MORE SOCIAL SECURITY, NOT LESS

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Abstract: This paper explores the feasibility of a government-sponsored insurance company, patterned after the government-sponsored mortgage agencies, that would be authorized to sell government-insured wage-indexed retirement annuities. This enterprise would assume the current obligations and cash flows of the social security system in exchange for the exclusive right to sell additional insurance contracts. It may or may not choose to finance itself through the issuance of equity shares. The empirical analysis in the paper focuses on the stochastic nature of the liabilities faced by such an agency and in particular examines the optimal portfolio of assets required to hedge wage-indexed liabilities.

JEL Classification: G22, H55

Keywords: Social Security, Wage Indexation

1 Thanks to Ibbotson Associates for providing access to their software and data used in this analysis. Please direct correspondence regarding this paper to william.goetzmann@yale.edu. Current versions of the paper can be found on the web at http://viking.som.yale.edu.
I. Introduction

The most striking feature of the current political and academic debate about social security is a lack of innovative thinking. The economic models and forecasts of the long-term viability of the social security system are treated in some ways as a zero-sum game. Either assets are effectively shifted from low-yielding U.S. government bonds to investments with higher expected returns and more risk, or liabilities are reduced by changing benefits, in effect deviating from the original intention of providing a basic economic governmental safety net for retirees. This grim calculus has been put to us repeatedly over the past few years: Something has to give, the system cannot be sustained in the long run. Sacrifices -- particularly by the generation of 40-somethings who have contributed via a professional lifetime of payroll taxes, but will not receive their expected benefits -- will be necessary to save the U.S. Government from future bankruptcy.

In the face of this rhetoric, one might easily argue that the real problem with social security is that it is far too limited in the future benefits it offers to savers, rather than being too expansive. Social security actually provides a unique kind of protection to retirees. It effectively guarantees a standard of living at retirement -- not simply growth in benefits in inflation-adjusted terms. Social Security benefits increase with the average wage level, which in turn is tied to growth in the U.S. economy as a whole. This innovation was intended to insure that retirees are not left behind while the rest of the economy enjoys the benefits of sustained national economic expansion.

One may argue whether this is fair or necessary, but that is not the purpose of this paper. If one accepts that wage indexation of benefits is broadly desirable, then assets
should be managed in such a way as to maximize the ability to achieve this goal – whether in the context of a government agency, or in the context of individual investor accounts. There is another important implication of wage indexation, however. To the extent that it provides a retirement annuity benchmarked to the national standard of living, people may be willing to pay for more of it. Suppose for example, that instead of facing the prospect of losing your currently promised wage-indexed future benefits, you could increase them throughout your lifetime by buying more government insurance? This paper argues that the Social Security Trust Fund could be turned into a viable endowment that would do two things. First, it would manage its assets to cover all currently promised obligations by shifting its portfolio to investments with long-term growth potential. Second, it would issue more wage-indexed insurance contracts at market rates. This would not only generate profits to help cover the costs of the basic social security liabilities, but would provide a realistic means for savers to plan for the future and avoid the risk of failed personal savings plans.

The empirical analysis in the paper focuses on the stochastic nature of the liabilities faced by such an agency and in particular examines the optimal portfolio of assets required to hedge wage-indexed liabilities. The results of the analysis suggest that the optimal investment portfolio for meeting long-term wage-indexed liabilities contains assets that move with inflation, particularly real estate. Even if a government pension agency were not established, these findings suggest a number of things. First, the current investment policy of funding vested liabilities with long-term bonds is not optimal. Second, if private accounts were offered as an alternative to the current system, savers should be given the option of investing in assets such as real estate and commodity funds.
which track the growth of inflation and wages. Third, it would be difficult for a private insurance company to hedge wage-indexed liabilities in the same manner as the government. Our current income tax structure provides a natural means to meet these liabilities in the event of the failure of the financial markets.

II. Hedging the Wage Index

In this section we examine the feasibility of hedging the wage index using currently available investment assets. The approach relies upon the basic tools of modern portfolio theory. We use estimates of the return, risk and correlations of a number of asset classes to form two potential investable portfolios. The first is chosen to minimize the “tracking error” between the fund return and the percentage change in the wage-index liability probability. The second is chosen to maximize the probability of exceeding the wage index though taking higher risk in anticipation of higher return.

II.1 Asset Choice

Our choice of assets is based upon the availability of an investable index and likelihood of this asset class having some correlation to the change in the wage index. Our analysis focuses on stocks, bonds, real estate and commodities. The latter two asset classes are included because of their well-known correlation to inflation. We consider two stock market indices: the S&P 500 index and an index of real estate investment trusts [NAREIT-Equity]. Our bond indices include measures of the total return to long term government bonds (the current asset holdings of the social security trust fund) and returns to a rolling investment in 30 day Treasury Bills. Commodity returns are captured by the
S&P Commodity index and non-public real estate returns are captured by the NCREIF NPI index of commercial real estate returns. This is a broad-based index of unlevered institutionally held properties for which the total return calculation is based upon actual cash flows and professional appraisals. The conservative nature of the appraisal process smooths returns and thus has the effect of reducing the measured volatility of the NPI index.

Table 1 reports the summary statistics for these asset classes as well as some macro-economic variables of interest: U.S. GDP growth and U.S. inflation over the period for which data on the growth in the wage index is available (1952 through 2003). Some series’ began later than 1952, so the statistical measures are not strictly comparable, but are included in the table anyway. The series’ are ranked according to their relative annualized growth rates. Notice that the growth in the wage index is higher than the growth in the CPI and just less than the return to investment in U.S. Treasury bills over the same period. The returns to stocks, bonds, commodities and real estate exceeded the growth in the wage index.

Table 2 reports the correlation among all of the series. Notice that wage growth is correlated to GDP growth, inflation, Treasury Bill returns and real estate returns as measured by the NCREIF index. It has negative correlation to stocks, long-term government bonds and real estate as measured by the NAREIT index. It has modest but positive correlation to growth in commodity prices. These correlations are largely driven by the high period of growth in real asset values and inflation during the 1970’s and 1980’s. Figure 1 shows the result of a regression of wage growth on TBill returns. The observations are numbered by year and boxes on each observation indicate relative
proximity in time. Although 1972 was somewhat of an outlier -- a year when T-bill returns lagged far behind wage growth – there is a consistent positive correlation between the variables throughout the period. If anything, this picture suggests that the Social Security Trust Fund should be holding short-term instruments rather than long-term government bonds. Bond values are actually likely to drop when wages are increasing causing the difference between the value of fund assets and fund liabilities to increase.

II.2 Constructing a Hedge

To construct a portfolio that hedges wage growth we consider a problem in which we have assets equal in present value to liabilities – that is, our investment portfolio is neither over-funded or under-funded. Our goal in this stage of analysis is to find what mixture of asset holdings on a dollar-for-dollar basis would provide the lowest “tracking error.” We define the wage variable as the liability and construct an efficient frontier in terms of surplus returns in excess of the liability. In effect, we require the portfolio to hold one dollar of liabilities characterized by the mean, variance and covariance of the wage index, and we construct a set of undominated investment portfolios from the remaining set of assets. The minimum-variance portfolio is that portfolio which minimizes the tracking error with respect to changes in the wage index.

Since the volatility of the NCREIF index is almost certainly underestimated, we increase its annual standard deviation to 12%. Also, because the expected returns to property investment via REITs is unlikely to be higher than the return to S&P 500 stocks, we reduce the expected return of the NAREIT index to 10% per year. Finally, we use the average long-term rate of return to bonds as opposed to current bond yields. This is
conservative to the extent that it may over-estimate the attractiveness of bonds as investments. For all other asset classes we retain their historical statistical values.

Figure 2 shows the efficient frontier. Note that since we prohibit short-sales for the purposes of this analysis, the frontier has a slightly flattened shape. T-Bills are near the minimum-variance point, but they are not precisely on it. A portfolio of 90% T-Bills, 4% S&P 500 stocks, 2% long-term government bonds, 3% commodities and 1% equity REITS is actually the optimal hedge portfolio from the perspective of minimizing the tracking error with respect to wage index growth – it minimizes the variance of the expected difference between fund liabilities and assets. In this framework it provides an expected surplus of .8% (80 basis points) per year, with an annual standard deviation of 3.5%. This point is not necessarily the optimal choice. It does not, for example, maximize the probability of achieving a positive surplus. That point on the frontier can be found by finding the portfolio with the highest ratio of expected surplus to standard deviation. It is represented by holdings of 23% S&P, 19% long term government bonds, 30% NCREIF, 13% commodities and 14% REITs. It has an expected surplus of 5.2% per year and an annual standard deviation of 7.9%. Under the assumption that returns are normally distributed, this results in an annual probability of the surplus portfolio achieving a positive return of 74%. Thus, at least under the basic assumptions of this analysis, we find that a fairly well-diversified portfolio of assets, including stocks, bonds, real estate and commodities provided the best potential for the portfolio return to exceed growth in the wage index. Figure 3 shows how portfolio weights vary across the entire range of the efficient frontier.
Why not simply invest in the highest returning asset, given the long-term nature of the liabilities? After all, shouldn’t stocks provide the best chance of long-term growth? The reason why an all stock portfolio is not optimal as a hedge against the liabilities is explained by the negative correlation between stocks and wage growth. Historically stock returns have been low when wage growth has been high, even though on average stock returns are high, there is a chance that such a scenario will repeat itself in the future and equity returns will fail to keep pace with wages. At the time when equity assets are most needed to meet obligations they may be worth less.

In sum, our analysis using the tools of modern portfolio theory demonstrates a few basic results. First, long term government bonds are not the ideal hedge against growth in the wage index. Second, while a portfolio mostly comprised of Treasury Bills is the best proxy for the wage index variable in that it is most highly correlated to it, it is not an ideal choice if the investment goal is to maximize the probability that assets will exceed liabilities in the long run. Third, investment options such as real estate and commodities may be important components of an optimal hedging portfolio designed to maximize the probability of exceeding wage growth. Yet these alternative asset classes have not been seriously discussed as options for individual investor accounts. Should we be asking investors to try and exceed liability growth without giving them the full tool-kit to do so?

II.3 Known Limitations

Modern portfolio theory has some important limitations that temper the interpretation of our analysis. Most important is the problem that the risk of many of
the variables may be improperly estimated. A standard assumption is that the returns of each asset class are independently distributed through time. They may be correlated to each other contemporaneously, but they are assumed to follow a random walk with respect to past returns. If this assumption is violated, then the standard deviation as measured from historical returns is a poor representation of the risks over a multi-year horizon. The intuition is that when a highly autocorrelated variable begins to decrease in value, it has a downward momentum that will cause it to have lower values on average for a period of time, and vice-versa. So, for example, if wages started on an upwards trajectory, they might continue that trend for a while. This tendency will result in wider distributions of outcomes at multi-year investment horizons.

One troublesome implication of this problem is that wage growth, which is highly autocorrelated, may in fact be riskier than historical statistics would suggest – in other words the current risk of future social security payouts compared to social security trust fund assets is higher than current economic projections if they have been made in the absence of a time-series model. One approach to this limitation is to construct an econometric model that includes inter-temporal dependency. This would allow a simulation of a variety of inter-temporal investment strategies, however it would no longer provide an “optimal” strategy in the sense we use the term here. Such a simulation should be used in any serious analysis of social security portfolio choice as a way to check the recommendations provided by modern portfolio theory models. If and when the social security liability becomes a national investment problem, economists should provide guidance about the implications of investor choice. Individual account holders will need to know the true risks they face.
III. Offering a Wage-Indexed Pension Annuity

The analysis to this point ignores the issue of whether the portfolio of assets designed to meet the wage index liability belongs to the government. It may instead belong to individual savers in private accounts, or to a private insurance company seeking the most efficient means to hedge itself against promised future payments. In this section we argue that a government-owned or sponsored corporation might in fact be ideal for this purpose.

Given the level of financial innovation in the United States over the past few decades, it is perhaps somewhat surprising that a wage-indexed retirement annuity is not currently offered by the private sector. Although the modern insurance market offers an astounding array of annuities for savings, no firm currently offers a contract tied to the wage-index – but why not? After all, one of the primary reasons to save for retirement is provide a future income that allows a decent quality of life in comparison to the rest of society. If the government decides to eliminate the wage indexation it would be an ideal marketing opportunity for the private insurance sector to step in and offer contracts to make up the difference between revised expectations and what the government once guaranteed. If the private sector provided these contracts it would certainly be enhancing public welfare. In the parlance of modern finance, they would be “completing” the market. Indeed, even if the wage indexation remained unchanged there would likely be a demand for such contracts.

II.1 A Government-Sponsored Pension Agency
It is becoming fashionable in the financial press in recent years to criticize the national mortgage agencies for taking a free ride on a governmental guarantee, or more properly on the right to borrow from the government at a preferred rate. The intellectual foundation for this critique is that the special relationship enjoyed by GNMA, Fannie Mae and Freddie Mac in effect represents unfair competition – the private sector is competitively disadvantaged. The argument continues that the private sector can offer everything the government agencies can offer.

A natural question to ask in response to this question is “Would it?” That is, would the private sector have created this extraordinary market for mortgages, had it been left the opportunity to do so? Looking around at the comparative lack of such a securitized market in most other countries in the world, it is doubtful that such an equilibrium would have been achieved without a governmental decision to do so. Granted, the agencies have now demonstrated the financial feasibility – indeed the lucrativeness – of meeting the broad national demand for guaranteed mortgages and for mortgage-backed securities, but neither of these features characterized the U.S. money markets before the creation of the mortgage agencies. Our capacity to finance the individual purchases of homes in this country and to offer mortgage-backed debt is the envy of the world. By any measure, the creation of quasi-public agencies to address the latent demand for housing finance was an overwhelming success, and has positively affected a large number of the individual households in the United States. If anything, Americans are “over-housed” in that a mortgage-financed home is typically the single largest investment in the household portfolio, and home-ownership in this country is at its highest level ever in absolute and in percentage terms. The same cannot be said for any
other country in the world – developed or undeveloped. Would this great leap in financial architecture have occurred is we had left housing finance entirely to local savings and loans and asset-backed securitization to the investment banks? Unlikely.

Another recent critique of the mortgage agencies is that they concentrate mortgage risk, and raise the potential of a systemic financial failure. In other words, they have loaned too much money to American home buyers. If interest rates increase, the value of these mortgages will drop, potentially weakening the firms. The alternative is to disperse these mortgages widely in the financial markets, so that the effects of any shock to the system is diffused. Rather than sanctioning their current use of derivative contracts to hedge interest rate risk, the argument goes, they should be limited in their scope. In short, they should not be allowed to lend so much money for homes, but should step aside and allow the private sector to issue and hold mortgages. This constraint would likely have the effect of raising mortgage rates. Raising costs to homeowners in order to reduce systemic risk might be a reasonable decision, but it is not costless.

Would a government pension agency share the same problem of concentrating financial risk in one major entity? Is it better to break up the issuance of annuities across a spectrum of private issuers? The argument in favor of putting current pension liabilities into an agency is that this makes it easier to set an investment policy geared to meeting them, rather than one geared to maximizing shareholder value in a private enterprise. The concentration of these liabilities makes it easier to fix problems of payment if they arise, rather than shifting this risk to householders. A shock to savings assets (or monetary values) that broadly threatens retirement pensions will be systemic – regardless of the financial architecture.
Given the success of the government-sponsored mortgage agencies, it would seem natural to use them as a model for offering another financial product that has the potential to improve the economic life of every citizen of the United States. The financial architecture of such a plan is straightforward. Create a company with a separate budget and initially, complete governmental ownership. The company (let’s call it GSPA) will assume all the current obligations of the social security system, and all the current assets of the social security fund (i.e. the U.S. government bonds it holds), and future payroll contributions. GSPA will have the immediate goal of using these assets to meet all specified obligations due to current beneficiaries. It will do this by re-balancing its portfolio of assets – perhaps in the manner explored in this paper. The current governmental guarantee of future social security payments will flow through to beneficiaries. This is not a plan that allows the U.S. Government to “off-load” its promises.

The one significant additional benefit that GSPA should be given by the government is the exclusive right to sell additional wage-indexed retirement annuities with a governmental promise. These would be offered at market rates that provide a profit to the agency. This profit may be sufficient to defer part of the costs of social security benefits currently offered. It is also this feature which might allow the agency to become a public company along the lines of the current mortgage agencies. If the market for “extra Social Security” develops into a profitable enterprise, it might attract private equity capital that would allow for further development. It might also enhance public scrutiny and regulatory oversight if the agency were subject to the legal framework
governing public corporations. Such future privatization is not necessary, however, for a
government sponsored pension agency to work.

While private companies could also offer annuity contracts, there is good reason
to believe that government annuities would be much more attractive than annuities
offered by private firms. The U.S. Government has a life expectancy beyond the life of
an individual. One of the gravest concerns of a long-term insurance purchaser is default
risk. A U.S. Government promise of payment is worth more than the same promise from
a corporation because the government is a stable one, and in addition it has the power of
taxation. Although no-one wants to raise taxes to meet pension obligations, it can be
done in the future in the event that asset growth does not match future liability growth.
Indeed, in the case of wage-indexed retirement annuities, the possibility of directly taxing
future wages exists, offering the potential for a close match between the fluctuations in
liabilities and the value of a future tax.

III.2 Alternate Structures

If the proposal for a government sponsored pension agency proved to be
politically infeasible, then there are alternative structures that might be used to achieve
the two goals proposed in this paper – namely, (1) maximizing the probability of meeting
wage-indexed retirement payouts and (2) meeting the demand for more wage indexed
pension annuities. The first goal may be partly addressed by adjusting the asset
allocation of the current Social Security Trust Fund, and/or offering individual account
owners the option of investing in a portfolio that is engineered to maximize the
probability of meeting the liabilities. Neither of these are included in the current plans discussed by the major parties but they should be.

The second goal can theoretically be addressed by the private insurance sector, however it needs some catalyst to set the process in motion. This catalyst might be the offer of a government guarantee in return for government oversight of risk-based capital requirements, or the creation of a government corporation whose equity shares can eventually be purchased by major insurance providers – effectively privatizing the entity rather than establishing a government sponsored competitor. As long as we focus on the goals of creating an endowment designed to match future liabilities, and a marketplace for additional wage indexed retirement annuities, the final structure can be left to the interplay of politics and the marketplace.

IV. Conclusion

As politicians debate what amounts to a default on future pension promises to U.S. citizens – disguised as a restructuring or reconfiguration of the social security system, it is important to think more broadly about solutions that would allow the U.S. government to keep its promises. Governments have faced fiscal crises many times in the past. Restructuring has always been an option, but it has adverse reputational consequences both to investors and to citizens. The low interest rates enjoyed by the U.S. government today is a direct consequence of its nearly perfect record of paying its bills. Although one may argue that the reduction of social security benefits does not represent a default, such semantics are lost on a large sector of the population who have planned their future economic lives around the expectation of a guaranteed, indexed minimum income
conditional upon their past contributions. Rather than risk a permanent shift in public trust, it is advisable to consider some creative alternatives. The model of the extraordinarily successful government sponsored mortgage agencies is one that could work.

As all Americans begin to face the prospect of mastering modern portfolio theory in order to manage their individual retirement accounts, it is worth considering that some would prefer to delegate the responsibility of understanding means, variances, correlations, autocorrelations, input uncertainty, efficient frontiers and probabilities to an agency that can deliver what they are expecting from their investment: a future income that maintains a reasonable standard of living. Wage-indexed pension annuities might just be an attractive alternative to the complexities of personal portfolio management.
Table 1: Summary Statistics of the Series’

All data based upon annual percentage changes. They are taken from Ibbotson Associates EnCorr program, except the wage growth which is a U.S. Government statistic. All mean and standard deviation values are in annual percentage terms.

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Years of Data</th>
<th>Geometric Mean</th>
<th>Arithmetic Mean</th>
<th>Standard Deviation</th>
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<tbody>
<tr>
<td>NAREIT-Equity TR</td>
<td>32</td>
<td>12.9</td>
<td>14.2</td>
<td>17</td>
</tr>
<tr>
<td>S&amp;P 500 TR</td>
<td>52</td>
<td>11.5</td>
<td>12.9</td>
<td>17.6</td>
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<tr>
<td>NCREIF Property TR</td>
<td>26</td>
<td>9.3</td>
<td>9.5</td>
<td>6.2</td>
</tr>
<tr>
<td>S&amp;P Commodity TR</td>
<td>33</td>
<td>7.1</td>
<td>9.5</td>
<td>25.1</td>
</tr>
<tr>
<td>IMF U.S. GDP</td>
<td>52</td>
<td>7</td>
<td>7</td>
<td>2.8</td>
</tr>
<tr>
<td>U.S. LT Gvt TR</td>
<td>52</td>
<td>6.3</td>
<td>6.8</td>
<td>10.8</td>
</tr>
<tr>
<td>U.S. 30 Day TBill TR</td>
<td>52</td>
<td>5.1</td>
<td>5.2</td>
<td>2.9</td>
</tr>
<tr>
<td>Wage Index Growth</td>
<td>52</td>
<td>4.9</td>
<td>4.9</td>
<td>2.2</td>
</tr>
<tr>
<td>U.S. Inflation</td>
<td>52</td>
<td>3.8</td>
<td>3.8</td>
<td>3.1</td>
</tr>
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</table>
Table 2: Correlation Among Series’

This correlation matrix is constructed using annual percentage changes in each of the variables. The correlation is measured over the maximum period of pair-wise overlap in each series. All data are from Ibbotson Associates EnCorr program, except the wage growth which is a U.S. Government statistic.

<table>
<thead>
<tr>
<th>Series</th>
<th>Wage Gr.</th>
<th>GDP</th>
<th>S&amp;P 500</th>
<th>LTG</th>
<th>TBill</th>
<th>INFL</th>
<th>NAREIT</th>
<th>NCREIF</th>
<th>Commodity</th>
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<td>Wage Gr.</td>
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<td>-0.13</td>
<td>-0.17</td>
<td>0.58</td>
<td>0.63</td>
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<td>0.63</td>
<td>0.12</td>
</tr>
<tr>
<td>U.S. GDP</td>
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<td>-0.17</td>
<td>-0.22</td>
<td>0.37</td>
<td>0.61</td>
<td>0.05</td>
<td>0.55</td>
<td>0.25</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td>-0.13</td>
<td>-0.17</td>
<td>1</td>
<td>0.12</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.46</td>
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<td>-0.25</td>
</tr>
<tr>
<td>LT Govt</td>
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<td>-0.22</td>
<td>0.12</td>
<td>1</td>
<td>0.23</td>
<td>-0.14</td>
<td>0.21</td>
<td>-0.23</td>
<td>-0.24</td>
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<tr>
<td>30 Day TBill</td>
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<td>0.37</td>
<td>-0.02</td>
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<td>1</td>
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<td>0.61</td>
<td>-0.02</td>
<td>-0.14</td>
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<td>NAREIT</td>
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<td>-0.13</td>
<td>1</td>
<td>0.04</td>
<td>-0.07</td>
</tr>
<tr>
<td>NCREIF</td>
<td>0.63</td>
<td>0.55</td>
<td>0.12</td>
<td>-0.23</td>
<td>0.52</td>
<td>0.51</td>
<td>0.04</td>
<td>1</td>
<td>0.15</td>
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<tr>
<td>Commodity</td>
<td>0.12</td>
<td>0.25</td>
<td>-0.25</td>
<td>-0.24</td>
<td>-0.07</td>
<td>0.3</td>
<td>-0.07</td>
<td>0.15</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 1: Regression of Annual Wage Growth (in percentage terms) on annual return to U.S. 30 Day Treasury Bills. The estimated model estimates are reported below the X axis. The coefficient estimate is statistically significant at greater than the 1% level. Observations are indicated by boxes. Larger boxes are more proximate in time. Date labels indicate the month and year of the observation.
The frontier is constructed using the Ibbotson Associates EnCorr Optimizer. Inputs are taken from historical values except for the two real estate series. The NCREIF volatility is set to 12%, while the NAREIT return is set to 10%. Weights are constructed using non-negativity constraints. The optimization is performed using the assumption of a liability stream matching the U.S. wage growth variable. The present value of assets and liabilities are set equal to match the assumption that liabilities are fully funded.
Figure 3: Portfolio Weights

This figure represents the change in the composition of portfolios along the efficient frontier. The X scale corresponds to increasing risk along the frontier divided into 100 equal increments. Cross-hatching indicates Treasury Bills, Gray (along the top) represents NAREIT, Black indicates S&P 500, Light Gray indicates Long Term Government Bonds, Diagonal Lines represent NCREIF and Horizontal lines represent the S&P Commodity index.