Yale ICF Working Paper No. 08-11

First Draft: February 21, 1992
This Draft: June 29, 1992

Safety First Portfolio Insurance

William N. Goetzmann, International Center for Finance, Yale School of Management,
Mark Broadie, Graduate School of Business, Columbia University

This paper can be downloaded without charge from the Social Science Research Network Electronic Paper Collection:
http://ssrn.com/abstract=1102328
Safety First Portfolio Insurance

Will Goetzmann
Mark Broadie
Columbia University
Graduate School of Business

First draft: February 21, 1992
This draft: June 29, 1992

Abstract

In this study, we show how a dynamic insurance program can be implemented within a mean-variance framework. The approach combines elements of the single period safety first idea suggested by Telser and developed by Leibowitz with multiperiod insurance strategies like CPPI and TIPP. The insurance program allows the user to set a probability of hitting a specified floor or target and also allows for changing risk attitudes through time. When the insurance strategy is tested on historical data, the insured portfolio achieves high long-term returns while mostly avoiding long bear markets. In order to understand how the insurance strategy might perform in the future, we simulate returns of the stock market and compare the insurance strategy to buy and hold strategies. An additional benefit of the safety first approach is that it specifies a strategy for underfunded portfolios as well as overfunded portfolios.
1. Introduction

In this study, we show how a dynamic insurance program can be implemented within a mean-variance framework. The approach combines elements of the single period safety first idea suggested by Telser (see Elton and Gruber (1991), pp. 216–222) and developed by Leibowitz and Kogelman (1991) with multiperiod insurance strategies like constant proportion portfolio insurance (CPPI) and time invariant portfolio protection (TIPP). A description of CPPI can be found in Black (1987) and Perold (1986). TIPP was proposed by Estep and Kritzman (1988); see also Choie and Seff (1989). The insurance program proposed here allows the user to set a probability of hitting a specified floor and also allows for changing risk attitudes through time.

The dynamic insurance program examined in this study does not restrict the investor to one form of floor. It can be used to insure a fluctuating level of liabilities (like CPPI), a ratcheting floor (like TIPP), as well as a traditional fixed floor for a fixed time period (like a synthetic put). It does not require an investor to have a constant risk attitude through time. It differs from previous portfolio insurance strategies in that it explicitly recognizes that possibility that the portfolio value can violate the floor. In fact, the investor assigns a probability to that event which reflects the investor's goals and risk attitude. The investor can select a convenient rebalancing frequency, e.g., monthly, quarterly, or annually, or the adjustment dates can be chosen to coincide with or to avoid expiration dates of futures contracts. Between adjustment dates, the portfolio is not rebalanced, and so it does not require immediate reaction to fluctuations in the market, or the constant attention of a trader.

The model we propose is based upon the safety first technique devised by Telser and recently revisited by Leibowitz and Kogelman (1991). The safety first portfolio insurance procedure improves on previous portfolio insurance methods by explicitly addressing the probability of dropping below the floor. This is useful information to investors in the post-1987 crash environment, since virtually all portfolio insurance programs violated their floors in October of that year. With safety first portfolio insurance, the investor (or the insurance provider) can specify the probability of hitting the floor, and this probability can be updated using any pertinent information about the expected volatility of the markets. For instance, the implied volatility of the S&P 500 estimated from the futures options markets can be used by the model. The safety first approach is highly flexible: since all inputs may be varied through time, it is possible to change the risk parameters through the course of the investor's life cycle.
For institutions, it may be adapted to the “asset liability” framework in which the investor has preferences about the level of underfunding of liabilities.

2. The Safety First and Target First Criteria

In the safety first approach, an investor specifies a maximum probability of a return less than some floor return. Among all portfolios that satisfy this safety criterion, the method chooses the portfolio with the maximum expected return. To illustrate the safety first method, consider a simple setting which includes two assets: a risky asset and a riskless, or cash, asset. The return of the risky asset is $R_t$ and the return of the riskless asset is $R_c$. Denote the mean returns by $\mu_t$ and $\mu_c$, respectively. Denote the standard deviation of return of the risky asset by $\sigma_t$ and note that the return of the riskless security is zero, i.e., $\sigma_c = 0$. In mean-standard deviation space shown in Figure 1, the risky asset is plotted as point $T$ and the riskless asset is plotted as point $C$. Let $x$ represent the fraction of wealth invested in the risky asset, and $(1 - x)$ the remaining fraction invested in the riskless asset. Then the return of the portfolio is $R_p = xR_t + (1 - x)R_c$. The mean portfolio return is $\mu_p = x\mu_t + (1 - x)\mu_c$ and the standard deviation is $\sigma_p = x\sigma_t$. Each nonnegative $x$ corresponds to a point on the ray emanating from $C$ and passing through $T$. In this simple case, the ray $\overrightarrow{CT}$ is the mean-variance efficient frontier.

Telser’s criterion is to choose a portfolio that maximizes expected return, but has a small probability, say $\alpha$, of a return less than a floor return $F$. So the safety first criterion is to maximize $\mu_p$ subject to $P(R_p \leq F) \leq \alpha$. If returns are normally distributed, this amounts to choosing $R_p$ so that

$$F = \mu_p + z_\alpha \sigma_p,$$

where $z_\alpha$ is the value which satisfies $P(Z \leq z_\alpha) = \alpha$ (and $Z$ is a standard normal random variable).1 As shown in Figure 1, portfolio returns that satisfy equation (1) form a ray emanating from $F$ with a slope of $-z_\alpha$. Typically an investor will choose $\alpha < 0.5$ so that $z_\alpha < 0$, i.e., the slope of Telser’s ray will be positive. If an investor wants to maximize expected return, subject to a constraint on floor return, and be on the mean-variance efficient frontier, then the portfolio must lie on the intersection of the Telser ray and the ray $\overrightarrow{CT}$. The intersection point is denoted by $X$ in Figure 1.

Substituting the expressions for $\mu_p$ and $\sigma_p$ into equation (1) gives

$$F = x\mu_t + (1 - x)\mu_c + z_\alpha x\sigma_t \Rightarrow x = \frac{F - \mu_c}{\mu_t - \mu_c + z_\alpha \sigma_t}.$$  

So $x$ is the fraction of wealth invested in the risky asset which gives a portfolio return identified as $X$ in Figure 1.

---

1 The assumption of normally distributed returns is not crucial to the development of the strategy. As discussed in Elton and Gruber (1991), Tchebyshev’s inequality can be used when weaker assumptions about the distribution of returns is desired.
When the investor’s probability of a return below the floor is very small, i.e., $\alpha \approx 0$ and $z_\alpha \ll 0$. Then Telser’s ray will have a large positive slope and will intersect the mean-variance ray $\overrightarrow{CT}$ close to the vertical axis. The portfolio will be nearly fully invested in the riskless asset. The same result holds if the investor’s floor return is close to the return of the riskless asset. In some cases, e.g., if the floor return is much less the the return of the riskless asset, the proportion in the risky asset may exceed one, i.e., the investor may lever the risky asset.

The same technology can be applied to selecting a portfolio in the absence of a riskless asset. However, the solution is no longer given by the intersection of two rays, but is the intersection of the Telser ray with the mean-variance efficient frontier. If the ray $\overrightarrow{CT}$ is upward sloping (reflecting the positive risk premium for the risky asset) and if $F > \mu_c$, then the ray $\overrightarrow{CT}$ and the Telser ray might not intersect. In other words, if the investor demands too high of a floor return, then no portfolio satisfies the safety first criterion. For example, if a pension fund is underfunded, then it can also happen that no portfolio satisfies the safety first criterion. In this case, the investor might want to choose a portfolio that has a chance of at least $\beta$ of achieving some return. When this happens, we refer to the return under consideration as the target return, rather than the floor return.
The Target First Criterion

The target first criterion is to choose the least risky portfolio that has a probability of at least $\beta$ of achieving a return of $F$ or more. So the target first criterion is to minimize $\sigma_p$ subject to $P(R_p \geq F) \geq \beta$. If returns are normally distributed, this amounts to choosing $R_p$ so that

$$F = \mu_p + z_\beta \sigma_p,$$

where $z_\beta$ is the value which satisfies $P(Z \geq z_\beta) = \beta$ (and $Z$ is a standard normal random variable). As shown in Figure 2, portfolio returns that satisfy equation (3) form a ray emanating from $F$ with a slope of $-z_\beta$. If the probability $\beta$ is chosen too large, then the target first problem can be infeasible, i.e., no portfolio satisfies the requirements. Typically an investor will choose $\beta < 0.5$ so that $z_\beta > 0$ and the target first ray will have a negative slope. If an investor wants to minimize risk, subject to a constraint on floor return, and be on the efficient frontier, then the portfolio must lie on the intersection of the target first ray and the ray $\overrightarrow{CT}$. The intersection point is denoted by $Y$ in Figure 2.

Following a similar reasoning, the optimal fraction $y$ to invest in the risky asset using the target first criterion is

$$y = \frac{F - \mu_c}{\mu_t - \mu_c + z_\beta \sigma_t}.$$

So $y$ is the fraction of wealth invested in the risky asset which gives a portfolio return identified as $Y$ in Figure 2.

![Figure 2. Illustration of the Target First Criterion](image-url)
When the investor desires a floor return greater than the riskless return, the target first approach entails additional risk to achieve this objective. In the next period, the investor can be farther below the floor, and the target first approach can select an even more risky strategy to achieve a floor return. This risk can be reduced by choosing $\beta$ very small, and can be eliminated by choosing $\beta = 0$. In this case, the optimal value of $\gamma$ is zero, i.e., the portfolio is fully invested in the riskless security.

As currently described, the safety first procedure applies to a single period investment problem. The solution becomes time varying if the investor’s floor return $F$, the probability threshold $\alpha$, or the probability threshold $\beta$ change through time. An example of this dynamic strategy is given in the next section.

3. Empirical Investigation of the Safety First Portfolio Insurance Strategy

We tested the safety first portfolio insurance strategy over the period 1926 through 1991, assuming an investor rebalances between Treasury bills and the S&P 500 at monthly intervals. Although it is rare, the portfolio value occasionally falls below the floor, as allowed in the model. When this occurs the target first portfolio strategy is employed. In a fashion similar to Estep and Kritzman (1987), we specify a floor that rises with the value of the portfolio. The floor wealth, denoted $W^f_t$, is defined to be a constant fraction $f$ of the maximum wealth achieved thus far. The floor return at time $t$, denoted $F_t$, is given by

$$F_t = \frac{W^f_t - W_t}{W_t},$$

where $W_t$ is the current wealth level. For example, if $f = 0.9$ and the initial wealth is $W_0 = 1$, then the current floor is $W^f_t = 0.9$ and the floor return is $F_0 = -0.1$. If wealth declines in period 1 to $W_1 = 0.95$, then $W^f_1 = 0.9$ and $F_1 = -0.0555$. If wealth increases in period 1, then $F_1 = -0.1$.

Although the mean and standard deviation of returns of T-bills and the S&P 500 can change over time, we selected the long term 1926–1991 mean and standard deviation of monthly returns (provided by Ibbotson Associates) as a basis for identifying the parameters $\mu_t$, $\mu_c$ and $\sigma_t$. As described above, we chose a floor wealth parameter of $f = 0.9$, allow a 10% chance of hitting the floor each month ($\alpha = 0.1$), and assign a 1 in 1,000 chance of reaching the target from below the floor ($\beta = 0.001$). Even though $f$ is constant, the floor return can vary through time because of the dependence of the floor on the maximum wealth achieved.

Figure 3 shows the path of the all stock portfolio, the all cash portfolio, the safety first portfolio and the ratcheting floor. Notice that the final wealth of the safety first portfolio exceeds the wealth of a portfolio fully invested in the stock market. Over the historical sample period, the (geometric) average annual rate of return of the safety first portfolio was 10.1%, while the stock market’s average annual return was 9.8% and Treasury bills returned 3.6%. In addition, the safety first strategy avoided the dramatic bear markets of the 1930’s and the early 1970’s. This striking result occurs because the safety first portfolio is levered, i.e., the fraction invested in the stock market is greater than one, for much of the sample period. Figure 4 shows the fraction invested in the risky asset over time.
4. Simulation Results

Although the historical performance of the insurance strategy is attractive, we cannot expect it to dominate the all stock portfolio in the future. To better understand the possible
outcomes of the strategy, we simulated the future course of the S&P 500 over ten year hori-
zons and repeatedly applied the safety first procedure. The value of the safety first portfolio
is compared to the value of the all stock portfolio at various time horizons. For the simu-
lation, the annual average return of the risky (i.e., stock) asset was taken to be 13% with a
standard deviation of 20%. The annual average return of the riskless security was set to 5%.
The safety first parameters were chosen as before, $f = 0.9$, $\alpha = 0.1$, and $\beta = 0.001$. In order
to compare the two strategies, a 120 month investment horizon was simulated 10,000 times.
The comparison of the two strategies is summarized by plotting percentiles of the resulting
distributions of wealth. The resulting 1–10–50–90–99 graph is given in Figure 5.

![Figure 5. Simulation of Safety First and All Stock Strategies](image)

The main result of Figure 5 is that the probability of a low return after ten years is much
less with the safety first strategy than the all stock strategy. This result is indicated by the 1%
point of the safety first strategy being much greater than the 1% point of the all stock strategy.
The cost of the safety first strategy is indicated by a lower median (50% point), a lower 90%
point, and a lower 99% point. In short, the cost of lowering the risk of low returns is to reduce
the average return of the portfolio and to reduce the magnitude of the larger returns.

For short time horizons, i.e., under three years, the safety first portfolio has lower down-
side risk (as indicated by the 1% points), higher upside potential (as indicated by the 99% points), but lower median wealth.
5. Choices of Parameters

The results of the historical analysis and the simulated future performance of the strategy are crucially dependent on the choice of the parameters. For overfunded portfolios, the investor is required to identify a floor and a probability of hitting the floor each time period. If the floor wealth and the probability parameter $\alpha$ are constant, the strategy can lead to uninteresting results. After $n$ time periods, the probability of hitting the floor is $1 - (1 - \alpha)^n$. This quantity approaches 1 as $n$ grows large, implying that the floor will eventually be hit. If the floor wealth is not increased over time, the investor will surely be disappointed in this result. It seems that the floor should rise in some fashion to “lock in” equity gains over time.

Because it is a dynamic investment program, the investor may alter his preferences at any stage in the process. For instance, younger investors may be willing to take on greater risk than older investors. Similarly, a pension fund manager can change the floor (or target) as the liabilities of the pension fund change.

6. Conclusions

We illustrated how the single period safety first idea could be implemented as a multiperiod portfolio insurance strategy. A historical evaluation of the strategy for the period 1926–1991 showed that the insured portfolio achieved high returns while mostly avoiding large losses. A simulation of the strategy shows that the safety first strategy changes the distribution of portfolio returns (compared to an all equity portfolio) in a way that may be attractive to some investors. As with other insurance strategies, it limits downside exposure without completely giving up the potential for large positive returns. Unlike other portfolio insurance strategies, the safety first approach explicitly models the possibility of dropping below a specified floor value and it does not rely on continuous trading. In addition, the same methodology provides a strategy for target oriented investors. The target first method explicitly minimizes the risk of an investor whose wealth is below the stated goal, while providing a positive probability of achieving that goal.

7. References


