 (4.8, 18.2)	1567.8 (1373.8, 1856.1)
VZ	3.47 (0.49, 10.77)	7.0 (1.9, 19.8)	0.43 (0.17, 1.08)	0.00 (0.00, 0.01)	0.01 (0.00, 0.06)	0.95	6.7 (2.1, 13.9)	198.3 (114.1, 343.5)	88.4 (12.2, 279.8)
SBC	0.00 (0.00, 0.02)	340.5 (238.4, 488.9)	0.04 (0.04, 0.04)	1.37 (0.74, 2.30)	0.69 (0.03, 2.03)	0.67	0.7 (0.3, 2.6)	16.0 (11.8, 21.5)	2.5 (0.6, 10.1)
HD	2.48 (1.11, 3.90)	8.6 (5.3, 16.1)	0.21 (0.15, 0.33)	0.00 (0.00, 0.00)	0.18 (0.12, 0.29)	0.39	25.4 (18.4, 31.7)	61.8 (47.8, 82.3)	118.0 (53.9, 183.3)
AIG	1.42 (0.32, 3.32)	16.8 (6.9, 36.4)	1.90 (0.87, 3.93)	0.01 (0.00, 0.02)	0.05 (0.00, 0.19)	0.96	2.5 (1.0, 4.6)	278.0 (184.1, 420.9)	13.3 (3.0, 31.4)
C	5.76 (3.11, 8.34)	8.1 (5.9, 12.2)	0.50 (0.38, 0.72)	0.00 (0.00, 0.00)	0.00 (0.00, 0.01)	0.99	22.3 (15.2, 28.6)	193.3 (165.3, 237.7)	86.0 (46.4, 125.8)

This table shows the estimates of selected parameters and quantities of the realized volatility-liquidity model. The model is described by the mean return equation in (18) with the price impact coefficient (19), conditional variance in (29), conditional information variance in (21)-(24), noise trading variance in (25), and the cost of private information in (27). Following Zhang, Mykland, and At-Sahalia (2005), the realized variance, $\sigma^2_{Re,t}$, is computed as the bias-corrected average of subsampled realized variances based on log price differences, as described in Subsection 2.6. The zero-profit condition (26) is numerically solved with equality at each node in each iteration. The variances of the two signal noises and the signed share turnover are restricted as $\gamma_{t-1} \equiv \gamma$, $\phi_{t-1} \equiv \phi$, and $\sigma^2_{stov,t} \equiv \sigma^2_{stov}$. b_t is computed as a generic entry of B_t using the relations in Theorem 1. For parameters without a time subscript, the mean of the Bayesian MCMC sampling distribution is shown along with the 2.5 and 97.5 percentiles in parentheses. For quantities with a time subscript, the time-series average of the sampling mean is shown along with the time-series averages of the sampling 2.5 and 97.5 percentiles in parentheses. $Corr$ is the correlation between the private signals of two informed traders, calculated as $(\sigma^2_{t+1} + \gamma)/(\sigma^2_{t+1} + \gamma + \phi)$ at mean parameter estimates.

Table 4: **Price impact regression**

Ticker	Stochastic vol-liq model g_1	g_2	Realized vol-liq model g_1	g_2
MSFT	5.2 ***(11.9)	14.6 ***(6.6)	5.5 ***(10.9)	24.3 ***(8.1)
HON	66.9 ***(15.1)	114.4 ***(12.1)	24.2 ***(11.6)	92.2 ***(13.0)
KO	86.0 ***(12.8)	242.8 ***(14.0)	2.2 ***(5.5)	48.9 ***(19.2)
DD	12.7 ***(12.8)	26.6 ***(12.5)	6.4 ***(7.3)	33.5 ***(15.8)
XOM	20.0 ***(7.4)	645.4 ***(18.4)	6.7 ***(2.7)	375.9 ***(18.0)
GE	12.0 ***(8.7)	30.7 ***(12.3)	5.0 ***(6.9)	36.7 ***(16.7)
GM	16.1 ***(15.3)	64.6 ***(14.5)	10.9 ***(10.0)	58.2 ***(13.1)
IBM	9.8 ***(8.3)	155.2 ***(12.8)	4.0 ***(5.2)	128.5 ***(12.6)
MO	5.3 ***(3.4)	64.2 ***(8.3)	1.4 ***(9.8)	12.4 ***(14.9)
UTX	9.6 ***(12.6)	91.4 ***(14.0)	8.5 ***(9.3)	104.7 ***(12.1)
PG	43.2 ***(15.9)	172.1 ***(11.5)	23.9 ***(7.7)	167.0 ***(10.9)
CAT	2.8 ***(6.4)	20.0 ***(14.1)	6.2 ***(3.9)	89.5 ***(13.6)
BA	39.9 ***(14.3)	119.7 ***(14.6)	0.8 ***(8.2)	18.0 ***(18.6)
PFE	116.5 ***(8.1)	383.1 ***(12.5)	1.4 ***(3.4)	23.6 ***(14.4)
JNJ	9.9 ***(11.8)	36.2 ***(10.9)	12.0 ***(3.7)	310.0 ***(15.6)
MMM	7.6 ***(14.1)	121.4 ***(13.0)	1.4 ***(6.4)	34.8 ***(8.5)
MRK	34.4 ***(11.3)	156.9 ***(13.1)	19.3 ***(6.5)	285.3 ***(16.1)
AA	31.9 ***(10.8)	56.9 ***(14.3)	3.8 ***(5.0)	14.8 ***(14.1)
DIS	268.3 ***(12.7)	178.3 ***(11.8)	9.7 ***(4.7)	20.7 ***(12.9)
HPQ	13.4 ***(10.8)	30.0 ***(12.7)	1.3 ***(5.5)	-0.5 (-1.0)
MCD	22.2 ***(15.1)	14.8 ***(12.4)	4.9 ***(9.7)	7.6 ***(13.7)
JPM	9.9 ***(13.0)	19.2 ***(11.0)	43.2 ***(11.9)	102.3 ***(11.5)
WMT	262.2 ***(15.3)	701.1 ***(11.9)	24.4 ***(5.2)	377.5 ***(15.2)
AXP	57.2 ***(10.1)	176.1 ***(12.5)	1.6 **(2.2)	37.9 ***(16.1)
INTC	22.6 (0.8)	14.4 (0.2)	3.2 (0.2)	8.4 (0.2)
VZ	71.1 ***(13.3)	283.6 ***(14.3)	58.3 ***(7.7)	326.2 ***(13.7)
SBC	28.6 ***(14.2)	48.9 ***(11.9)	4.4 ***(4.0)	63.6 ***(17.3)
HD	7.8 ***(9.7)	27.0 ***(13.7)	25.8 ***(6.8)	81.9 ***(6.6)
AIG	19.6 ***(9.4)	408.2 ***(16.7)	8.7 ***(6.1)	447.8 ***(16.5)
C	15.3 ***(15.6)	331.9 ***(19.9)	10.8 ***(7.1)	228.5 ***(15.3)

This table shows the estimated slope coefficients of the price impact regression, $\underline{a}_t = g_0 + g_1\delta^2_{t-1} + g_2\delta^2_{t-1,M} + \epsilon_t$, for both the stochastic and realized volatility-liquidity ("vol-liq") models. t-statistics are shown in parentheses. *, **, and *** represent significance at 10, 5, and 1%, respectively.

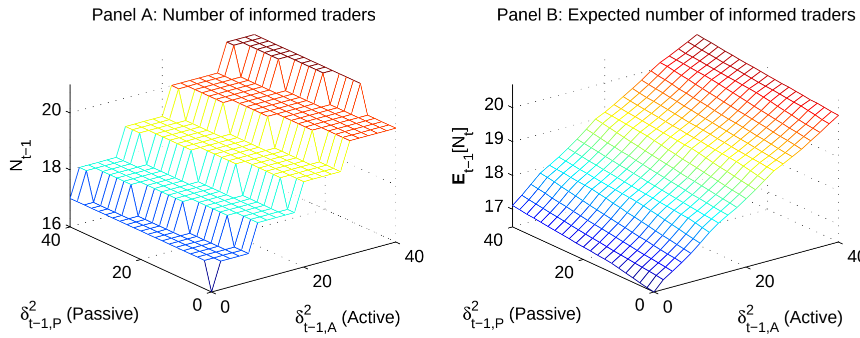

Figure 1: **The number of informed traders**. This figure plots the number of informed traders at time $t-1$ (Panel A) and the expected number of informed traders at time t (Panel B) against the time $t-1$ squared shocks to the informationally active and passive securities in Example 1.

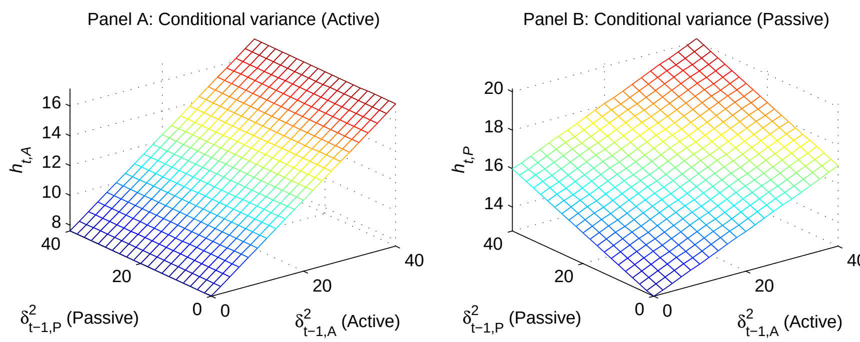

Figure 2: **News impact surfaces**. This figure plots the conditional variances of the informationally active ($h_{t,A}$, Panel A) and passive ($h_{t,P}$, Panel B) securities in Example 1 against their squared shocks at time $t-1$. $h_{t,A}$ and $h_{t,P}$ are the diagonal elements of H_t in Equation (11).

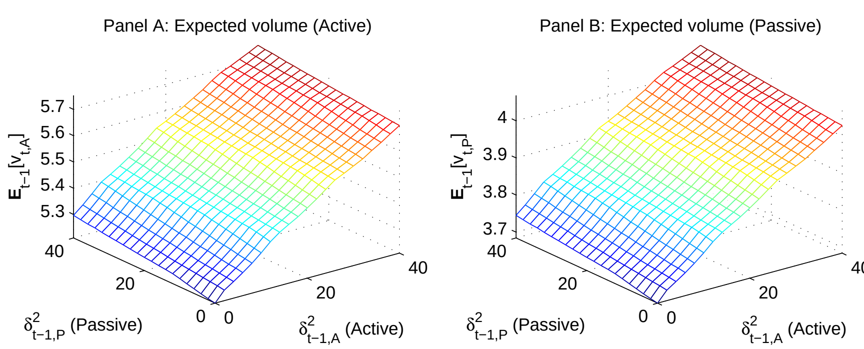

Figure 3: **Expected trading volume**. This figure plots the expected trading volume of the informationally active ($\mathbf{E}_{t-1}[v_{t,A}]$, Panel A) and passive ($\mathbf{E}_{t-1}[v_{t,P}]$, Panel B) securities in Example 1 against their squared shocks at time $t-1$. $\mathbf{E}_{t-1}[v_{t,A}]$ and $\mathbf{E}_{t-1}[v_{t,P}]$ are the elements of $\mathbf{E}_{t-1}[V_t]$ in Equation (13).

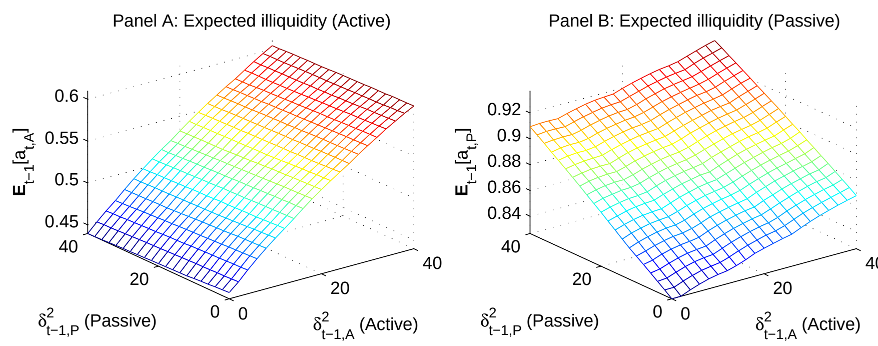

Figure 4: **Expected illiquidity**. This figure plots expected the illiquidity of the informationally active ($\mathbf{E}_{t-1}[a_{t,A}]$, Panel A) and passive ($\mathbf{E}_{t-1}[a_{t,P}]$, Panel B) securities in Example 1 against their squared shocks at time $t-1$. $\mathbf{E}_{t-1}[a_{t,A}]$ and $\mathbf{E}_{t-1}[a_{t,P}]$ are the diagonal elements of $\mathbf{E}_{t-1}[A_t]$ in Equation (14).

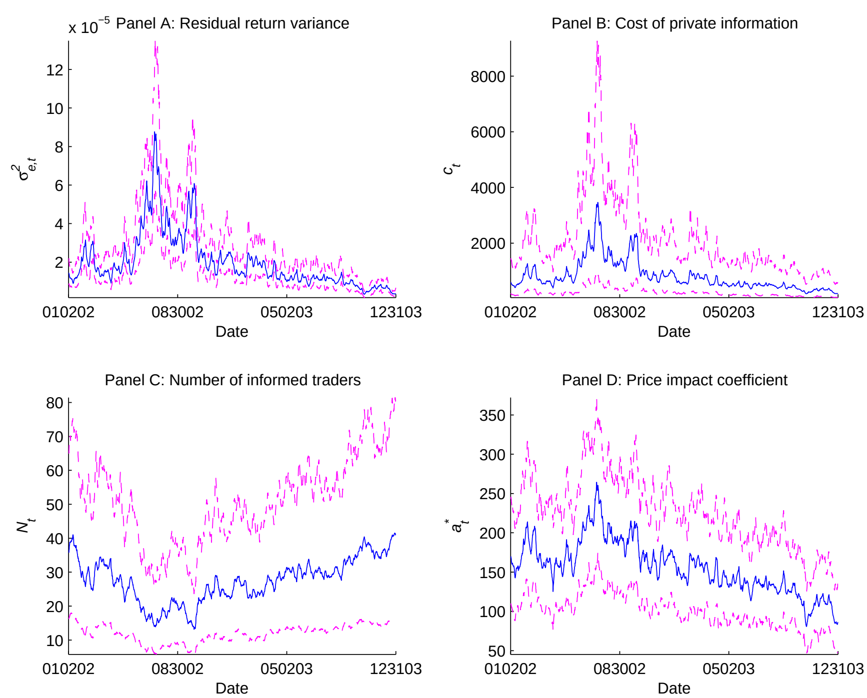

Figure 5: **Estimates for IBM**. This figure plots selected parameter estimates of the stochastic volatility-liquidity model for IBM from the Bayesian MCMC procedure. The means of the following quantities are plotted along with the 95% confidence intervals in dashed lines: Panel A: the residual return variance, $\sigma^2_{e,t}$; Panel B: the cost of private information, c_t; Panel C: the number of informed traders, N_t; Panel D: the price impact coefficient, $\underline{a}_t$ (shown as a^*_t in the vertical label). The horizontal axis represents time at the hourly frequency. Only the dates of several periods are shown in the *mmddyy* format.