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MACROECONOMIC FACTORS AND THE CORRELATION OF STOCK AND BOND RETURNS

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Abstract

This paper examines the correlation between stock and bond returns. It first documents that the major trends in stock-bond correlation for G7 countries follow a similar reverting pattern in the past forty years. Next, an asset pricing model is employed to show that the correlation of stock and bond returns can be explained by their common exposure to macroeconomic factors. The link between the stock-bond correlation and macroeconomic factors is examined using three successively more realistic formulations of asset return dynamics. Empirical results indicate that the major trends in stock-bond correlation are determined primarily by uncertainty about expected inflation. Unexpected inflation and the real interest rate are significant to a lesser degree. Forecasting this stock-bond correlation using macroeconomic factors also helps improve investors' asset allocation decisions. One implication of this link between trends in stock-bond correlation and inflation risk is the Murphy's Law of Diversification: *diversification opportunities are least available when they are most needed.*

Keywords: Stock-Bond Correlation, Asset Allocation, Macroeconomic Factors
JEL Codes: G12, G15, E44

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Abstract

This paper examines the correlation between stock and bond returns. It first documents that the major trends in stock-bond correlation for G7 countries follow a similar reverting pattern in the past forty years. Next, an asset pricing model is employed to show that the correlation of stock and bond returns can be explained by their common exposure to macroeconomic factors. The link between the stock-bond correlation and macroeconomic factors is examined using three successively more realistic formulations of asset return dynamics. Empirical results indicate that the major trends in stock-bond correlation are determined primarily by uncertainty about expected inflation. Unexpected inflation and the real interest rate are significant to a lesser degree. Forecasting this stock-bond correlation using macroeconomic factors also helps improve investors' asset allocation decisions. One implication of this link between trends in stock-bond correlation and inflation risk is the Murphy's Law of Diversification: *diversification opportunities are least available when they are most needed.*

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Introduction

This paper studies the comovement between stock returns and long-term government bond returns, and attempts to explain the economic driving forces behind this relationship. The correlation of stock and bond returns plays a pivotal role in investors' diversification and asset allocation decisions. How do stock and bond returns co-move? Given that stocks, long-term government bonds and other high grade long-term fixed income products account for a dominant share in all traded financial assets, one might think that economists would have already answered this fundamental question. However, despite its ultimate importance, the nature of this correlation remains elusive.[1]

Many academics have tried to understand the comovement of stock and bond returns. Using the dynamic present value model, Shiller and Beltratti (1992) study annual data of the U.S. and the U.K. They conclude that the observed stock-bond correlations are too high to be justified by theory. Campbell and Ammer (1993) use the same framework to decompose the variances and covariance of monthly stock and bond returns in the post-war U.S. Both studies implicitly assume that the stock-bond correlation is time invariant. However, from a theoretical perspective, Barsky (1989) argues that the stock and bond comovement is state dependent. In particular, he points out that low productivity growth and high market risk are likely to lower both corporate profits and the real interest rate, which propels stock and bond prices in opposite directions.

The most recent studies have moved in the direction of recognizing and investigating time-varying comovement between stock and bond returns. Scruggs and Glabadanidis (2001) strongly reject models which impose a constant correlation restriction on the covariance matrix between stock and bond returns. Fleming, Kirby and Ostdiek (1998) find a strong volatility linkage across stock-bond-bill markets, and attribute it to the information flow in these markets. However, they associate the information flow with volatility and do not identify the exact information that causes the comovement. David and Veronesi (2001) show that the uncertainty about inflation and firm earnings

[1] In the first version of *The Intelligent Investor*, published in the 1950s, the author, then investment guru Benjamin Graham, claims that the correlation between stock and bond returns is negative. His argument provides the basis for the asset allocation advice of 50-50 split in stocks and bonds. However, in the second version of this book published in the 1970s, the correlation structure has changed and the argument is dropped. Today, one can randomly search the term "stock and bond correlation" on the internet, and easily find sharply contradictory opinions among market participants. When it comes to story-telling, one man's story is just as good as others. Most of these opinions are based on causal observations and lack the support of concrete evidence.

explains some of the changes in the variances and covariance of stock and bond returns. Stivers and Sun (2002) use regime-switching models to study the short-run dynamics of the stock-bond comovement. In particular, they investigate the "flight to quality" issue by examining the effect of stock market volatility on bond returns. Using a similar methodology, Gulko (2002) finds dramatic changes in the comovement patterns of stock and bond returns around market crashes.

A related line of literature looks at high frequency data and examines how news announcements affect short-run stock and/or bond price movements. Examples include Cutler, Poterba and Summers (1989), Fleming and Remolona (1999), Balduzzi, Elton and Green (2001), and Fair (2001). Although they provide important insights into the price adjustment and formation process, they do not seem to explain the long-run comovement.

The primary contribution of this paper is that it tests the link between macroeconomic factors and the stock-bond correlation. We also expand the usual scope of this literature from U.S. (and occasionally U.K.) markets to all G7 markets, which enhances the robustness of the conclusions. Moreover, we evaluate the economic significance of this link from the perspective of a risk averse investor. This paper focuses on the stock-bond correlation at monthly frequency.

Our main findings are, first, that uncertainty about long-term expected inflation plays an important role in determining the major trends of stock-bond correlations. Greater concerns for future inflation are likely to result in stronger comovement between stock and bond returns. Secondly, we demonstrate that the uncertainty about other macroeconomic factors, such as the real interest rate and unexpected inflation, also affects the comovement of stock-bond returns, but to a lesser degree. Finally, we show that forecasting stock-bond correlations based on macroeconomic factors helps improve investors' asset allocation decisions.

The first three sections of this paper lay out the foundations for our empirical analysis. Section 1 describes the data of stock and bond returns, and documents some stylized facts about G7 stock-bond correlations during the last forty years. We observe a sharply reverting trend and a seemingly convergent trend in G7 stock-bond correlations. The potential concern for the conditioning bias of correlation is also addressed. Section 2 develops a simple model in which stock and bond returns can be derived endogenously under a unified framework. It provides the theoretical guide to help distill the cause of their comovement and select the candidates for empirical analysis. This model suggests that the comovement between stock and bond returns is induced by their common exposure to macroeconomic factors: expected inflation, the real interest rate, and unexpected inflation. Section 3 constructs the measures for the uncertainty about macroeconomic factors. We construct two measures for the uncertainty about expected inflation: a short-term measure derived using the generalized Phillips Curve, and a long-term measure derived from the term structure. We also construct two measures for the uncertainty about the real interest rate: a short-term measure derived as its conditional volatility, and a long-term measure which takes into account the concerns for regime shifts.

Section 4 contains the main analysis of this paper. We examine the link between macroeconomic factors and the comovement of stock and bond returns using three formulations, each in succession allowing for greater flexibility in modeling the dynamics of stock and bond returns, and taking us one step further into the cause of their comovement. The first formulation uses a linear regression model to link the unconditional stock-bond correlation with the uncertainty about macroeconomic factors. The second formulation specifically models the autocorrelations in the mean and volatility of stock returns. The third formulation recognizes autocorrelations in both stock and bond returns, and jointly models them using a vector autoregression model with conditional heteroscedastic volatility. The empirical results are consistent with the predictions of the theoretical model. Although all three factors are able to explain the stock-bond correlation to a certain extent, the uncertainty about long-term expected inflation dominates the other factors.[2] The effect of the uncertainty about the real interest rate and unexpected inflation is more visible when expected inflation is partially removed from stock and bond returns.

Section 5 evaluates whether accounting for time-varying stock-bond correlations helps investors improve their asset allocation decisions. We form a short-term dynamic strategy which allows investors to rebalance their portfolios using the forecast of stock-bond correlations based on macroeconomic factors. We show that, in the U.S., this dynamic strategy commands an annual premium of about 0.5% over a moving average strategy.

Finally, Section 6 concludes this paper.

[2]This is similar to the conclusions reached by David and Veronesi (2001) about the variances and the covariance of stock and bond returns. The link between the stock-bond correlation and inflation is also explored in a recent article by Ilmanen (2002).

1 Data Description and Stylized Facts

In this section, we describe the data of stock and bond returns, and document some stylized facts about stock-bond correlations in G7 countries (the U.S., the U.K., France, Germany, Japan, Canada and Italy).

We analyze the data from 1958 to 2001. Choosing 1958 as the starting point reflects two considerations. First, hyperinflation in some of the war torn countries did not subside until the early 1950s. During the 1950s, financial markets resumed normalcy, and financial data started to reflect free market movements rather than state control. Second, the U.S. Treasury-Federal Reserve Accord of 1951 formally relieved the Federal Reserve of the obligation to support the U.S. government bonds and allowed it to pursue independent monetary policy. Although Germany issued the first post-war bonds in 1948, it was only after the creation of the Bundesbank in 1957 that government bonds regained the safe-haven status for investors. In Japan, the trading of bonds resumed in 1956.

1.1 Data of Stock and Bond Returns

We use stock and bond indices at two frequencies: monthly (1958-2001) and daily (1980 or later -2001). Stock indices are value-weighted broad market indices, and bond indices are long-term (usually 10-year) benchmark government bond indices. Early monthly stock indices (1958-1969) are from Global Financial Data [GFD] and later data series are from MSCI.[3] Daily MSCI stock capital appreciation [CA] returns start from January 1980. Daily stock total returns [TR] are interpolated using daily CA returns and monthly dividend income.[4]

Early monthly bond indices (1958-1980 or later) are also from GFD and later data series are from Datastream.[5] Daily TR indices of 10-year treasury bonds of the U.S., the U.K. and Germany are available from January 1980. The bond series of France, Japan, Canada and Italy start from 1985, 1984, 1985 and 1991 respectively. We replace

[3]To insure the quality and consistency of the data from different sources, we compare GFD data with MSCI data during their common period. GFD and MSCI stock returns have correlations of about 0.95 for all countries except France. Two sources of French data series have a correlation of only 0.6 in the 1970s, which rises to 0.98 later.

[4]The purpose of the interpolation is to keep the analysis consistent since we always use total returns of bonds. It does not affect the stock-bond correlation since the monthly dividend income is equally allocated to each trading day within a month.

[5]Comparisons of GFD bond return data with Datastream and IFS-IMF (1960-) data also indicate high similarities.

monthly GFD bond series with Datastream series whenever the latter are available.

Table 1 shows the sample statistics of stock and bond returns at both frequencies. This table also includes the sample statistics of inflation rates. In the last 40 years, the mean returns of stocks vary from 11.74% for Canada to 14.78% for the U.K. For monthly data, the stock-bond correlations are generally within the range of 0.2 to 0.3, except for Japan and Italy. For daily data, most correlations also fall in the range of 0.2 and 0.3 except for Japan and Canada. Since the monthly data cover a longer period than the daily data, we also compute monthly stock-bond correlations for the same time period as covered by the daily data. Just by looking at the correlation coefficients, one may conclude that the 0.2 to 0.3 range seems to describe well the comovement of stock and bond returns. Nevertheless, we show below that this observation masks some dramatic changes in stock-bond correlations over the last forty years. The following subsections examine different measures of correlation and the potential problems of using correlations to measure comovement. Table 2 shows the autocorrelation of stock and bond returns. Autocorrelations are generally low except for monthly bond returns. For daily returns, autocorrelations disappear very quickly as the number of lags increases. This table, combined with the fact that we are using value-weighted broad market stock indices and liquid benchmark bond indices, should eliminate any concern for non-synchronous trading problems of daily data.

1.2 Stylized Facts about Stock-Bond Correlations

Figure 1 shows the 60-month rolling correlation of U.S. stock and bond returns, along with the 90% error bands.[6] The dates on the X-axis are the ending dates of each rolling window. We can see that U.S. stock-bond correlation is much higher between the 1970s and the early 1990s than in the beginning and the end of the sample period. The upper panel of Figure 2 shows the 60-month stock-bond rolling correlations of G6 countries (G7 excluding Italy). Observations of Italy and of October 1987 are removed to keep the picture clean and readable. Note that these data are not excluded in the empirical analysis part of this paper.

These pictures show that there are two trends in the rolling stock-bond correlations. First, we can see a persistent upward trend in stock-bond correlations across all countries until the mid 1990s. The highest correlation reaches a peak of 0.7 in the

[6]Error bands are computed using the asymptotic distribution of correlation coefficient. See Anderson (1984) for details.

mid-1990s in the U.K. During this period, the average correlation is as high as 0.5. Recently, these correlations have decreased dramatically to around 0. Particularly, there is a sudden drop in Japan in the early 1990s, pushing it into negative territory.[7] We call this the *reverting trend*. Second, there also seems to be a *converging trend* since the correlation curves are moving closer to each other, with the exception of Japan. These two trends are further highlighted in the lower panel of Figure 2. The average stock-bond correlation rises steadily from 0.1 in 1963 to 0.5 in 1994, then declines sharply to 0.05 in 2001. Excluding Japan and Italy, the dispersion of these curves, measured as the cross sectional standard deviation, has been decreasing steadily. If the rolling correlation accurately reflects how stock and bond returns co-move over time, then the idea of diversifying between stocks and bonds has very different implications in the 1960s than in the 1990s. Ignoring the change in stock-bond correlations, a well positioned portfolio for one period can be ill-fated for another.

The availability of recent daily data, from 1980 to 2001, allows us to compute stock-bond correlations within each month using non-overlapping data. Figure 3 shows the non-overlapping stock-bond correlations of all G7 countries. For better visualization, the Hodrick-Prescott filter is applied to remove the high frequency variations.[8] This picture confirms what is illustrated in the latter parts of Figures 1 and 2. We can see that the stock-bond correlations of the U.S., the U.K., France and Germany cluster together. Canada is significantly lower than this group for most of this period. All G7 countries have witnessed significant reduction in their stock-bond correlations since the mid-1990s. In particular, Japan deviates from the other countries in the early 1990s and remains in the negative region throughout the entire period.

1.3 Conditioning Bias

The recent literature on the *conditioning bias* (Boyer, Gibson and Loretan 1997, Longin and Solnik 2001, Forbes and Rigobon 2002) raises concerns about using correlation to measure comovement. These authors point out that the correlation coefficient computed for a period with extreme volatility can be seriously biased upwards relative to the full sample correlation coefficient (presumably the true measure of comovement).

[7] Japan's prolonged bear market started in the early 1990s. From July 1992 to June 1993, the monthly stock-bond correlation in Japan is negative in 10 out of 12 months. Monthly stock and bond returns sharply switch signs.

[8] We apply the conventional smoothness parameter of Hodrick-Prescott filter for monthly data, 14400.

In other words, time-varying correlations may simply be an artifact of fluctuations in volatility in certain instances, rather than reflecting any genuine changes in the comovement patterns. Nevertheless, our results are not seriously affected by this criticism for the following reasons. First, as shown by Corsetti, Pericoli and Sbracia (2001), conditioning bias is overestimated due to the restrictive settings in the aforementioned papers. In a more general factor model, this concern can be greatly alleviated. Second, conditioning bias is a serious problem if one is particularly interested in the comovement patterns under "extreme" market conditions, which, by definition, are rare. However, the purpose of this paper is to account for the changes of stock-bond correlation over a long period, most of which is considered "normal". Third, a term, the stock unique component, is included in our analysis of unconditional correlation. It serves as an adjustment term for the fluctuations in stock return volatility, similar to those used by Loretan and English (2000) and Forbes and Rigobon (2002). Moreover, the conclusions reached in this paper do not depend exclusively on the results of unconditional correlation, which is subject to the criticism of conditioning bias.

2 Driving Forces of Stock-Bond Correlations: Theoretical Background

Stock and bond prices are the discounted sums of their future cash flows. Assuming there is no default risk, a stock's cash flow is an infinite stream of uncertain dividends, while a bond's cash flow is a fixed number of payments of pre-determined coupon income. Evidently, factors that exclusively affect the discount rates are likely to move stocks and bonds in the same direction, while those affecting only stock dividends will reduce their comovement. The influential empirical studies by Chen, Roll and Ross (1986) and Fama and French (1993) provide the guide to identify the pricing factors of stocks and bonds. Building on their insights, this section presents a simple model in which stock and bond returns can be endogenously derived. This model provides a unified framework to analyze consistently the economic driving forces behind the stock-bond correlation.

Jointly pricing stocks and bonds is a non-trivial task.[9] Two recent papers have made

[9]The is primarily because any realistic assumption about dividend processes easily renders closed-form solutions unattainable. Campbell (1986) and Abel (1988) derive equilibrium models to price stocks and bonds under very restrictive distributional assumptions. Bakshi and Chen (1997) achieve

progress in this field. Bekaert and Grenadier (2001) derive an affine pricing model which has clear economic interpretations. However, a closed-form solution for stock price can only be derived in some special cases. Mamaysky (2002) provides an innovative way of tackling stock price by directly modeling the dividend yield process, which allows one to derive conveniently an analytical solution for stock returns. However, the pricing factors in his model are unobserved latent factors. We show that, by combining the contributions of these two papers, one can derive the affine representations for stock and bond returns based on observable economic factors. Our setting is also similar to Brennan and Xia (2002), except that the stock return process in their model is exogenously assumed.

2.1 An Affine Model of Stock and Bond Returns

As a simple case of Bekaert and Grenadier (2001), we assume that the real interest rate r_{t+1} and inflation rate π_{t+1} both follow affine mean reverting processes.

$$r_{t+1} = \overline{r} + \rho_r \left(r_t - \overline{r}\right) + \sigma_r \varepsilon_{t+1}^r \tag{1}$$

$$\begin{aligned} \pi_{t+1} &= \overline{\pi} + \rho_\pi \left(\pi_t - \overline{\pi}\right) + \sigma_\pi \varepsilon_{t+1}^\pi \\ &= \widehat{\pi}_t + \sigma_\pi \varepsilon_{t+1}^\pi \end{aligned} \tag{2}$$

where $\overline{r}$ and $\overline{\pi}$ are the long-run equilibrium levels of the real interest rate and inflation rate, ε_{t+1}^r and ε_{t+1}^π are the shocks to these two variables, ρ_r and ρ_π are the speed of adjustment. $\widehat{\pi}_t$ represents expected inflation. The real interest rate process is just a discrete-time version of the Vasicek (1977) process.

Following Mamaysky (2002), we assume that the log dividend yield of stocks, $\delta = \ln\left(1 + \frac{D}{P^S}\right)$, follows a mean reverting process.

$$\delta_{t+1} = \overline{\delta} + \rho_\delta \left(\delta_t - \overline{\delta}\right) + \sigma_\delta \varepsilon_{t+1}^\delta \tag{3}$$

where $\overline{\delta}$ is the long-run level of dividend yield. ε_{t+1}^δ represents the shocks to the dividend yield process and ρ_δ is the speed of adjustment. All shocks are assumed to have the standard normal distribution.

the similar goal by invoking direct utility function.

The modeling of dividend yield process, rather than the conventional dividend growth rate process, results in a more tractable pricing formula. As stressed by Mamaysky (2002), every affine dividend yield process corresponds uniquely to a non-linear dividend growth process, and vice versa.

Next, we specify the log real pricing kernel of the economy, m, which prices all assets. The existence of the pricing kernel is ensured by the arbitrage-free assumption and the uniqueness conditions are derived by Harrison and Kreps (1979).

$$
\begin{aligned}
m_{t+1} &= \ln\left(M_{t+1}\right) = -\mu_m - r_t + \phi_r \sigma_r \varepsilon_{t+1}^r + \phi_\pi \sigma_\pi \varepsilon_{t+1}^\pi + \phi_\delta \sigma_\delta \varepsilon_{t+1}^\delta \\
&= -\mu_m - r_t + \phi' \Sigma \varepsilon_{t+1}
\end{aligned} \tag{4}
$$

where $\phi = [\phi_r, \phi_\pi, \phi_\delta]'$, $\varepsilon = \left[\varepsilon^r, \varepsilon^\pi, \varepsilon^\delta\right]'$, Σ is a 3×3 matrix with $[\sigma_r, \sigma_\pi, \sigma_\delta]$ on its diagonal and zeros elsewhere.

Since the real interest rate is the return on one-period real bond, the pricing kernel must satisfy the following non-arbitrage condition.

$$
r_t = -\ln\left(E_t\left(M_{t+1}\right)\right) = \mu_m + r_t - \frac{1}{2}\phi' \Sigma \rho \Sigma \phi \tag{5}
$$

where $\rho = E\left(\varepsilon\varepsilon'\right)$. Therefore,

$$
\mu_m = \frac{1}{2}\phi' \Sigma \rho \Sigma \phi \tag{6}
$$

We can derive stock and bond returns from this model. They are summarized in the following two propositions.

Proposition 1 (Nominal returns of n-period bond) *Consider, at time t, a nominal zero coupon bond which pays one dollar in n periods. Its log return at time $t+1$ is the sum of the real interest rate, term premium, expected inflation plus inflation shocks and interest rate shocks. Specifically,*

$$
\begin{aligned}
B_{t+1}^{n-1} = &\underbrace{r_t}_{Real\,Interest\,Rate} + \underbrace{\widehat{\pi}_t}_{Expected\,Inflation} + \underbrace{\frac{1}{2}\phi' \Sigma \rho \Sigma \phi - \frac{1}{2}\phi^{*\prime} \Sigma \rho \Sigma \phi^*}_{Term\,Premium} \\
&+ \underbrace{A_{n-1}^r \sigma_r \varepsilon_{t+1}^r}_{Interest\,Rate\,Shocks} + \underbrace{A_{n-1}^\pi \sigma_\pi \varepsilon_{t+1}^\pi}_{Inflation\,Shocks}
\end{aligned} \tag{7}
$$

where $\phi^* = \phi + \left[A^r_{n-1}, \left(A^\pi_{n-1} - 1\right), 0\right]'$, $A^r_{n-1} = -\frac{\left(1-\rho_r^{n-1}\right)}{1-\rho_r}$, *and* $A^\pi_{n-1} = -\frac{\rho_\pi\left(1-\rho_\pi^{n-1}\right)}{1-\rho_\pi}$

Proof. see Appendix ■

The upper part of (7) indicates that expected bond return is the sum of the real interest rate, expected inflation and term premium. The lower part of (7) indicates how bond returns respond to interest rate and unexpected inflation shocks. Under normal conditions $|\rho_r| < 1$ and $|\rho_\pi| < 1$, we have $A^r_{n-1} < 0$ and $A^\pi_{n-1} < 0$, which means positive shocks to unexpected inflation and the real interest rate cause bond returns to fall. Both A^r_{n-1} and A^π_{n-1} are increasing functions of maturity n in absolute value, indicating that bonds with longer maturity are more vulnerable to these shocks.

Proposition 2 (Nominal stock returns) *The log total return of a stock which maintains a dividend yield process* δ_t *can be represented as the sum of the real interest rate, risk premium, expected inflation plus inflation shocks, interest rate shocks and dividend yield shocks.*

$$\begin{aligned} S_{t+1} \quad = \quad & \underbrace{r_t}_{Real\ \ interest\,rate} + \underbrace{\widehat{\pi}_t}_{Expected\ Inflation} + \underbrace{\frac{1}{2}\phi'\Sigma\rho\Sigma\phi - \frac{1}{2}\bar{\phi}'\Sigma\rho\Sigma\bar{\phi}}_{Risk\ Premium} \\ & + \underbrace{a^r\sigma_r\varepsilon^r_{t+1}}_{Interest\ Rate\ Shock} + \underbrace{\sigma_\pi\varepsilon^\pi_{t+1}}_{Inflation\ Shock} + \underbrace{\left(a^\delta + 1\right)\sigma_\delta\varepsilon^\delta_{t+1}}_{Dividend\ Shock} \end{aligned} \tag{8}$$

where $\bar{\phi} = \phi + \left[a^r, 0, \left(a^\delta + 1\right)\right]'$, $a^r = -\frac{1}{1-\rho_r}$ *and* $a^\delta = \frac{\rho_\delta}{(1-\rho_\delta)}$

Proof. see Appendix ■

The upper part of (8) shows the expected stock returns. Expected stock returns share two components with expected bond returns: the real interest rate and expected inflation. The lower part of (8) shows that unexpected stock returns are subject to all three shocks in the economy. Under normal conditions, we have $a^\delta > 0$ and $a^r < 0$, which means that positive dividend shocks raise stock returns, and positive interest rate shocks reduce stock returns. Unexpected shocks of the price level also raise stock returns.

2.2 Implications for The Stock-Bond Correlation

It is important to note that the exogenous shocks in this model are all homoscedastic, which implies that all moments of stock and bond returns, including the correlation, are constant. Models with heteroscedastic shocks, which generate time-varying moments, are often intractable or technically complicated. Given the empirical nature of this paper, we do not pursue those models here. Instead, using comparative statics, we can derive from this simple model the implications for the driving forces of the stock-bond correlation. These implications are then empirically examined in the following sections.

First, let us look at the conditional covariance of stock and bond returns

$$
\begin{aligned}
cov_t\left(R_{t+1}^{n-1}, R_{t+1}^{s}\right) = & \; A_{n-1}^{r} a^{r} \sigma_{r}^{2} \\
& + A_{n-1}^{\pi} \sigma_{\pi}^{2} + A_{n-1}^{\pi}\left(a^{\delta}+1\right) \rho_{\delta\pi} \sigma_{\delta} \sigma_{\pi} + \left[A_{n-1}^{\pi} a^{r} + A_{n-1}^{r}\right] \rho_{\pi r} \sigma_{r} \sigma_{\pi} \\
& + A_{n-1}^{r}\left(a^{\delta}+1\right) \rho_{\delta r} \sigma_{r} \sigma_{\delta} \qquad (9)
\end{aligned}
$$

The first row shows that, since $A_{n-1}^{r} a^{r} \sigma_{r}^{2} > 0$, higher uncertainty about the real interest rate tends to increase the comovement of stock and bond returns. This is intuitive because the real interest rate determines how an investor discounts stock and bond cash flows. Therefore, interest rate shocks are likely to move stock and bond prices in the same direction. The second row summarizes the effect of unexpected inflation on the comovement through the nominal channel, $A_{n-1}^{\pi} \sigma_{\pi}^{2}$; the cash flow channel, $A_{n-1}^{\pi}\left(a^{\delta}+1\right) \rho_{\delta\pi} \sigma_{\delta} \sigma_{\pi}$; and the discount factor channel, $\left[A_{n-1}^{\pi} a^{r} + A_{n-1}^{r}\right] \rho_{\pi r} \sigma_{r} \sigma_{\pi}$. The nominal channel, $A_{n-1}^{\pi} \sigma_{\pi}^{2} < 0$, unambiguously reduces the stock-bond comovement. However, the effects of the other two channels are ambiguous and depend on parameter values ($\rho_{\delta\pi}$, $\rho_{\pi r}$, and $\left[A_{n-1}^{\pi} a^{r} + A_{n-1}^{r}\right]$). They are also among the most debated topics in finance and macroeconomics.[10] This line of literature is vast and the consensus is yet to emerge. Therefore, the only thing we can conclude is that, if the economy is neutral to unexpected inflation shocks (i.e., $\rho_{\delta\pi} = \rho_{\pi r} = 0$), then we expect the stock-bond correlation to decrease with higher uncertainty about unexpected

[10] The conventional wisdom is "stocks are real, bonds are nominal." Bodie (1976), and Fama and Schwert (1977) both find the counterevidence to this claim, while Boudoukh, and Richardson (1993) support this claim with long-run data. Fama (1981) attributes the negative correlation between inflation and stock return to the cash flow channel, while Goto and Valkanov (2002) report important effect of the discount factor channel.

inflation. Otherwise, the effect of unexpected inflation shocks cannot be determined. Similarly, the effect of the last component also depends on parameter value ($\rho_{\delta r}$). Despite of these indeterminant terms, we focus our investigation on the less ambiguous part of the link.

Secondly, expected components of the stock and bond returns (7 and 8) offer an additional source of positive comovement: expected inflation.[11] Similar to the real interest rate, expected inflation moves stock and bond returns in the same direction. Greater uncertainty about this factor is likely to cause higher comovement.

One source to reduce the comovement comes from the fact that dividend shocks are unique to stock returns. Consistent with the empirical work of Fama and French (1993), Mamaysky (2002) shows that the pricing factors for bonds are only a subset of those for stocks. That is to say, dividend shocks can be decomposed as the sum of three components:

$$\varepsilon_{t+1}^{\delta} = \kappa_r \varepsilon_{t+1}^{r} + \kappa_\pi \varepsilon_{t+1}^{\pi} + \kappa_u \varepsilon_{t+1}^{u} \tag{10}$$

where ε_{t+1}^{u} is unique to stock returns and, $cov_t\left(\varepsilon_{t+1}^{u}, \varepsilon_{t+1}^{r}\right) = cov_t\left(\varepsilon_{t+1}^{u}, \varepsilon_{t+1}^{\pi}\right) = 0$. Following the terminology of Mamaysky (2002), we call ε_{t+1}^{u} the stock unique component. Higher uncertainty about this component reduces the stock-bond correlation because it makes stock returns more volatile without affecting bond returns.[12]

In summary, this model points to the uncertainty of three macroeconomic factors, expected inflation, the real interest rate, and unexpected inflation as the explanatory factors of the stock-bond correlation. Greater uncertainty about expected inflation and the real interest rate increases this correlation. The effect of unexpected inflation is ambiguous and depends on whether the dividend yield and the real interest rate are affected by unexpected inflation shocks. In addition, uncertainty about the stock unique component reduces the stock-bond correlation by changing the volatility of stock returns.

[11] The risk premium and the term premium may also be the sources of comovement. However, as shown above, these two premia are the functions of exogenous risks. Therefore, this source of comovement is not independent from those caused by the risk in inflation, the real interest rate and the divided yield.

[12] Since $Corr\left(B_t, S_t\right) = \frac{Cov(B_t, S_t)}{Var(B_t)^{\frac{1}{2}} Var(S_t)^{\frac{1}{2}}}$, higher uncertainty about the stock unique component, κ_u, increases $Var\left(S_t\right)$ without affecting $Var\left(B_t\right)$ and $Cov\left(B_t, S_t\right)$. Therefore, it mechanically reduces the stock-bond correlation, $Corr\left(B_t, S_t\right)$.

3 Measuring the Uncertainty about Macroeconomic Factors

It is important to note that it is the uncertainty, rather than the levels, of the macroeconomic factors that affects the stock-bond correlation. As the final preparation for empirical analysis, this section explains how the macroeconomic factors and the measurements of their uncertainty are constructed in this paper.

3.1 Macroeconomic Data

Macroeconomic data are from the IFS-IMF database. Monthly inflation rate is the log-difference of the Consumer Price Index (CPI). Monthly industrial production growth rate is the log-difference of the Industrial Production Index (IP). We use the treasury bill (T-bill) yield as the short-term interest rate whenever it is available and the money market rate when otherwise. The mean spread between the money market rate and the T-bill yield is subtracted from the former to ensure consistency, when conversion is needed. Quarterly GDP growth rate is computed using IFS-IMF volume index. All macroeconomic data are monthly, except for GDP. Most of the macroeconomic data series start from 1960 and later, which limits the empirical analysis to the period of 1961-2001.

3.2 Defining Economic Variables

Expected Inflation (EXPINF)

Two measures of expected inflation are provided: one for the short-term (EXPINF_S) and one for the long-term (EXPINF_L). Motivated by the generalized Phillips Curve model, the short-term expected inflation (EXPINF_S) is estimated as the one month ahead forecast of a three-variable rolling Bayesian Vector Autoregression (BVAR) model. These three variables are the monthly inflation rate, the IP growth rate, and the T-bill rate.[13] This model is estimated using monthly data for each of the G7 countries from 1958 to 2001.

The long-term expected inflation (EXPINF_L) is needed because the concerned assets in this paper are both long-term assets. Their durations exceed the horizon of

[13]The number of lag is chosen to be 12 for BVAR. Hall and Krieger (2000), and Sims (2000) apply the same model to generate inflation forecast. Stock and Watson (1999) show that a generalized Phillips Curve based on real aggregate activity tends to outperform many alternative models.

short-term inflation expectations. Although the long horizon forecast of BVAR model is a natural choice for EXPINF_L, it quickly reverts to its long-term mean value for out-of-sample forecast. Besides, due to its linear structure, the BVAR model may not capture the potential non-linearity in long-term inflation expectations. An alternative source for obtaining long-term expected inflation is the yield curve. Mishkin (1989) and Fama (1990) both argue that the long end of the term structure is likely to reveal information about expected future inflation because the volatility of expected inflation outweighs that of the real interest rate in the long-run. Using real GDP growth rates as the proxy for the long-term real interest rates, we compute the long-term expected inflation by subtracting the average GDP growth rates of the past 5 years from the long-term bond yields, similar to Bordo and Dewald (2001).[14] We compare this measure with the long horizon forecast of BVAR and find that they bear a close resemblance. The correlation of these two measures is in the range of 0.7-0.9 for G7 countries.

Real Interest Rate (RINT)

The *Ex Ante* real interest rate is calculated as the difference between the T-Bill rate and short-term expected inflation: $RINT\,(t) = TBill\,(t-1) - EXPINF\_S\,(t)$.

Unexpected Inflation (UNINF)

Unexpected inflation rate is calculated as the forecasting error of the 3-variable BVAR model: $UNINF\,(t) = INF\,(t) - EXPINF\_S\,(t)$.

Stock Unique Component (STQ)

STQ is not a macroeconomic factor, but it is needed as a control variable for the volatility of stock returns due to the unique stock pricing factors. We cannot simply use the dividend yield data because it may correlate with inflation or the real interest rate. However, as suggested by (10), the stock specific component can be backed out by purging bond factors from stock returns. To do so, we regress daily stock returns onto daily bond returns within each month; then the standard deviation of the regression residuals is used as the STQ proxy.

3.3 Constructing Uncertainty Measures

Constructing uncertainty measures deserves special attention. As pointed out by David and Veronesi (2001), uncertainty about economic fundamentals may contain informa-

[14] According to Phelps's *Golden Rule* theory, the GDP growth rate can be viewed as the return on the existing capital in the economy and should be equal to the long-term interest rate in equilibrium.

tion that cannot be revealed by short data series, e.g. structural breaks and regime shifts. As a result, uncertainty can be high even when volatility is low.

The vast macroeconomic literature on inflation offers some clue as for the uncertainty about expected inflation. It has long been argued that inflation cannot stabilize at high levels (Okun 1971 and Friedman 1977) because higher inflation induces higher uncertainty. In an empirical study, Ball and Cecchetti (1990) show that the uncertainty about expected inflation (or the persistent trend) is positively related to its level,[15] while unexpected inflation is not. Therefore, we use the level of expected inflation as the proxy for its uncertainty. Figure 4 shows the uncertainty of long-term and short-term expected inflation in the U.S. The pictures of other countries are similar. Contrasting these two measures reveals an interesting phenomenon. Although inflation subsided in most of these countries since the early 1980s, the concern for future inflation persisted for a much longer period.[16]

Since we do not have *a priori* knowledge for the uncertainty about unexpected inflation, we proxy it with the conditional volatility of UNINF generated from a GARCH(1,1) model.

A natural method to derive the uncertainty about the real interest rate is also to compute the conditional volatility implied in a GARCH(1,1) model, which we call the short-term measure (RINT_S). However, this method may be subject to David and Veronesi's criticism of measuring uncertainty because there is strong evidence that the U.S. real interest rate has experienced regime shifts in the post-war period (Garcia and Perron 1996). To address this issue, we estimate the 3-state regime-switching model of Garcia and Perron (1996) for each of the G7 countries.[17] Then following David and Veronesi (2001), we use the root-MSE (Mean Squared Error) to capture the uncertainty about the actual regime of the real interest rate. We call it the long-term uncertainty measure of the real interest rate (RINT_L). Figure 5 contrasts these two measures for the U.S. Note that the base level of RINT_L is zero, while the base level of RINT_S is the unconditional volatility, which is greater than zero. Similar to the case of expected inflation, RINT_L has prolonged high uncertainty than RINT_S during the late 1970s

[15] We replicate the study by Ball and Cecchetti (1990) with the data of G7 countries. The results indicate that the inflation level can explain up to 90% of the cross sectional variations of its variability.

[16] Orphanides and Williams (2002) discuss the theoretical aspects of prolonged inflation uncertainty. David that Veronesi (2001) show empirically that the inflation uncertainty generated from a regime-switching model persists in the U.S. until the early 1990s.

[17] We use the Gauss code of Kim and Nelson (1999) to estimate the Garcia and Perron (1996) 3-state regime-switching model for the real interest rate.

and much of the 1980s.

Table 3 reports the correlations of different uncertainty measures about expected inflation and the real interest rate. The correlation of the two uncertainty measures of expected inflation, EXPINF_L and EXPINF_S, is generally between 0.4 and 0.7, except for Japan (-0.04). The correlation of the two uncertainty measures of the real interest rate, RINT_L and RINT_S, is generally between 0.4 and 0.6, except for France (0.09). To simplify notations, EXPINF_L, EXPINF_S, UNINF, RINT_L, RINT_S and STQ are used to denote the respective measures of their uncertainty hereafter.

4 Driving Forces of Stock-Bond Correlations: Empirical Analysis

The simple theoretical model in Section 2 is built upon the extreme assumptions of affine state variables and homoscedastic shocks. We derive its implications using comparative statics. As a result, this model offers no direct guide as to how its implications can be tested. In this section, we use three formulations to empirically examine the link between the stock-bond correlation and the uncertainty about macroeconomic factors suggested by the model. The first formulation is very intuitive. It directly tests this link using a linear regression of unconditional stock-bond correlations. The drawback of this formulation is that stock returns, and to a lesser extent bond returns, are known to be autocorrelated in both the first and the second moments. This may affect the time series properties of stock-bond correlations. To tackle this problem, the second formulation takes account of autocorrelations in stock returns. It looks at the contemporaneous effect of bond returns on stock returns, rather than the unconditional correlation. The third formulation takes one step further to specify the autocorrelations in both stock and bond returns and jointly estimates them. These three formulations are selected such that each in succession offers more realistic modeling of the dynamics of stock and bond returns. We are also interested in whether the results are consistent using different formulations and, if not, whether the more complicated formulations shed additional light on the link we are trying to elucidate.

Let us start this section by defining a vector of the uncertainty about macroeconomic factors which will be repeatedly referred to below,

$$UCTY = [1 \ EXPINF \ \ RINT \ \ UNINF]'.$$

4.1 Formulation 1: Unconditional Correlation

The first formulation directly links the unconditional stock-bond correlation with the uncertainty about macroeconomic factors.

$$\begin{aligned} YCorr_t &= \beta_c \cdot YCorr_{t-1} + \beta_s \cdot STQ_{t-1} + \beta' \cdot UCTY_t + \varepsilon_t \qquad (11) \\ &= \beta_0 + \beta_c \cdot YCorr_{t-1} + \beta_s \cdot STQ_{t-1} + \beta_e \cdot EXPINF_t + \beta_r \cdot RINT_t + \beta_u \cdot UNINF_t + \varepsilon_t \end{aligned}$$

Here, we regress the Fisher transformation of the stock-bond correlation coefficient ($YCorr_t$) onto its first lag ($YCorr_{t-1}$), the stock unique component (STQ), and the uncertainty about macroeconomic factors ($UCTY_t$). The monthly series of unconditional correlation are computed using daily stock and bond returns within each month. Since STQ measures the disparity of stock and bond volatility, it also serves as an adjustment term for the potential correlation bias, similar to those proposed by Loretan and English (2000) and Forbes and Rigobon (2002). Since STQ is a regressor generated using stock and bond returns, it may be correlated with the error term, which potentially invalidates the inference of the estimators. To avoid this problem, we always lag STQ by one period.

$YCorr$ is the Fisher's transformation of correlation coefficient, defined as

$$YCorr = \frac{1}{2} \ln \left(\frac{1 + Corr}{1 - Corr} \right) \qquad (12)$$

which transforms the correlation coefficient from the range of $[-1, 1]$ to $(-\infty, \infty)$ with a continuous and monotonic function.[18]

Table 4 shows the regression results of unconditional correlations. Both the U.S. regression and G7 panel regression results are presented. T-statistics are corrected for heteroscedasticity and serial correlation.[19]

These regressions only cover the period from 1980 to 2001 because we need daily return data, which are available only after 1980, to compute the monthly stock-bond correlation. The first five rows of Table 4 show the effect of each of the individual macroeconomic factors. Two long-term uncertainty measures, EXPINF_L and RINT_L,

[18]Fisher transformation of correlation coefficient is known to have standardized normal distribution asymptotically. It converges to its asymptotic distribution much faster than a lot of other alternative transformations. See Anderson (1984) for details.

[19]Serial correlation of error terms in a regression model with a lagged dependent variable as (11) can potentially bias the estimators. Fortunately, Durbin-Watson statistics in Table 4 give no indication of error terms being serial correlated.

are positive and highly significant for both the U.S. and the G7 panel. In contrast, the two short run uncertainty measures, EXPINF_S and RINT_S, are positive but generally insignificant except for EXPINF_S in the G7 panel. The statistical significance of the two long-term measures can be explained by the fact that they match the long durations of stocks and long-term government bonds. Besides, they are also designed to capture the information that is not present in the short-term measures, such as non-linearity and structural breaks.

The next two regressions examine the effect of the long-term measures and the short-term measures respectively. This offers an opportunity to look into the relative explanatory power of the three macroeconomic factors. It is evident from the results that, using long-term measures, the uncertainty about expected inflation strongly dominates the uncertainty about the real interest rate. Both the magnitude and the significance of RINT_L coefficient are reduced compared to the results in the second row. We can observe a similar dominance of EXPINF_S over RINT_S in the next row, despite the fact that they do not have much explanatory power overall. Throughout Table 4, the uncertainty about the unexpected inflation is shown to be consistently positive but insignificant, indicating that unexpected inflation shocks may affect the real side of stock returns.

The monthly stock-bond correlation is also positively autocorrelated at one lag. The coefficient for the lagged term is persistently around 0.4. Another interesting observation is that R^2 has very small variations across all regressions. This indicates that most of the high frequency fluctuations in the stock-bond correlation are explained by the first lag and STQ. The contribution of macroeconomic factors, especially EXPINF_L, lies in their effect on the major trends of the stock-bond correlation. Since the EXPINF_L is estimated using the level of inflation expectation, its coefficient has very intuitive economic explanation. All else being equal, during the period between 1980 and 2001, a 10% long-term inflation expectation would be likely to raise the stock-bond correlation by 0.17 in the U.S. and on average 0.26 for the G7 countries.[20]

[20] A similar exercise can be done at annual frequency from 1961 to 2001. An earlier version of this paper also reports its results, which strongly supports the explanatory power of EXPINF_L. These results are dropped due to two concerns. First, both the annual stock-bond correlation and STQ are computed using only 12 observations. Second, some additional assumptions have to be made in order to measure the uncertainty of the macroeconomic factors at annual frequency.

4.1.1 Business cycles

Many researchers argue that business cycles may have a strong effect on asset returns (e.g., Rouwenhorst 1995). Schwert (1989) shows that they can explain much of the time series variations in the stock return volatility. Therefore, a natural question is whether the stock-bond correlation varies at different stages of the business cycle. Table 5 replicates the previous regression augmented with a dummy variable for business cycle stages (1 for expansion, and 0 for recession). Here we use the business cycle dates of Economic Cycle Research Institute (ECRI),[21] who applies the same methodology used by NBER to the business cycles dating for major industrial countries. Clearly, the results do not indicate that the business cycle has any effect on the stock-bond correlation, either in the U.S. or in the G7 panel.

4.2 Formulation 2: Contemporaneous Effect

Three concerns may cast doubt on the validity of conclusions about the stock-bond correlation based on the results in the first formulation. First, it only covers the 1980-2001 period and does not fully utilize the entire dataset. Second, the regression of unconditional correlation needs an adjustment term, STQ, which is a generated regressor. Lagging it by one period does not guarantee that it is uncorrelated with the error term. Third, the presence of a strong lagged term may simply overshadow the explanatory power of other variables in the regression. Since stock returns and, to lesser extent bond returns, are known to have GARCH type of volatility, the strong autocorrelation of monthly stock-bond correlation can be a result of autocorrelated volatility. We can address these concerns with Formulations 2 and 3.

Formulation 2 models the stock return as a linear combination of its lagged return and the contemporaneous bond return.

$$\begin{aligned} S_{t+1} &= \alpha_0 + \alpha_S \cdot S_t + \left(\alpha' \cdot UCTY_{t+1}\right) \cdot B_{t+1} + \varepsilon_{t+1} \\ &= \alpha_0 + \alpha_S \cdot S_t + (\alpha_B + \alpha_e \cdot EXPINF_{t+1} + \alpha_r \cdot RINT_{t+1} + \alpha_u \cdot UNINF_{t+1}) \cdot B_{t+1} + \varepsilon_{t+1} \end{aligned} \quad (13)$$

The coefficient for the bond return, $\alpha' \cdot UCTY_{t+1}$, measures the contemporaneous effect of bond returns on stocks returns. It is time-varying and determined by the uncertainty about macroeconomic factors. Clearly, a higher coefficient, $\alpha' \cdot UCTY_{t+1}$, indicates a

[21] http://www.businesscycle.com/

higher correlation between stock and bond returns. Furthermore, the stock return volatility is assumed to follow a GARCH(1,1) process.

$$E_t\left(\varepsilon_{t+1}^2\right) = h_t; \quad and \quad h_{t+1} = \omega + \eta\varepsilon_{t+1}^2 + \gamma h_t$$

To write stock and bond returns in such a linear regression form (13), it is important that bond returns are uncorrelated with the error term. This formulation is theoretically plausible because (7) and (8) suggest that stock returns contain shocks that are orthogonal to bond returns. Note that this formulation represents only the statistical relationship and does not claim any causal relationship. This model also allows us to look into the link between the uncertainty about macroeconomic factors and the stock-bond correlation without having first to compute the correlation coefficient for each period. The immediate benefit is that we can now extend our analysis to the entire 1961-2001 period. Specifically modeling autocorrelated volatility of stock returns shields the results of our main investigation from this type of influence. An additional benefit of Formulation 2 is that we no longer need STQ to adjust for the volatility disparity of stock returns relative to bond returns.

This model is estimated using Maximum Likelihood Method (ML) jointly for all G7 countries using monthly data. We show the results for both the full sample period, 1961-2001, and the period used in the first formulation, 1980-2001. We restrict the coefficients to be the same across all G7 countries except for the constant terms $\alpha_0's$.[22]

Table 6 presents the estimation results. Constant terms $\alpha_0's$ are less important and are therefore omitted to keep the table easier to read. Across all the panels, we can find strong evidence that monthly stock returns are autocorrelated at the first lag (α_S), and its volatility follows a GARCH process (ω, η, γ). The conditional volatility of the stock return is very persistent since the coefficient for the lagged term (γ) is around 0.9. Besides, $\alpha' \cdot \overline{UCTY}$, which measures the average response of stock returns to the change in bond returns, is relatively stable across all panels. On average, every 1% increase in the bond return is likely to be accompanied by 0.5% increase in the stock return.

Panels A and B show the estimation results for the 1980-2001 period. When the two long-term measures, EXPINF_L and RINT_L are used (Panel A), EXPINF_L is marginally significant while RINT_L is not. When the two short-term measures are

[22]There may be some interest in allowing $\omega's$ to be heterogeneous as well. We estimate such model and find very little variation of $\omega's$ across G7 countries.

used (Panel B), none of the macroeconomic factors is significant. These results are consistent with those obtained from the unconditional correlation regressions (Table 4).

Panels C and D extend this exercise to the full sample period, 1961-2001. In contrast to Panel A, Panel C shows that both the statistical and the economic significance of EXPINF_L is higher when we look at the full sample. The uncertainty about long-term expected inflation plays a much stronger role in determining the comovement between stock and bond returns than that about the real interest rate. Replacing EXPINF_L and RINT_L with two short-term measures, Panel D shows that the uncertainty about inflation and the real interest rate is less economically significant but more statistically significant. Our explanation is that, although both the inflation risk and the interest rate risk play an important role in determining the comovement between stock and bond returns, the risk in expected inflation outweighs that in the real interest rate. Moreover, stock and long-term bond returns, both of which are long-term assets, are more affected by the uncertainty about long-term expected inflation than its short-term counterpart. Therefore, we see that EXPINF_L dominates RINT_L in the estimation results. When we replace EXPINF_L with EXPINF_S, the uncertainty about the real interest rate is more significant.

Another interesting observation in Panels C and D is that UNINF is significant but negative. This is very different from the previous results presented in the paper. This indicates that during the entire sample period, 1961-2001, the unexpected rises of the price level increase the nominal side of stock returns more than they adversely affect their real side.

4.3 Formulation 3: Conditional Correlation

The results of Formulation 1 and Formulation 2 suggest that the uncertainty about expected inflation (especially the long-term expected inflation) plays a dominant role in determining the trend of stock-bond correlation. As shown by (7) and (8), expected inflation induces comovement between stock and bond returns through their expected components. Therefore, we wonder whether accounting for the expected stock and bond returns may help distill the role of the other two macroeconomic factors: the real interest rate and unexpected inflation. This question can be answered using Formulation 3.

Formulation 3 extends Formulation 2 by allowing stock and bond returns to jointly follow a Vector Autoregression (VAR) process. It is similar to the model used by Longin and Solnik (1995) in their study of the comovement of international equity returns.

$$\begin{pmatrix} S_{t+1} \\ B_{t+1} \end{pmatrix} = \begin{pmatrix} \alpha^S \\ \alpha^B \end{pmatrix} + \begin{pmatrix} \beta_{SS} & \beta_{SB} \\ \beta_{BS} & \beta_{BB} \end{pmatrix} \cdot \begin{pmatrix} S_t \\ B_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{S,t+1} \\ \varepsilon_{B,t+1} \end{pmatrix}$$

If the past returns well capture the entire information set available at time t, then we expect the conditional correlation, $\rho_t^{S,B} = Corr_t\left(S_{t+1}, B_{t+1}\right) = Corr_t\left(\varepsilon_{S,t+1}, \varepsilon_{B,t+1}\right)$, to be relatively free from the influence of the uncertainty about expected inflation.

Let $H_t = E_t\left(\varepsilon_{t+1}\varepsilon_{t+1}'\right) = \begin{pmatrix} h_{S,t}^2 & h_{SB,t} \\ h_{SB,t} & h_{B,t}^2 \end{pmatrix}$ denote the conditional covariance matrix, where $\varepsilon_{t+1} = \left(\varepsilon_{t+1}^S, \varepsilon_{t+1}^B\right)'$. To specify the dynamics of conditional correlation, we first assume that the volatility of individual asset returns follows a univariate GARCH(1,1) process,

$$h_{S,t+1}^2 = \omega_S + \eta_S\left(\varepsilon_{S,t+1}\right)^2 + \gamma^S h_{S,t}^2; \quad h_{B,t+1}^2 = \omega_B + \eta_B\left(\varepsilon_{B,t+1}\right)^2 + \gamma^B h_{B,t}^2$$

The covariance term is then determined by $h_{S,B,t} = h_{S,t} \cdot h_{B,t} \cdot \rho_t^{S,B}$. The next assumption is that $\rho_t^{S,B}$ is the function of the uncertainty about macroeconomic factors

$$\rho_t^{S,B} = \frac{\exp\left(2A' \cdot UCTY_t\right) - 1}{\exp\left(2A' \cdot UCTY_t\right) + 1}$$

where the function form $f(x) = \frac{\exp(2x)-1}{\exp(2x)+1}$ is the reverse Fisher transformation which ensures that $\rho \in [-1, 1]$

Our approach differs from that of Longin and Solnik (1995) in the following two ways. First, our model is conditional on the past returns while their model is conditional on a set of information variables such as the dividend yield, interest rates, and the January dummy. Our choice is motivated both by the strong autocorrelations found in stock and bond returns (Table 2) and the limited forecastability of stock returns at monthly frequency. Second, we apply the reverse Fisher transformation to ensure $\rho \in [-1, 1]$, while Longin and Solnik (1995) do not.

This model is estimated jointly for the G7 countries using Maximum Likelihood Method (ML), allowing for the constant vector in the mean equation $\left(\alpha^S, \alpha^B\right)'$ to differ across countries. The estimation results are presented in Table 7. We only show the estimation results using EXPINF_L because those using EXPINF_S vary significantly with initial values and are not reliable. The constant terms α^S and α^B are less interesting and therefore are not shown here.

Panels A and B cover the period of 1980-2001 and 1961-2001 respectively. Both panels indicate that past bond returns increase both current stock and bond returns ($\beta_{SB} > 0$ and $\beta_{BB} > 0$), and the past stock returns reduce the current bond returns ($\beta_{BS} < 0$). The assumption of GARCH(1,1) volatility of the stock and bond returns is strongly supported by the estimation results, as both (η_S, γ_S) and (η_B, γ_B) are positive and highly significant. In Panel A, the coefficients of both EXPINF_L and RINT_L are shown to be positive and significant. Note that the coefficient for EXPINF_L (1.40) is smaller than that in Table 4 (2.623). The coefficient for RINT_L is much larger and more statistically significant. It increases to 10.07 ($tstat = 2.76$) from 0.20 ($tstat = 0.11$).[23] The results in Panel B are consistent with these findings. The coefficients for both EXPINF_L (1.58) and RINT_L (8.44) are positive and significant, and are comparable with those in Panel A. In addition, the effect of the uncertainty about unexpected inflation is similar to the results of Formulation 2. It does not seem to affect the conditional correlation during the 1980-2001 period. However, when the model is estimated for the 1961-2001 period, its coefficient becomes negative and significant.

One interpretation of these results is that the VAR model weakens, but does not completely eliminate, the effect of expected inflation uncertainty. Nevertheless, the strengthening of RINT_L coefficients relative to the weakening of EXPINFL_L coefficients suggests that RINT_L does play a limited role in shaping the comovement of stock and bond returns.

4.4 Summary of Empirical Results

Among all the macroeconomic factors that affect the stock-bond correlation, the uncertainty about expected inflation (especially the long-term measure) strongly dominates other factors. The uncertainty about the real interest rate and unexpected inflation also influence the comovement of stock and bond returns. However, their effect can be better observed when the uncertainty about expectation inflation is removed or weakened. It is also important to note that these macroeconomic factors affect the major trends, rather than the month-to-month variations, of the stock-bond correlation.

[23] The dramatic increase in RINT_L's coefficient is not surprising because the mean values of EXPINF_L is about 10-30 times larger than those of RINT_L in G7 countries.

5 Economic Value of Correlation Timing

Understanding the driving forces of stock-bond correlations offers an opportunity for investors to improve their asset allocation decisions. This section evaluates whether this opportunity generates significant economic value. Our approach is similar to the short-horizon dynamic strategy used by Fleming, Kirby and Ostdiek (2001) in their study of the economic value of volatility timing. Short-horizon dynamic strategy means that the hypothetical investor seeks to maximize her one-period utility and does not hedge against future changes in the investment opportunity set. Under Merton's (1973) framework, such a short-horizon strategy should underperform the optimal strategy because it ignores the hedging component. Therefore, a short-horizon strategy sets a higher bar for positive economic value than the optimal strategy. To separate the value of correlation timing from that of return and volatility forecastability, we further assume that investors take the expected returns and the volatility of stocks and bonds as constant. This can be interpreted as the perspective of an investor who saves for retirement and ignores the short-run variations of asset returns.

Since the setting of Fleming, Kirby and Ostdiek (2001) does not allow for an analytical solution for the optimal portfolio, they have to evaluate their strategy by examining two sub-optimal portfolios: maximum-mean and minimum-variance portfolios. We avoid this problem by assuming that investors have power utility function over terminal wealth and that asset returns are log-normally distributed.

$$U\left(W_T\right) = \frac{W_T^{1-\gamma}}{1-\gamma}$$

where γ is the risk aversion coefficient of the power utility function.

These assumptions enable us to obtain an analytical solution to investors' asset allocation problem. Investors can allocate their money among stocks, bonds and cash. Cash earns the risk-free rate each period. Campbell and Viceira (2002) show that the one-period optimal asset allocation can be expressed as,

$$\alpha_t = \frac{1}{\gamma}\Sigma_t^{-1}\left(E_t r_{t+1} - r_{f,t}\cdot I - \sigma_t^2/2\right) \tag{14}$$

where α_t is the vector of portfolio shares invested in stocks and bonds, Σ_t is the conditional covariance matrix of stock and bond returns, $E_t r_{t+1}$ is the expected return vector, $r_{f,t}$ is the risk free rate, $I = [1,1]'$, and σ_t^2 is the vector of stock and bond variances.

Below we compare two strategies using U.S. market data: a moving average (MA) strategy and a dynamic strategy. The MA strategy investor applies the 60-month moving average of stock-bond correlations as her forecast for the next month.[24] In contrast, the dynamic strategy investor takes into account current macroeconomic factors and bases her forecast on the following equation.

$$YCorr_t = \beta_0+\beta_c\cdot YCorr_{t-1}+\beta_s\cdot STQ_{t-1}+\beta_e\cdot EXPINF_{t-1}+\beta_r\cdot RINT_{t-1}+\beta_u\cdot UNINF_{t-1}+\varepsilon_t \tag{15}$$

This regression is ran every month with all the past data and the forecast is strictly out-of-sample. Both investors form their portfolios according to (14) and rebalance them at the end of each month. They start with 11 years of information (1980-1990), and the actual investment period is 1991-2001. The expected returns and volatility are set to be equal to the U.S. values during the 1991-2001 period. The mean returns are 15.01% and 7.26%, respectively. The standard deviations are 15.61% and 6.63%.

We use Certainty Equivalence (CE) as our measure for economic value. It is defined as the maximum fee that an investor would like to pay for holding a dynamic strategy versus a moving average strategy. $CE = \sup\left\{\delta | E\left(U\left(W^{MA}\right)\right) \leq E\left(U\left(W^{Dynamic} - \delta\right)\right)\right\}$

To compute the expected utility, we observe that final wealth is
$W_T = W_0 \cdot \prod_{t=1}^{T}(1+r_t)$, where W_0 is the initial wealth. Therefore, the log-utility is given by:

$$\begin{aligned} \log\left(U\left(W_T\right)\right) &= (1-\gamma)\sum_{t=1}^{T}\log\left(1+r_t\right) + (1-\gamma)\log W_0 - \log(1-\gamma) \\ &= (1-\gamma)\, T\cdot\overline{\log\left(1+r_t\right)} + (1-\gamma)\log W_0 - \log(1-\gamma) \end{aligned} \tag{16}$$

It is clear from the equation above that $U\left(W_T\right)$ is log-normally distributed. This allows us to estimate consistently the expected utility using its sample average, [25]

$$\overline{U\left(W_T\right)} = \exp\left(\left(1-\gamma\right)T\cdot\overline{\log\left(1+r_t\right)} + \frac{1}{2}\left(1-\gamma\right)^2 T^2\widehat{Var}\left(\overline{\log\left(1+r_t\right)}\right)\right)\cdot\frac{W_0^{1-\gamma}}{1-\gamma}$$

[24]The moving average length of 60-month is chosen to be consistent with the rolling correlation window in Section 1. Varying this length affects our results quantitatively, but not qualitatively.

[25]If x is log-normally distributed, i.e., $\log x \sim N\left(\mu,\sigma^2\right)$, then $E\left(x\right) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$

Figure 6 compares the out-of-sample correlation forecast with the actual correlation and the 60-month moving average. The forecast matches the trend of the actual correlation reasonably well, especially in the late 1990s. However, the 60-month moving average tends to overestimate stock-bond correlations during that period. Figure 7 contrasts the asset allocations of both investors for $\gamma = 10$ and risk-free rate equals 4.5%, the average U.S. T-Bill yield in the 1990s. We can see that they invest a similar portion of their portfolios in stocks and this portion has been remarkably stable. This is primarily due to the assumption of constant expected asset return and volatility. Bonds and cash become close substitutes, and both investors move heavily into bonds in the late 1990s. This is because when the stock-bond correlation sharply decreases in the late 1990s, investors respond to the improved diversification opportunities by shifting cash into bonds. Although cash bears zero risk, this attractive feature loses ground to the higher expected return of bonds.

Table 8 compares the performance of two strategies under various assumptions of risk aversion and the risk-free rate. The last column shows the bootstrapped P-values for the test $H_0 : CE \leq 0$. The following information can be summarized from this table. First, from the perspective for CRRA investors, the dynamic strategy strongly dominates the MA strategy. In most cases, the hypothesis $CE \leq 0$ can be rejected. For example, taking 4.5% as the risk-free rate, the dynamic strategy has a CE of 0.64% per annum for $\gamma = 10$. Second, the dynamic strategy portfolio is a more risky portfolio. Both its mean and volatility are greater than those of the MA portfolio. Higher Sharpe Ratios indicate that the dynamic strategy investor is better rewarded for the risk she takes. Third, CEs decrease with γ. This is because higher risk aversion discourages the holding of both stocks and bonds, making it harder for either strategy to make a difference. Fourth, CEs increase with risk-free rate. This is because, as the stock-bond correlation decreases, the dynamic strategy investor is able to seize the better diversification opportunity and capture the bond premium faster than the MA investor. Higher risk-free rate further discourages the MA investor from holding more bonds by reducing the share of cash.

The evidence in Table 8 is strongly in favor of the dynamic strategy. In this case, understanding the driving factors of the stock-bond correlation helps investors to better respond to changes of the stock-bond diversification opportunity. However, this exercise has some limitations. In order to separate the value of correlation timing from that of time-varying expected returns and volatility, we make the simplifying assumption that the expected returns and volatility are constant. We also evaluate the performance according to nominal wealth. Modifying these assumptions can potentially affect the results. Therefore, more comprehensive studies need to be done in order to fully assess the economic value of accounting for the stock-bond correlation.

6 Conclusion

Using the data of G7 countries from the past 40 years, this paper documents large variations in the stock-bond correlation. There has been a sharply reverting trend in stock-bond correlations across all G7 countries. They grew steadily upwards from around zero in the early 1960s to about 0.5 in the mid-1990s, and in recent years they reverted back to zero. There also seems to be a converging trend in stock-bond correlations across G7 countries.

A simple model which endogenously derives stock and bond returns reveals that the uncertainty about expected inflation and the real interest rate is likely to increase the comovement between stock and bond returns. The effect of unexpected inflation is ambiguous and depends on how dividends and the real interest rate respond to unexpected inflation shocks. Empirical analysis generally confirms these predictions. Among the macroeconomic factors considered here, the uncertainty about long-term expected inflation plays a dominant role in affecting the major trends of how stock and bond returns co-move. The effect of unexpected inflation and the real interest rate is significant to a lesser degree.

Our analysis sheds light on the reverting trend observed in G7 stock-bond correlations. The 1970s saw an oil crisis and a subsequent economic stagflation in major industrial countries, which caused high and persistent inflation expectations for over a decade. Investors' concern for inflation strongly affected the valuation of financial assets during this period and resulted in high comovement between stock and bond returns. The sharp decline in stock-bond correlations in the 1990s can be partially attributed to the lower inflation risk during this period.

Stocks and bonds are two major asset classes for ordinary investors. A lower stock-bond correlation indicates better diversification opportunities. The fact that stock-bond correlation is positively related to inflation risk is a disturbing message for investors. During the periods when inflation risk is high, asset returns tend to be more volatile. This gives investors a stronger incentive to diversify the investment risk.

Unfortunately, these are also the periods when stock-bond correlations are high and diversification opportunities are meager. This observation leads to the Murphy's Law of Diversification: *diversification opportunities are least available when they are most needed*.

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Appendix

Proof of Proposition 1. Suppose the price of a n-period nominal bond has the following exponential form.

$$P_t^n = \exp\left(A_n^0 + A_n^r r_t + A_n^\pi \pi_t\right) \tag{17}$$

The log pricing kernel for nominal asset is $m_{t+1} - \pi_{t+1}$. Therefore, bond price must satisfy the following recursive functional form.

$$P_t^n = E_t\left[\exp\left(m_{t+1} - \pi_{t+1}\right) P_{t+1}^{n-1}\right] \tag{18}$$

$$\begin{aligned} P_t^n &= E_t\left[\exp\left(m_{t+1} - \pi_{t+1}\right) P_{t+1}^{n-1}\right] \qquad (19) \\ &= E_t\left[\exp\left(\mu_m - r_t dt - \phi' \Sigma \varepsilon_{t+1} + A_{n-1}^0 + A_{n-1}^r r_{t+1} + \left(A_{n-1}^\pi - 1\right)\pi_{t+1}\right)\right] \\ &= E_t\left[\exp\begin{pmatrix} \frac{1}{2}\phi^{*\prime}\Sigma\rho\Sigma\phi^* - \mu_m + A_{n-1}^0 \\ +A_{n-1}^r\left(1-\rho_r\right)\overline{r} + \left(A_{n-1}^\pi - 1\right)\left(1-\rho_\pi\right)\overline{\pi} \\ +\left(A_{n-1}^r\rho_r - 1\right) r_t + \left(A_{n-1}^\pi - 1\right)\rho_\pi \pi_t \end{pmatrix}\right] \end{aligned}$$

where $\phi^* = \phi + \left[A_{n-1}^r, \left(A_{n-1}^\pi - 1\right), 0\right]'$

Equating (19) and (17), we can obtain the following recursive functions for parameters:

$$A_n^0 = \frac{1}{2}\phi^{*\prime}\Sigma\rho\Sigma\phi^* - \mu_m + A_{n-1}^0 + A_{n-1}^r\left(1-\rho_r\right)\overline{r} + \left(A_{n-1}^\pi - 1\right)\left(1-\rho_\pi\right)\overline{\pi} \tag{20}$$

$$A_n^r = \left(A_{n-1}^r\rho_r - 1\right) \Rightarrow A_n^r = -\frac{\left(1-\rho_r^n\right)}{\left(1-\rho_r\right)} \tag{21}$$

$$A_n^\pi = \left(A_{n-1}^\pi - 1\right) \Rightarrow A_n^\pi = -\frac{\left(1-\rho_\pi^n\right)}{\left(1-\rho_\pi\right)}\rho_\pi \tag{22}$$

Next, it is straightforward to derive the return of this bond from period t to period $t+1$

$$
\begin{aligned}
B_{t+1}^{n-1} &= \ln P_{t+1}^{n-1} - \ln P_t^n \\
&= \left(A_{n-1}^0 - A_n^0\right) + \left(A_{n-1}^r r_{t+1} - A_n^r r_t\right) + \left(A_{n-1}^\pi \pi_{t+1} - A_n^\pi \pi_t\right) \\
&= r_t + \widehat{\pi}_t + \mu_m - \frac{1}{2}\phi^{*\prime}\Sigma\rho\Sigma\phi^* \\
&\quad -\frac{\left(1-\rho_r^{n-1}\right)}{(1-\rho_r)}\sigma_r\varepsilon_{t+1}^r - \frac{\left(1-\rho_\pi^{n-1}\right)}{(1-\rho_\pi)}\rho_\pi\sigma_\pi\varepsilon_{t+1}^\pi
\end{aligned} \tag{23}
$$

where $\widehat{\pi}_t = \overline{\pi} + \rho_\pi\left(\pi_t - \overline{\pi}\right)$;
the unexpected return is:

$$
\left(E_{t+1} - E_t\right) B_{t+1}^{n-1} = -\frac{\left(1-\rho_r^{n-1}\right)}{(1-\rho_r)}\sigma_r\varepsilon_{t+1}^r - \frac{\left(1-\rho_\pi^{n-1}\right)}{(1-\rho_\pi)}\rho_\pi\sigma_\pi\varepsilon_{t+1}^\pi \tag{24}
$$

Q.E.D. ■

Proof of Proposition 2. Since stock pays real dividend, its pricing does not need to invoke the nominal pricing kernel as in the case of nominal bond pricing. Suppose there is a stock which stops paying dividend in n periods (Assuming transversality condition, we can extend $n \to \infty$ to obtain the price of infinitely dividend-paying stocks). The real stock price takes the following form:

$$
P_t^s = \exp\left(a_n^0 + a_n^r r_t + a_n^\delta \delta_t\right) \tag{25}
$$

It must satisfy the following recursive form:

$$
\begin{aligned}
P_t^s &= E_t\left[M_{t+1}\left(P_{t+1}^s + D_{t+1}\right)\right] = E_t\left[M_{t+1}P_{t+1}^s\left(1 + \frac{D_{t+1}}{P_{t+1}^s}\right)\right] \\
&= E_t\left[\exp\left(m_{t+1} + a_{n-1}^0 + a_{n-1}^r r_{t+1} + \left(a_{n-1}^\delta + 1\right)\delta_{t+1}\right)\right] \\
&= E_t\left[\exp\begin{pmatrix} \frac{1}{2}\bar{\phi}'\Sigma\rho\Sigma\bar{\phi} - \mu_m + a_{n-1}^0 \\ +a_{n-1}^r\left(1-\rho_r\right)\overline{r} + \left(a_{n-1}^\delta + 1\right)\left(1-\rho_\delta\right)\overline{\delta} \\ +\left(a_{n-1}^r\rho_r - 1\right)r_t + \left(a_{n-1}^\delta + 1\right)\rho_\delta\delta_t \end{pmatrix}\right]
\end{aligned} \tag{26}
$$

where $\bar{\phi} = \phi + \left[a_{n-1}^r, 0, \left(a_{n-1}^\delta + 1\right)\right]'$;
Equating 25 and 26, we can obtain the following recursive functions for parameters:

$$a_n^0 = \frac{1}{2}\bar{\phi}'\Sigma\rho\Sigma\bar{\phi} - \mu_m + a_{n-1}^0 + a_{n-1}^r(1-\rho_r)\,\overline{r} + \left(a_{n-1}^\delta + 1\right)(1-\rho_\delta)\,\overline{\delta} \tag{27}$$

$$a_n^r = \left(a_{n-1}^r\rho_r - 1\right) \Rightarrow a^r = \lim_{n\to\infty} a_n^r = -\frac{1}{(1-\rho_r)} \tag{28}$$

$$a_n^\delta = \left(a_{n-1}^\delta + 1\right)\rho_\delta \Rightarrow a^\delta = \lim_{n\to\infty} a_n^\delta = \frac{\rho_\delta}{(1-\rho_\delta)} \tag{29}$$

therefore, the nominal stock return is:

$$\begin{aligned} S_{t+1} &= \ln\left(P_{t+1}\exp\left(\delta_{t+1}\right)\right) - \ln P_t + \pi_{t+1} \\ &= \left(a_{n-1}^0 - a_n^0\right) + a^r\left(r_{t+1} - r_t\right) + a^\delta\left(\delta_{t+1} - \delta_t\right) + \delta_{t+1} + \pi_{t+1} \\ &= r_t + \widehat{\pi}_t + \mu_m - \frac{1}{2}\bar{\phi}'\Sigma\rho\Sigma\bar{\phi} \\ &\quad -\frac{1}{(1-\rho_r)}\sigma_r\varepsilon_{t+1}^r + \sigma_\pi\varepsilon_{t+1}^\pi + \frac{1}{(1-\rho_\delta)}\sigma_\delta\varepsilon_{t+1}^\delta \end{aligned} \tag{30}$$

the unexpected stock return is

$$\left(E_{t+1} - E_t\right) S_{t+1}^s = -\frac{1}{(1-\rho_r)}\sigma_r\varepsilon_{t+1}^r + \sigma_\pi\varepsilon_{t+1}^\pi + \frac{1}{(1-\rho_\delta)}\sigma_\delta\varepsilon_{t+1}^\delta \tag{31}$$

Q.E.D. ■

Tables and Figures

Table 1: Sample Statistics of Stock and Bond Returns

Stock returns are calculated using value-weighted broad market stock indices. Bond returns are calculated using long-term government bond indices. Monthly stock and bond returns are from January 1958 to December 2001. Daily stocks returns are from January 1, 1980 to December 31, 2001. Daily bond returns of the U.S., the U.K. and Germany start from 1980. Daily bond returns of France, Japan, Canada and Italy start from 1985, 1984, 1985, and 1991, respectively. Note that sample statistics of daily bond returns and daily stock-bond correlations have different periods. Inflation rates are calculated using monthly CPI indices. All returns are nominal and annualized.

Panel A: Monthly Data (1958-2001)	US	UK	France	Germany	Japan	Canada	Italy
Stock Returns							
Mean (%)	11.88	14.78	12.55	12.15	11.96	11.74	12.62
S.D. (%)	14.58	19.38	19.17	18.97	18.31	16.18	23.43
Bond Returns							
Mean (%)	5.97	8.77	7.67	7.24	8.56	6.52	9.82
S.D. (%)	7.94	9.51	8.46	5.44	9.40	9.91	10.10
Stock-Bond Correlation							
(1958-2001)	0.26	0.31	0.24	0.23	0.13	0.28	0.19
(1980-2001)	0.28	0.39	0.34	0.22	0.11	0.26	0.18
Inflation							
Mean (%)	4.29	6.32	5.11	3.00	4.12	4.39	7.21

Panel B: Daily Data *(1980-2001)*	US	UK	France	Germany	Japan	Canada	Italy
Stock Returns							
Mean (%)	15.19	16.33	16.81	13.72	7.28	11.53	19.50
S.D. (%)	16.45	15.80	18.53	19.26	18.68	15.59	22.30
Bond Returns							
Mean (%)	8.99	11.86	9.49	7.25	6.39	9.71	12.32
S.D. (%)	8.30	8.51	6.47	5.20	5.98	7.41	7.28
Stock-Bond Correlation	0.23	0.26	0.26	0.22	0.06	0.09	0.28
Inflation							
Mean (%)	3.42	4.32	3.59	2.40	1.19	3.55	5.98

Table 2: Autocorrelation of Stock and Bond Returns

Stock returns are calculated using broad market stock indices. Bond returns are calculated using long-term government bond indices. Monthly stock and bond returns are from January, 1958 to December 2001. Daily stocks returns are from January 1, 1980 to December 31, 2001. Daily bond returns of the U.S., the U.K. and Germany start from 1980. Daily bond returns of France, Japan, Canada and Italy start from 1985, 1984, 1985 and 1991 respectively. ρ_i is the autocorrelation coefficient for the $i'th$ lag. Ljung-Box Q-statistic(12) tests for higher order autocorrelation up to 12 lags. The critical values for significance levels 1%, 5%, 10% are 26.22, 21.03, 18.55 respectively.

	US	UK	France	Germany	Japan	Canada	Italy
Panel A: Monthly Stock Return (1958-2001)							
ρ_1	0.012	0.102	0.107	0.110	0.065	0.050	0.085
ρ_2	-0.038	-0.084	-0.041	0.017	0.038	-0.066	-0.013
ρ_3	0.024	0.074	0.050	0.008	0.039	0.072	0.041
$Ljung - Box$ $Q - Stat(12)$	9.181	25.975	16.888	25.580	9.980	16.332	20.301
Panel B: Daily Stock Return (1980-2001)							
ρ_1	0.032	0.062	0.095	0.030	0.062	0.130	0.133
ρ_2	-0.036	-0.024	-0.003	-0.053	-0.057	-0.012	-0.012
ρ_3	-0.033	-0.006	-0.015	0.002	-0.009	0.000	0.016
$Ljung - Box$ $Q - Stat(12)$	30.339	46.874	89.894	43.358	79.001	109.340	140.490
Panel C: Monthly Bond Return (1958-2001)							
ρ_1	0.080	0.173	-0.010	0.267	0.222	0.136	0.276
ρ_2	-0.015	0.010	-0.030	0.061	0.048	0.018	0.030
ρ_3	-0.087	-0.064	0.091	0.047	0.053	0.023	0.112
$Ljung - Box$ $Q - Stat(12)$	19.382	32.115	21.536	54.601	45.446	31.839	82.874
Panel D: Daily Bond Return (1980 *-2001)							
ρ_1	0.000	0.054	0.036	0.021	0.079	0.028	0.028
ρ_2	-0.004	0.024	0.012	-0.032	0.010	0.005	0.000
ρ_3	0.003	-0.016	0.011	-0.001	-0.008	0.010	0.000
$Ljung - Box$ $Q - Stat(12)$	0.8474	45.217	18.707	33.213	66.13	37.890	11.235

Table 3: Correlation of Different Measures of Expected Inflation and Real Interest Rate Uncertainty

The upper panel shows the correlations of two uncertainty measures of expected inflation, EXPINF_L and EXPINF_S. The lower panel shows the correlations of the two uncertainty measures of the real interest rate, RINT_L and RINT_S.

	US	UK	France	Germany	Japan	Canada	Italy
Correlation of EXPINF_L and EXPINF_S	0.467	0.676	0.468	0.376	-0.036	0.386	0.570
Correlation of RINT_L and RINT_S	0.598	0.362	0.092	0.534	0.446	0.603	0.204

Table 4: Regression of stock-bond correlation (1980-2001, monthly)

$$YCorr_t = \beta_0 + \beta_c \cdot YCorr_{t-1} + \beta_s \cdot STQ_{t-1} + \beta_e \cdot EXPINF_t + \beta_r \cdot RINT_t + \beta_u \cdot UNINFN_t + \varepsilon_t$$

$YCorr$ is the Fisher transformation of the stock-bond correlation. STQ is the uncertainty about the stock unique component. EXPINF_L and EXPINF_S are the uncertainty about long-term and short-term expected inflation respectively. RINT_L and RINT_S are the long-term and short-term uncertainty measures of the real interest rate respectively. UNINF is the uncertainty about unexpected inflation. Stock-bond correlations are computed using daily stock and bond returns within each month. All variables are monthly. See Section 3 for the definitions of these terms. The table presents the results of both U.S. regression and G7 panel regression. Panel constants are not shown here. T-statistics are in the parentheses and are corrected for serial correlation and heteroscedasticity.

	Constant	$Ycorr_{t-1}$	STQ	EXPINF_L	EXPINF_S	RINT_L	RINT_S	UNINF	R^2	DW-Stat
US	0.214	0.406	-11.838	1.755					0.282	2.207
	(3.039)	(7.636)	(-2.138)	(2.843)						
Panel		0.361	-11.623	2.666					0.257	2.186
of G7		(13.901)	(-5.960)	(7.015)						
US	0.278	0.431	-12.822		0.722				0.267	2.240
	(4.293)	(7.484)	(-2.191)		(1.077)					
Panel		0.394	-13.413		1.218				0.237	2.231
of G7		(15.392)	(-6.542)		(3.300)					
US	0.284	0.417	-13.843			12.244			0.276	2.233
	(4.715)	(7.165)	(-2.345)			(2.597)				
Panel		0.398	-13.470			4.186			0.235	2.234
of G7		(15.445)	(-6.533)			(2.757)				
US	0.265	0.437	-12.708				4.592		0.265	2.247
	(2.944)	(7.360)	(-2.127)				(0.559)			
Panel		0.401	-13.730				3.231		0.242	2.223
of G7		(15.458)	(-6.579)				(1.425)			
US	0.196	0.436	-13.243					4.725	0.266	2.243
	(1.671)	(7.285)	(-2.057)					(1.041)		
Panel		0.400	-13.795					1.716	0.237	2.227
of G7		(15.099)	(-6.559)					(1.500)		
US	0.205	0.404	-12.492	1.452		3.703		0.854	0.285	2.205
	(1.661)	(7.494)	(-2.152)	(1.811)		(0.517)		(0.180)		
Panel		0.360	-11.655	2.623		0.201		0.711	0.257	2.185
of G7		(13.756)	(-5.947)	(6.275)		(0.111)		(0.631)		
US	0.192	0.428	-13.541		0.885		-7.063	6.143	0.269	2.242
	(1.586)	(7.395)	(-2.099)		(1.003)		(-0.670)	(1.351)		
Panel		0.393	-13.449		1.141		0.095	0.937	0.246	2.215
of G7		(15.147)	(-6.513)		(2.762)		(0.037)	(0.746)		

Table 5: Stock-Bond Correlation and Business Cycle

$YCorr_t = \beta_0 + \beta_c \cdot YCorr_{t-1} + \beta_s \cdot STQ_{t-1} + \beta_y \cdot CYCLE_t + \beta_e \cdot EXPINF_t + \beta_r \cdot RINT_t + \beta_u \cdot UNINF_t + \varepsilon_t$.

$YCorr$ is the Fisher transformation of the stock-bond correlation. STQ is the uncertainty about the stock unique component. CYCLE is the dummy variable for business cycle (1 for expansion and 0 for recession). EXPINF_L and EXPINF_S are the uncertainty about long-term and short-term expected inflation respectively. RINT_L and RINT_S are the long-term and short-term uncertainty measures of the real interest rate respectively. UNINF is the uncertainty about unexpected inflation. Stock-bond correlations are computed using daily stock and bond returns within each month. All variables are monthly. See Section 3 for the definitions of these terms. The table presents the results of both U.S. regression and G7 panel regression. Panel constants are not shown here. T-statistics are in the parentheses and are corrected for serial correlation and heteroscedasticity.

	Constant	$Ycorr_{t-1}$	STQ	CYCLE	EXPINF_L	RINT_L	UNINFN	R^2	DW-Stat
(1980-2001)									
US	0.305	0.438	-12.550	-0.004				0.265	2.246
	(3.710)	(7.359)	(-2.112)	(-0.072)					
Panel		0.403	-13.766	-0.005				0.232	2.239
of G7		(15.553)	(-6.591)	(-0.250)					
US	0.098	0.400	-11.968	0.061	1.668	3.220	2.665	0.286	2.203
	(0.641)	(7.486)	(-2.147)	(1.107)	(2.026)	(0.452)	(0.569)		
Panel		0.358	-11.577	0.040	2.682	0.884	1.269	0.258	2.184
of G7		(13.690)	(-5.921)	(1.678)	(6.326)	(0.479)	(1.068)		

Table 6: Contemporaneous Effect of Stock and Bond Returns

This table presents the estimation results of the following model in Section 4.2

$$\begin{aligned} S_{t+1} &= \alpha_0 + \alpha_S \cdot S_t + (\alpha' \cdot UCTY_{t+1}) \cdot B_{t+1} + \varepsilon_{t+1} \\ &= \alpha_0 + \alpha_S \cdot S_t + (\alpha_B + \alpha_e \cdot EXPINF_{t+1} + \alpha_r \cdot RINT_{t+1} + \alpha_u \cdot UNINF_{t+1}) \cdot B_{t+1} + \varepsilon_{t+1} \end{aligned}$$

$$Var_t\left(\varepsilon_{t+1}^2\right) = h_t \;\; and \;\; h_t = \omega + \eta \varepsilon_t^2 + \gamma h_{t-1}$$

EXPINF, RINT and UNINF are the uncertainty about expected inflation, the real interest rate, and unexpected inflation respectively. See Section 3 for the definitions of these terms. All data are monthly. Panels A and Panel B cover different time periods. The first two columns of both panels use EXPINF_L and RINT_L; the other two columns use EXPINF_S and RINT_S. α_e_L and α_r_L denote the coefficients for EXPINF_L and RINT_L respectively. α_e_S and α_r_S denote the coefficients for EXPINF_S and RINT_S respectively. $\alpha' \cdot \overline{UCTY}$ measures the average response of stock returns to the change in bond returns. The model is estimated using Maximum Likelihood Method (ML) jointly with all G7. The constant term α_0 is allowed to differ across countries. The estimates of the constant term are not shown below.

	Panel A:	1980-2001			Panel B:	1961-2001		
α_S	0.049	(1.960)	0.051	(2.021)	0.069	(4.092)	0.070	(3.781)
α_B	0.369	(2.321)	0.611	(4.151)	0.461	(5.653)	0.686	(9.125)
α_e_L	2.332	(1.839)			4.108	(5.796)		
α_e_S			-1.108	(-0.974)			1.674	(2.395)
α_r_L	2.812	(0.499)			2.046	(1.016)		
α_r_S			-4.907	(-0.499)			1.540	(4.115)
α_u	1.722	(0.450)	4.796	(0.967)	-4.664	(-2.706)	-7.162	(-8.842)
ω	0.000	(2.626)	0.000	(2.663)	0.000	(2.953)	0.000	(11.003)
η	0.096	(4.816)	0.098	(4.971)	0.105	(4.776)	0.104	(8.717)
γ	0.879	(34.211)	0.878	(35.848)	0.873	(33.979)	0.874	(89.883)
$\alpha' \cdot \overline{UCTY}$	0.581		0.653		0.497		0.529	
Log-Likelihood	2810.800		2809.200		5460.300		5449.400	

Table 7: Conditional Correlation of Stock and Bond Returns

This table presents the estimations results of the bi-variate GARCH model in Section 4.3. Readers are referred to the text for details of this model. The conditional correlation of stock and bond returns, $\rho_t^{S,B}$, is determined by the uncertainty of macroeconomic factors, $UCTY_t$, through the Fisher transformation of correlation.

$$\rho_t^{S,B} = \frac{\exp(2A' \cdot UCTY_t) - 1}{\exp(2A' \cdot UCTY_t) + 1}$$

$A' \cdot UCTY_t$ denotes the linear combination of the uncertainty of macroeconomic factors.

$$A' \cdot UCTY_t = A_0 + A_e \cdot EXPINF_{t+1} + A_r \cdot RINT_{t+1} + A_u \cdot UNINF_{t+1}$$

EXPINF, RINT and UNINF are the uncertainty about expected inflation, the real interest rate, and unexpected inflation respectively. See Section 3 for the definitions of these terms. All data are monthly. Only the estimation results using long term measures, EXPINF_L and RINT_L, are shown here because those using short-term measures, EXPINF_S and RINT_S ,fail to converge to a stable global maximum. A_e_L and A_r_L denote the coefficients for EXPINF_L and RINT_L respectively. The model is estimated using Maximum Likelihood Method (ML) jointly with all G7 data. The constant terms in the mean equations are allowed to differ across countries. The estimates of the constant terms are omitted to keep the table concise.

	Panel A:	1980-2001	Panel B:	1961-2001
β_{SS}	0.001	(0.045)	0.047	(1.083)
β_{SB}	0.151	(2.982)	0.139	(3.717)
β_{BS}	-0.064	(-7.565)	-0.022	(-0.819)
β_{BB}	0.156	(6.352)	0.194	(9.919)
ω_S	0.000	(3.387)	0.000	(0.390)
η_S	0.075	(4.955)	0.089	(2.347)
γ_S	0.903	(50.020)	0.895	(9.536)
ω_B	0.000	(9.581)	0.000	(1.009)
η_B	0.103	(5.364)	0.175	(4.582)
γ_B	0.873	(52.881)	0.825	(21.614)
A_0	0.118	(2.626)	0.120	(4.061)
A_e_L	1.305	(2.220)	1.579	(4.620)
A_r_L	10.070	(2.757)	8.444	(4.643)
A_u	-9.927	(-1.084)	-6.807	(-3.655)
Log-Likelihood	7166.4		13726	

Table 8: Economic Value of Timing Stock-Bond Correlations

This table compares the performance of the MA strategy and the dynamic strategy during 1991-2001 using U.S. data. Both strategies rebalance the portfolios at the end of each month. Expected returns and volatility of stock and bond returns are assumed to be constant and equal to U.S. values during this period. The expected stock and bond return are 15.01% and 7.26% respectively. Their standard deviations are 15.61% and 6.63%. The annualized mean, standard deviation, and the Sharpe Ratio of both strategies are show below. Certainty Equivalence is the maximum amount that a dynamic strategy investor would like to pay for holding it rather than a MA strategy. The investors are assumed to have a power utility function and γ is the constant relative risk aversion coefficient. The last column shows the bootstrapped P-values for the test, $H_0 : CE \leq 0$.

	MA Strategy			Dynamic Strategy			Certainty	P-Value
	Mean (%)	Std (%)	Sharpe Ratio	Mean (%)	Std (%)	Sharpe Ratio	Equivalence (%)	
$\gamma = 5$								
3%	18.86	17.26	0.92	20.92	18.19	0.99	0.59	0.206
4%	16.24	15.48	0.79	18.03	16.01	0.88	1.06	0.067
4.5%	15.28	14.74	0.73	16.95	15.02	0.83	1.31	0.024
5%	14.55	14.13	0.68	16.10	14.12	0.79	1.58	0.011
6%	13.79	13.33	0.58	15.12	12.65	0.72	2.14	0.001
$\gamma = 10$								
3%	10.94	8.63	0.92	11.97	9.10	0.99	0.25	0.245
4%	10.13	7.74	0.79	11.03	8.00	0.88	0.51	0.073
4.5%	9.91	7.37	0.73	10.74	7.51	0.83	0.64	0.026
5%	9.79	7.06	0.68	10.57	7.06	0.79	0.79	0.012
6%	9.92	6.66	0.59	10.58	6.32	0.72	1.09	0.001
$\gamma = 15$								
3%	8.30	5.75	0.92	8.99	6.06	0.99	0.16	0.254
4%	8.10	5.16	0.79	8.70	5.34	0.88	0.33	0.076
4.5%	8.12	4.91	0.74	8.67	5.01	0.83	0.43	0.027
5%	8.21	4.71	0.68	8.73	4.70	0.79	0.52	0.011
6%	8.63	4.44	0.59	9.07	4.22	0.73	0.73	0.000

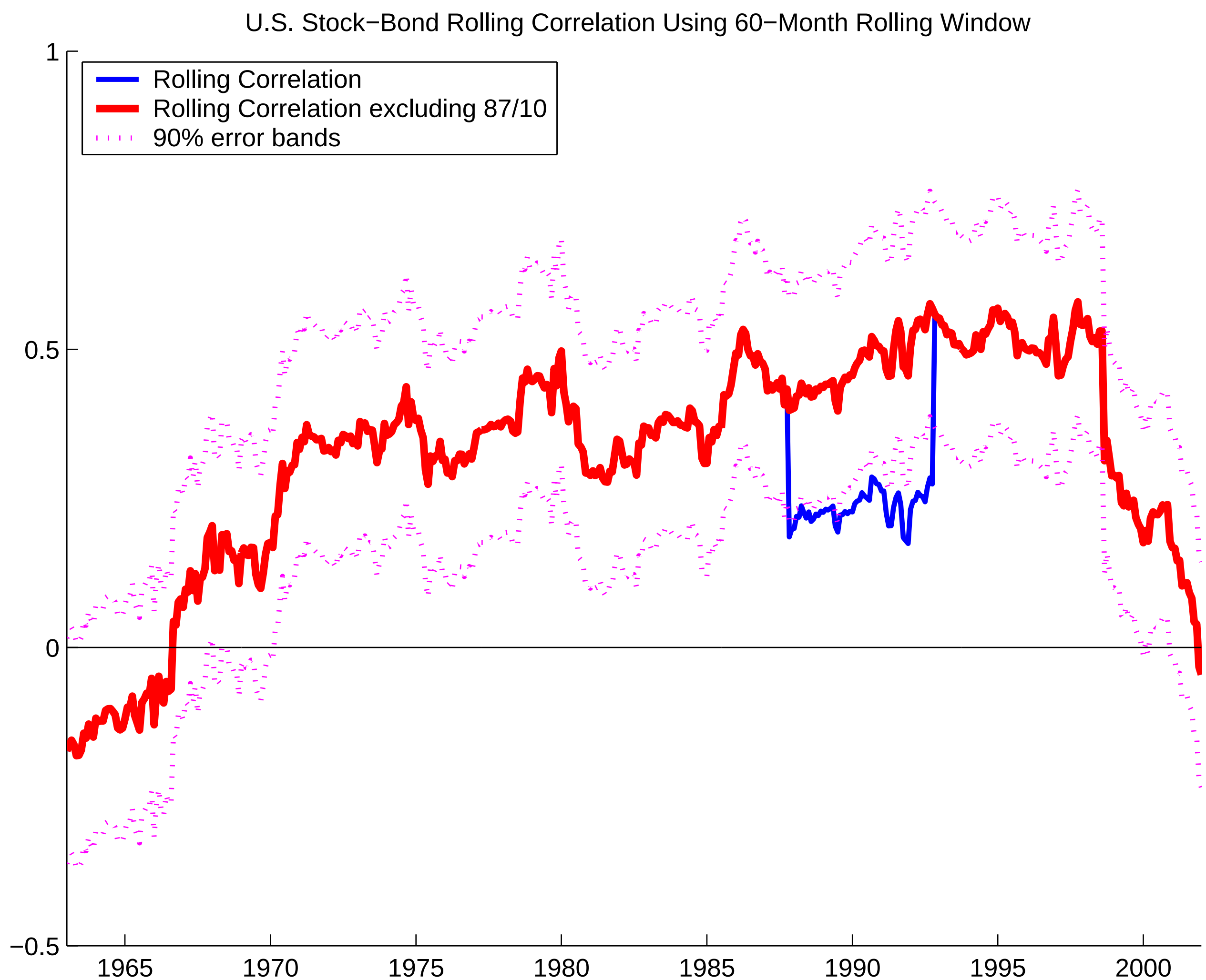

Figure 1: U.S. Rolling Stock-Bond Correlations, 1961-2001

This figure shows U.S. rolling stock-bond correlations using 60-month rolling window. The thick line is the rolling correlation excluding October 1987. The two dashed lines are the 90% upper and lower error bands. The thin line includes October 1987. Stock returns are calculated using the broad market total return indices. Bond returns are calculated using the long-term government bond total return indices. All returns are monthly, from January 1958 to December 2001.

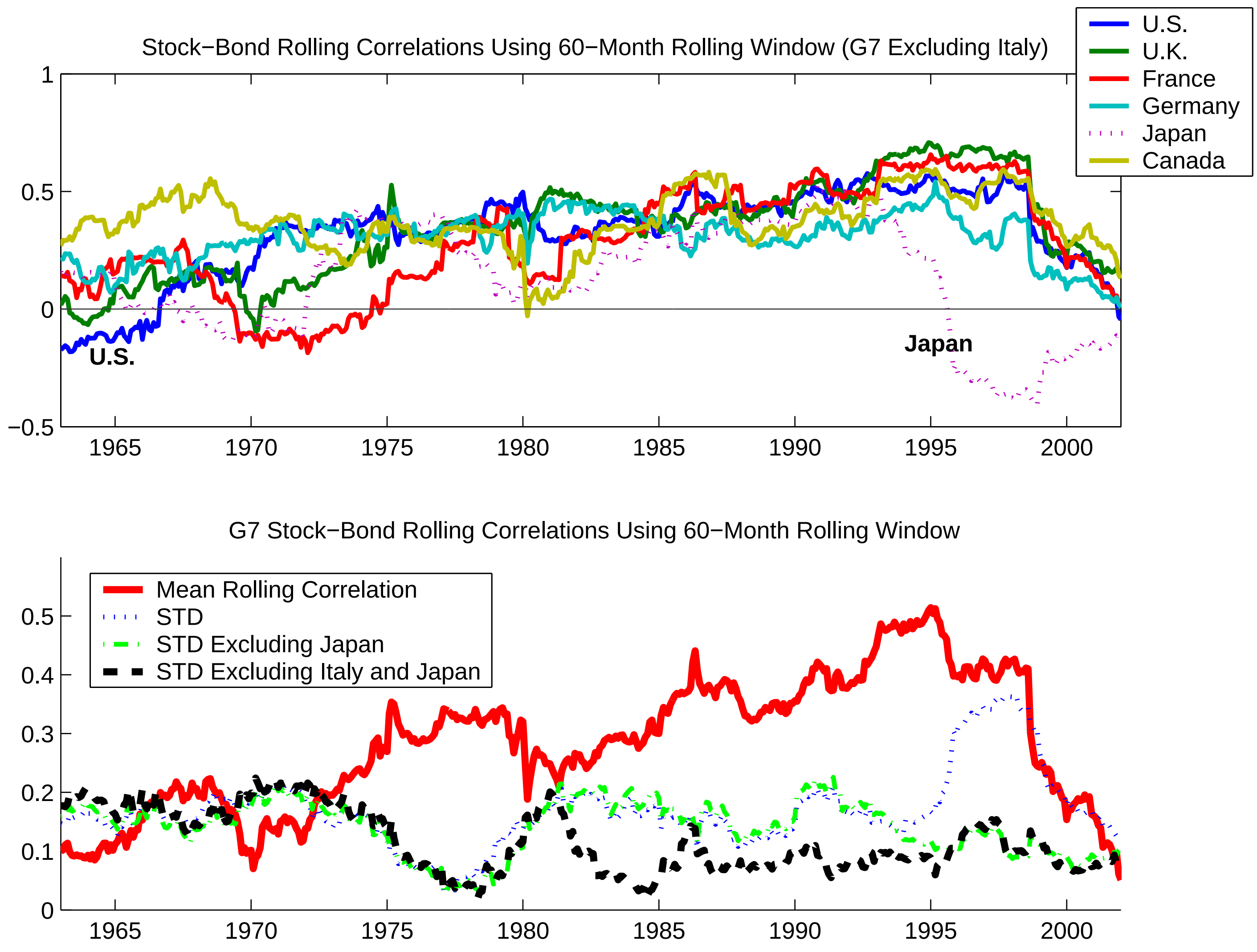

Figure 2: Rolling Stock-Bond Correlations of G7 Countries, 1961-2001
The upper panel shows the rolling stock-bond correlations of G6 countries (Italy is excluded to keep the figure readable). The data of October 1987 is also excluded for the same reason. The lower panel shows the cross-sectional mean and standard deviation of the G7 stock-bond correlations. All returns are monthly, from January 1958 to December 2001.

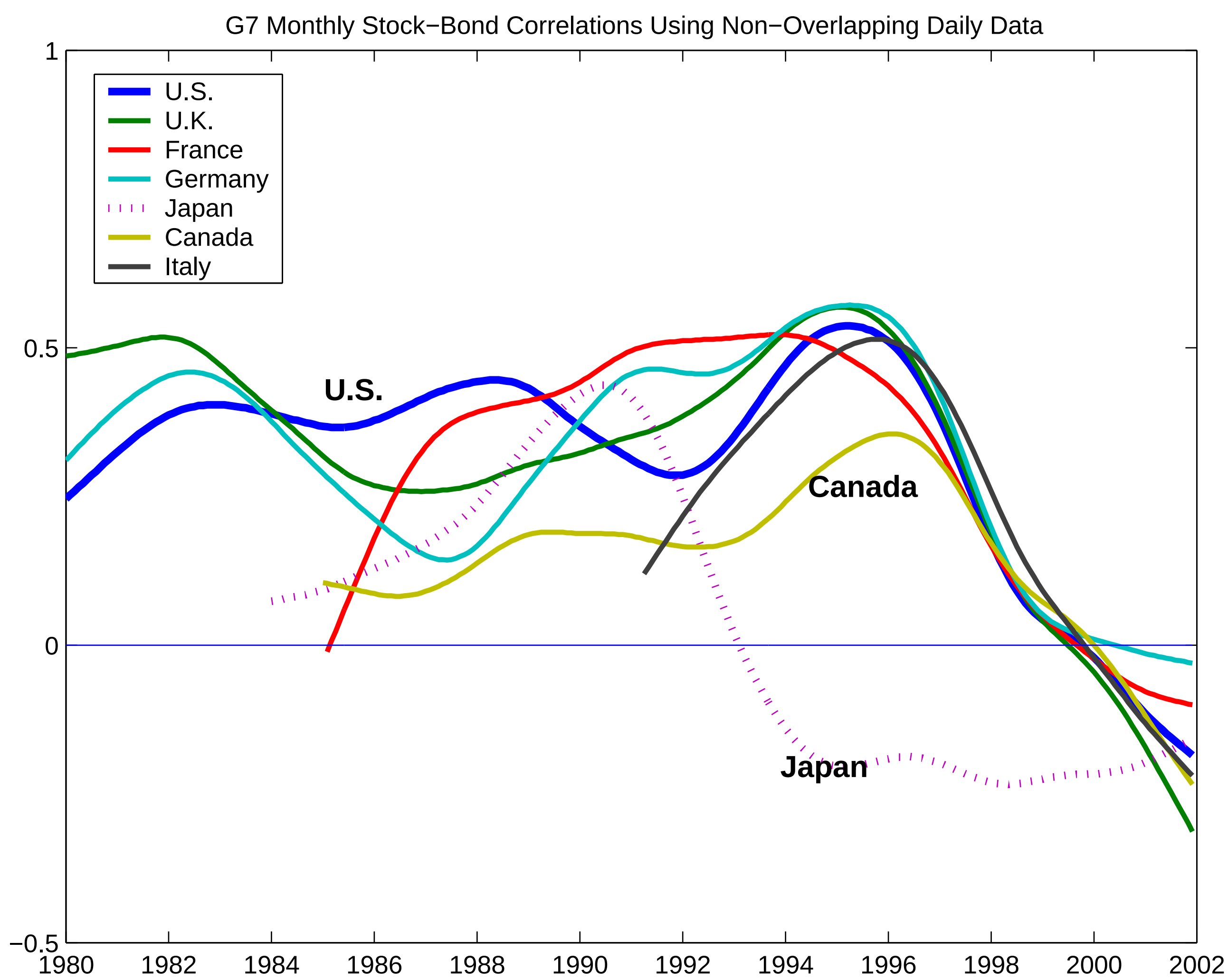

Figure 3: Non-Overlapping Stock-Bond Correlations Using Daily Data, 1980-2001

The figure shows the non-overlapping monthly stock-bond correlations of G7 countries. Monthly correlations are calculated using daily stock and bond returns within each month. All graphs are smoothed by Hodrick-Prescott filter with a smoothing parameter of 14400. Daily stock and bond returns are available only from or after January 1980.

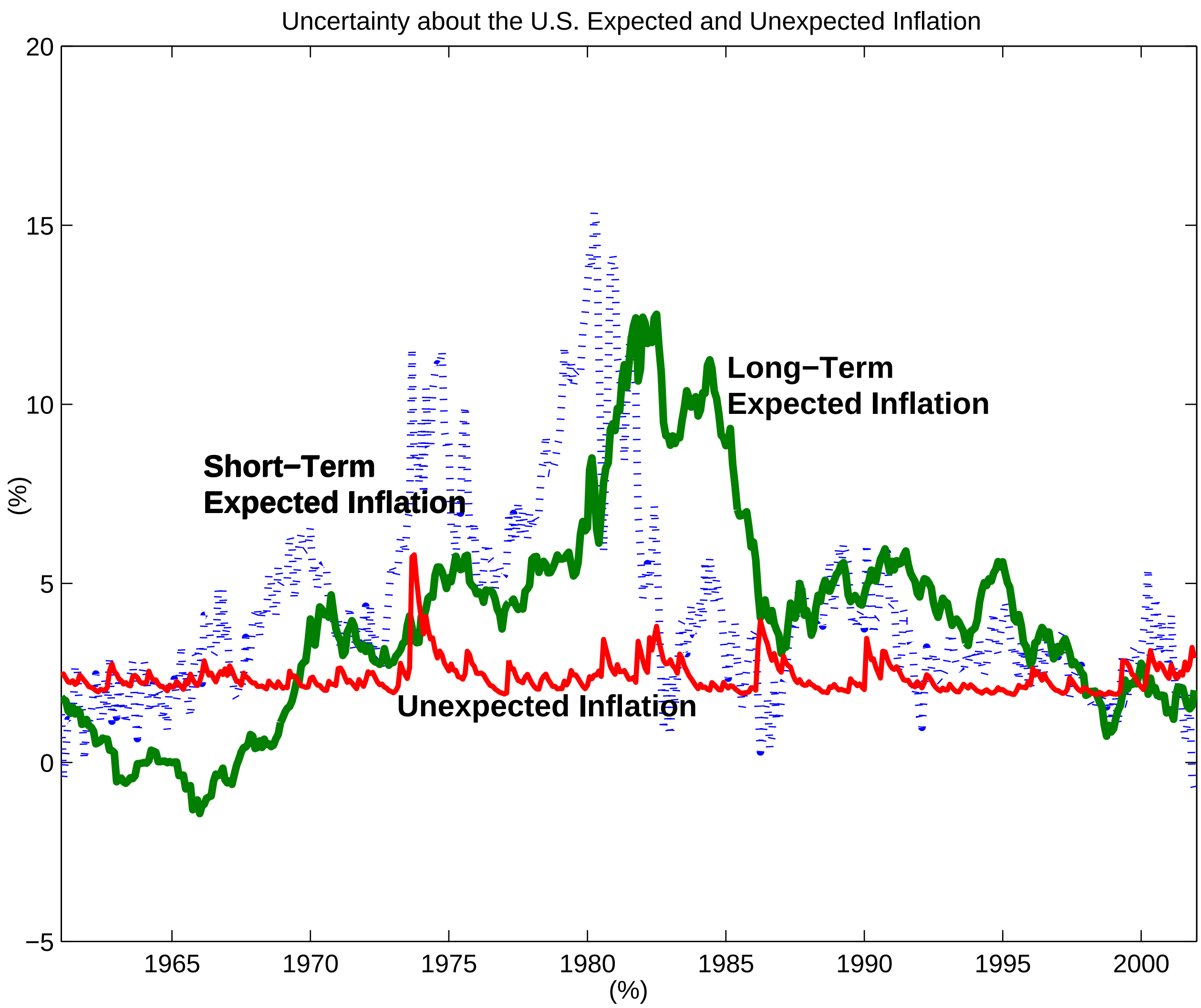

Figure 4: Uncertainty about the U.S. Expected and Unexpected Inflation
Long-term expected inflation is the difference between the long-term government bond yields and the 5-year moving average of real GDP growth rates. Short-term expected inflation is the one-month ahead forecast of a rolling BVAR system of three variables: log-difference of CPI, log-difference of IP, and short-term government bond yield. Unexpected inflation is the forecast error of BVAR system. The levels of the short-term and long-term expected inflation are used as their measures for uncertainty. The uncertainty about unexpected inflation is measured as the conditional volatility generated by a GARCH(1,1) model. All data are monthly, from January 1961 to December 2001.

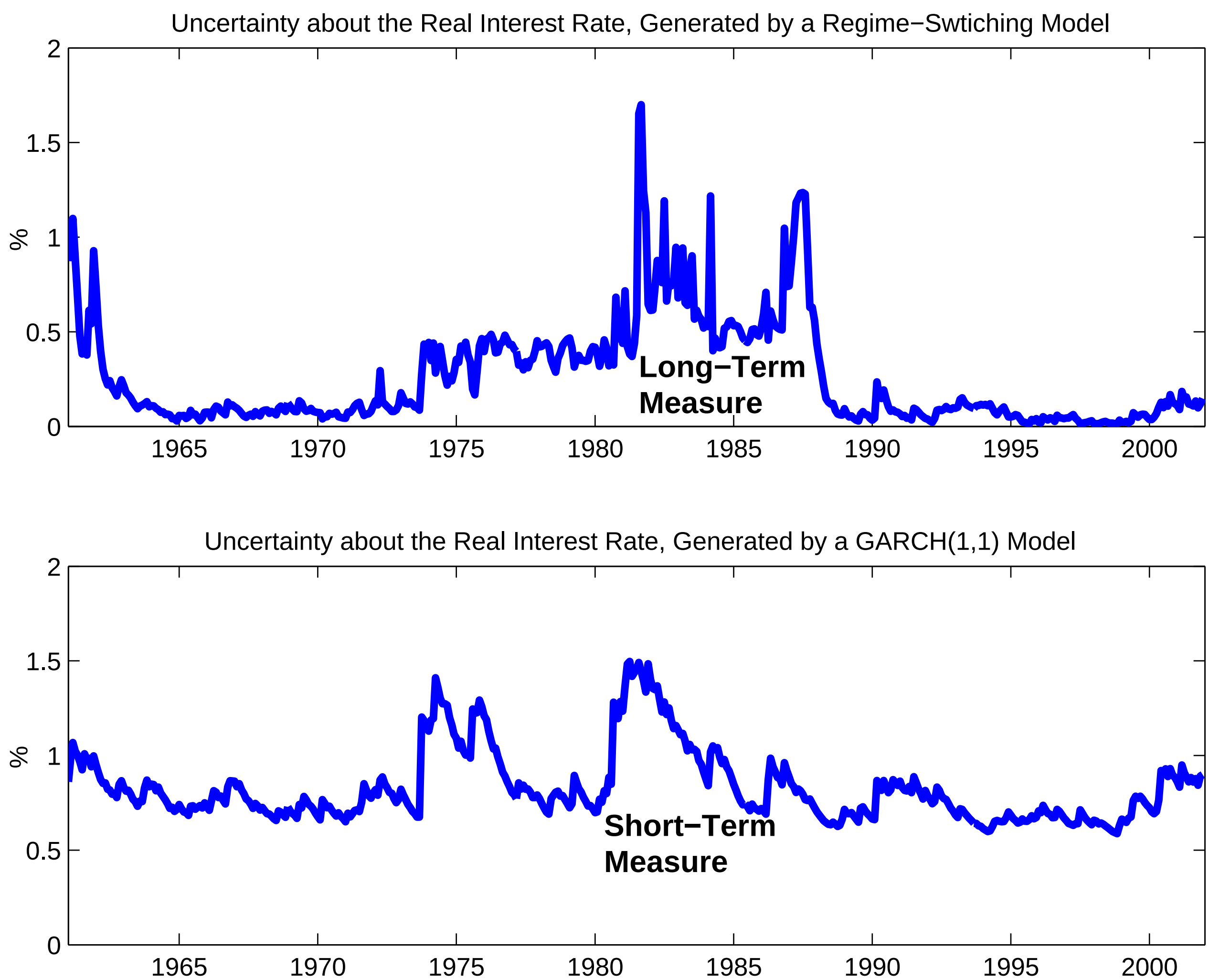

Figure 5: The Uncertainty about the U.S. Real Interest Rate

The upper panel shows the long-term uncertainty about the real interest rate, generated as the root mean squared error (RMSE) of a 3-state Markov regime-switching model. The lower panel shows the short-term uncertainty of the real interest rate, measured as the conditional volatility generated by a GARCH(1,1) model. All data are monthly, from January 1961 to December 2001.

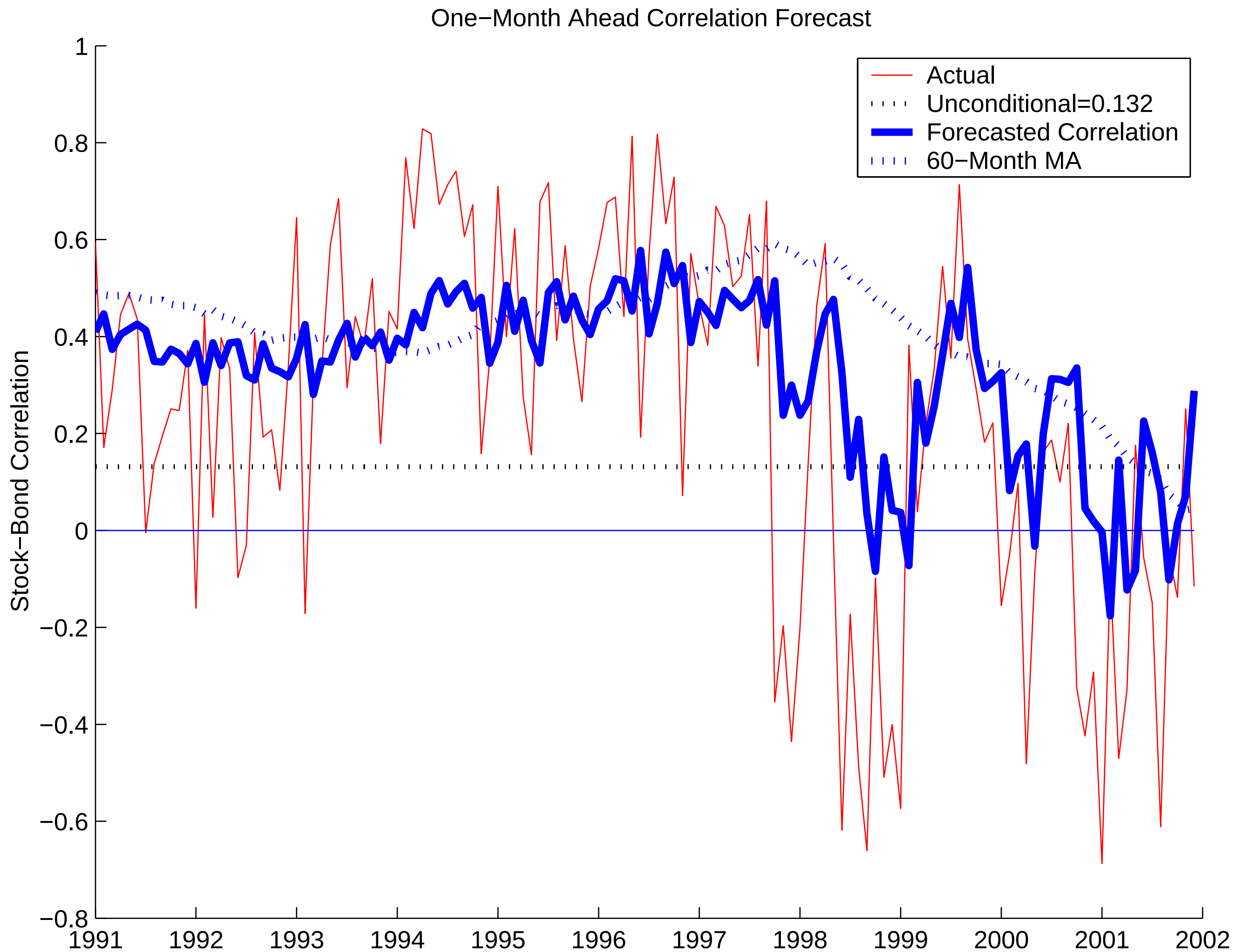

Figure 6: Forecasting U.S. Stock-bond Correlations

The thin line is the actual U.S. stock-bond correlation computed using daily stock and bond returns within each month. The thick line is the one-month forecast using the forecasting equation (15). The dashed line is the 60-month moving average of past correlations. The unconditional stock-bond correlation during the 1991-2001 period is 0.132.

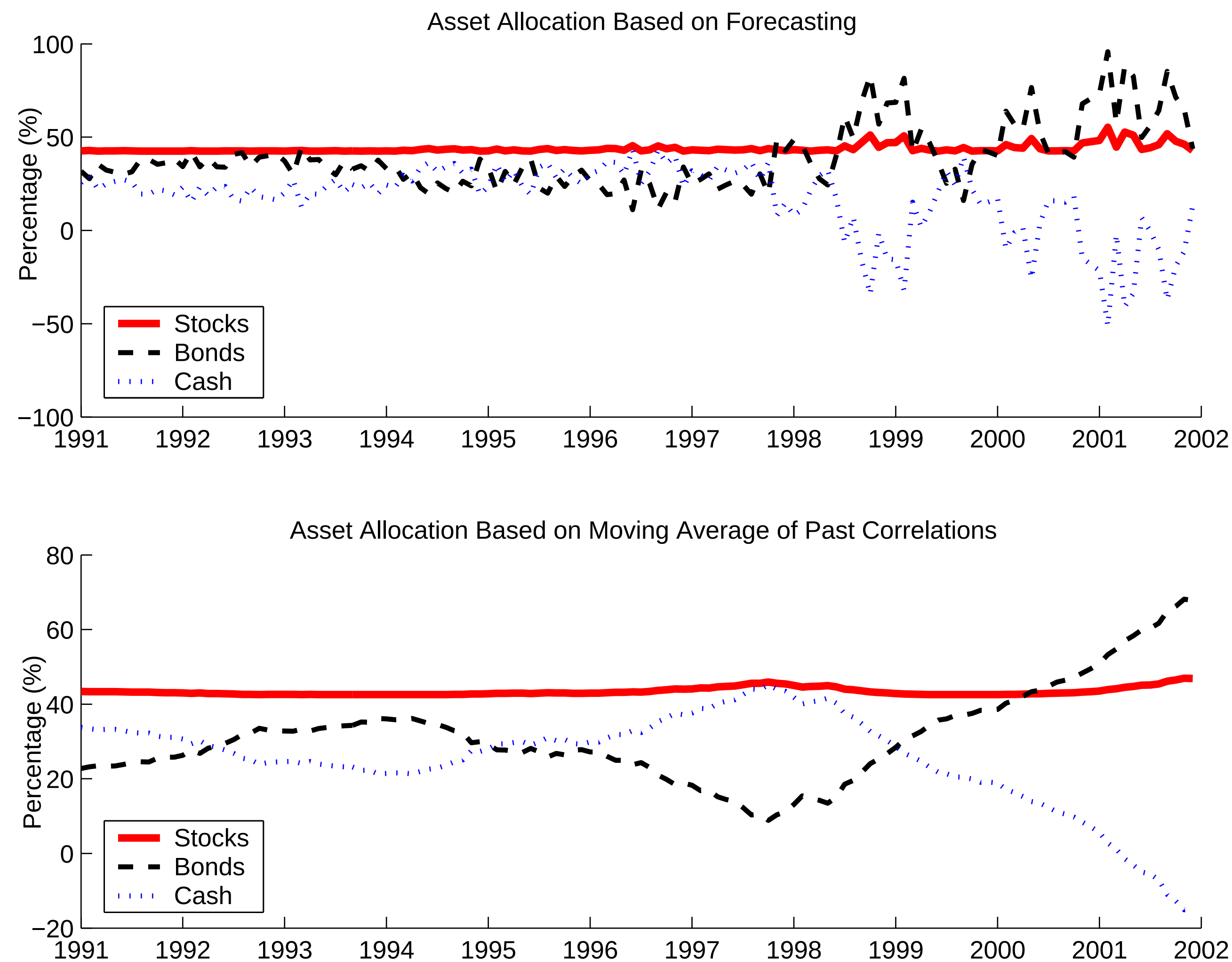

Figure 7: Asset Allocation among Stocks, Bonds and Cash

The upper panel shows the optimal asset allocation of a dynamic strategy investor. The lower panel shows the optimal asset allocation of a MA strategy investor. Both investors are assumed to have a CRRA utility function with a risk aversion coefficient of 10. The interest rate is assumed be constant and equal to 4.5%, the average U.S. T-Bill yields between 1991 and 2001