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Knowing What Others Know: Common Knowledge, Accounting, and Capital Markets

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Knowing What Others Know: Common Knowledge, Accounting and Capital Markets¹

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Abstract

The concept of common knowledge concerning higher orders of knowledge has seen exciting new developments in the fields of philosophy, game theory, statistics, economics and cognitive science in the recent decades. Even though information lies at the heart of accounting and capital markets research, these new developments have had only a faint echo in these fields. Common knowledge thinking may significantly advance our understanding of financial reporting, analysis, securities valuation, managerial control, auditing and information systems. Such accounting and business applications will also make important contributions in the form of concrete real life examples and applications to the basic fields where the idea of common knowledge originated. This paper is an overview of common knowledge and its actual and potential applications to accounting and capital markets research.

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1. Introduction

Emperor's Clothes

In Hans Christian Andersen's fable "The Emperor's New Clothes" two scoundrels convinced a vain emperor that they could make a special gorgeous-looking cloth of silk and gold threads that is invisible only to the incompetent and the stupid. The emperor gave them money and materials to make the royal garments, and they dressed him in nothing at all. No one at the court, including the emperor himself, dared admit that they could not see the clothes for the fear of being branded stupid and incompetent. When the emperor went out in a parade to show off his new "clothes" to the public, there was great applause and praise for how splendid his new outfit was. Then a child cried: the emperor has no clothes at all. After a moment of stunned silence, so did everyone else.

What difference did the child's cry make? Apparently none. Everybody saw after the cry exactly what he or she saw before—the same emperor in the same "invisible clothes"—seen with the same eyes. But not quite; something had changed. What, why, and how?

Let us explore answers to such questions. Is this phenomenon real, or just idle imagination? If it is real, how important is it in the world of accounting and business? Are there interesting accounting, capital markets and business phenomena for which the fable might hold some valuable lessons?

The Stock Market

Before returning to our emperor and his magnificent clothes, let us look at two familiar stories from the world of business. John Maynard Keynes' classic description of the stock market comes to mind first. He wrote:

Professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which the average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligence to anticipating what the average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees (Keynes, 1936, Chapter 12).

The LIFO Method of Accounting

A second story from the world of business concerns one of the perennial controversies of post World War II accounting in the U.S. For over half a century now, U.S. tax law has permitted business firms to use LIFO method of inventory valuation for tax purposes provided that the same method is also used for preparation of public financial reports . During periods of inflation, this method of accounting can save substantial amount of cash by delaying tax payments to the government (Sunder 1976a,b). In spite of significantly positive rates of inflation for more than forty years after World War II, large numbers of firms in industries with significant inventory inflation failed to adopt LIFO accounting. When surveyed about their reasons for reluctance to adopt LIFO, corporate financial officers express their apprehension that the adoption of LIFO, and the consequent rise in cash flows and fall in earnings, may cause a drop in stock prices. Yet, empirical studies fail to reveal any such drop. In fact most empirical studies show that the stock prices of firms who adopt LIFO rise (Sunder, 1973, 1975). How do we reconcile these facts?

Beliefs about Others

What is common to the stories about the emperor's clothes, stock markets, and LIFO accounting? In all three, what we believe about others and about their beliefs plays a crucial role. Since people had been led to believe that the emperor's clothes will be invisible only to the stupid and the incompetent, even as they saw the naked emperor, each could hide the supposed stupidity and incompetence of themselves by pretending to see the clothes, and joining in the general applause. Under the maintained hypothesis about the link between the visibility of clothes and the wisdom of the beholder, louder

applause became evidence of one's own stupidity. Fortunately, this evidence remained private, and could be concealed from others by joining in the applause. It is possible that everyone in the crowd may have been privately convinced of his own stupidity and incompetence, and cheered loudly to deny it publicly.

But there is a second possibility. Perhaps the cheering masses included some who did not believe that they were stupid and incompetent because they could not see any clothes on the emperor's body. Even such people would have known that if they said that the emperor had no clothes, others around them would falsely regard them as stupid and incompetent. They found it wise to keep their doubts to themselves. Note that such people did not have to be few, or in a minority. In fact the results would have been the same even if no one of the crowd believed in the maintained hypothesis. All we need to understand what they did is that the people believed that others around them believed in the maintained hypothesis. This line of argument can be extended to higher levels of beliefs without limit.

Fortunately, there was this child in the crowd, too young to know or understand the maintained hypothesis—and blurted out what he saw. Now comes the crucial link in the chain of events. Had the child simply been innocent of the maintained hypothesis, his words would have been ignored by the others. At worst, those who heard the child would have concluded that he was a late bloomer.

The child was not only innocent, others in the crowd knew children, in general, to be innocent. This knowledge of children's innocence made all the difference because his words could not be ignored. If innocent observation matched the observation of the adult, it raised doubts about the maintained hypothesis in the minds of the adults. The

maintained hypothesis was a stretch to begin with. Self-esteem guards against negative conclusions about one's own wisdom. Combine this with the most unusual spectacle of the emperor parading in the buff and an innocent voice that said so, the weight of evidence turned against the maintained hypothesis. And each such doubt reinforced the doubt in others. Child's innocence, people's belief in child's innocence, and people's belief in the other peoples' belief in his innocence all played a role in revelation of the truth.

The higher the orders of beliefs on which the applause was based, the easier it would have been for the child's remark to change their minds. As the house of cards gets taller, it takes an ever so slight a nudge to bring it down.

What about the stock market? John Maynard Keynes, besides being one of the best-known economists of the twentieth century, was also a famous and successful stock market speculator. He sets up this hierarchy of decision rules, and suggests that as we go down the list, the decision rules become better at describing how the stock market works. We can examine the logical validity of his argument by asking ourselves the following question. Suppose the price of Microsoft shares today is \$100. I believe that, given the future plans and prospects of Microsoft, its fundamental value six months from now will be \$125. Should I buy the shares now? If my cost of capital is less than \$25 for this six month period, the answer is yes.

Now let us go to Keynes' second level of analysis. Suppose that I also believe now, in addition to the above, that six months from now, people will believe the fundamental value of Microsoft to be \$90. Is it still a good idea to buy the shares at \$100 now? Not really because in this case, we should expect to incur a loss of \$10 by acting on

our own beliefs about the fundamental value. Unless we can influence those beliefs to coincide with our own, the market will follow the beliefs of the vast majority. Rule 1 for investment looks good only until we look at Rule 2. Similarly, Rule 2 is dominated by Rule 3, and so on.

In the stock market, as in the fable of the emperor, our beliefs about others play a key role. Why were so many people so jittery whenever Mr. Greenspan (the Chairman of the Board of Governors of the Federal Reserve Bank of the United States) uttered the phrase “irrational exuberance” for the first time? Perhaps many people are not quite sure of what others think about what others think of which face is the prettiest. If they believe that others in the market will pay attention to what Mr. Greenspan thinks, they cannot afford to ignore their words. There is nothing irrational about that.

Now let us return to our accounting example of LIFO method of inventory valuation. Remember that in designing a business organization we must solve the agency problem of inducing managers to take actions that will maximize shareholder value. We try to solve this problem by linking managerial compensation to shareholder value. But shareholder value reported by the manager is subject to self-interested manipulation. Therefore, we replace accounting values by stock market values, which are, one would hope, less subject to managerial manipulation. In an efficient stock market, this solution to the agency problem should work reasonably well.

So, let us consider a manager who has been induced, through such a carefully designed compensation scheme, to try to maximize shareholder value as measured by stock prices. What should a manager do if he knows that the adoption of LIFO accounting will save cash for the firm but will lower its reported income? If the objective

of the manager is to maximize the present value of firm's cash flows, there is no question about what the right course of action for the manager is. Just as in the case of Keynes' stock market example, investment decision should be made on the basis of the fundamental value of the firm under Rule 1. Here the manager should adopt LIFO if it increases the net present value of cash flows. But there is another layer to the story.

Our shareholders, recognizing the difficulty of observing either managers' actions or future cash flows have linked the manager's compensation to stock prices. Such a manager must consider not only the direct effect of his actions on the cash flows of the firm (i.e. on the fundamental value of the firm), but also on the stock prices. If the stock prices are the fundamental value (Keynes Rule 1), we have no problem, and we should expect such a manager to adopt LIFO.

However, suppose the stock prices are determined by Rule 2 instead. Now the manager must also consider how the investors may react to his LIFO decision. If he believes the investors to be the followers of Rule 1 (fundamental value), he should adopt LIFO. But what if the manager has some doubts, and believes that some or all investors use accounting income as the basis of stock price valuation. In such a case, a manager may well decide that his own interests will be best served (stock prices will be maximized) if he does not adopt the income reducing LIFO decision. Amershi and Sunder (1987) introduced the common knowledge consequences to accounting by proving that once we allow for the possibility that the managers' beliefs about how shareholders value shares are in error, an efficient stock market fails to discipline the managers from making wrong decisions. Managers may rationally fail to adopt LIFO

knowing fully well that LIFO saves cash and is in the best interests of the shareholders. Their beliefs about how investors act is a crucial part of this apparent paradox.

To summarize, emperor's clothes, stock market behavior and LIFO adoptions are but three examples of problems that can arise when we consider *common knowledge*.

What is common knowledge?

2. Common Knowledge

Common knowledge has come to be used as a technical term in philosophy, statistics, game theory and economics to denote knowledge that includes knowledge about what others know . Simply put, a piece of information is common knowledge between agents A and B if both A and B have the information, and both A and B know that the other has the information, and both A and B know that the other knows that the other has the information, and so on, *ad infinitum* . Lewis's and Aumann's definitions are worded somewhat differently, but they amount to the same.

It is sometimes useful to subdivide common knowledge into its parts. When A and B have the information, it is called first order knowledge or mutual knowledge; when they both know that the other has the information, it is labeled second order knowledge, and so on to the higher orders of knowledge. Common knowledge is the combination of all orders of knowledge from the first to the highest conceivable.

Common knowledge can be knowledge about anything. For example, it can be knowledge about events, actions, strategies, institutions, beliefs, behavior, goals, rationality, processes, or some combinations of these. Once we consider the possibilities of all there is for each of us to know of what others know of all of these, attaining common knowledge would appear to be a challenge indeed.

Its mathematical definition, like many other mathematical concepts (e.g., a point or a line in geometry), can be visualized but not realized in practice. Cognitively, it is difficult to imagine any of us consciously thinking about orders of knowledge beyond the first few. Yet, the concept of common knowledge, unachievable as it may sound, has proved to be a valuable theoretical benchmark. Like the character in Moliere's play, who was forty years old before he realized that he had been speaking prose all his life, more people are now recognizing, that in a great deal of our theoretical work, we have been assuming the existence of common knowledge all along. Why this realization, and what are its implications for accounting and business research?

But first, a capsule history of the concept . It is not clear who came up with the idea first. In his Treatise of Human Nature (1740) David Hume argued that all agents have to know what behavior to expect from one another in order that they can engage in coordinated activity. Littlewood (1953) described one of the best known examples of common knowledge reasoning which later appeared in Martin Gardener's Mathematical Games. Schelling (1960) and Harsanyi (1967-68) discussed common knowledge type behavior. Lewis (1969) gave the first known formal definition of common knowledge as nested orders of knowledge. Alternate definitions have been given by Schiffer (1972), Aumann (1976), and Gilbert (1989).

Arriving at Common Knowledge

Even though common knowledge has been mathematically defined, there exist different views on what it takes for knowledge to become common. Must we have explicit cognition of each of the infinite number of nested layers of knowledge in order to arrive at common knowledge? This remains a point of debate.

There used to be a television show in which a volunteer married couple was brought to the stage and each spouse was asked to answer a question independently. If their answers coincided, they won a prize. Suppose they were asked: which will be the next restaurant you would go to for dinner together. Assuming that they wanted to win the prize, each spouse was in the same position as Keynes' stock market investors or the newspaper beauty-contest participants. They could name their personal favorite, or what they believe to be the favorite of their spouse, or what they thought their spouse thinks to be their own personal favorite, and so on, knowing full well that their spouse will also have to make a similar decision from his or her point of view.

Even if their personal favorite restaurant is common knowledge between the spouses, such knowledge is not sufficient to win the prize. They also need to have between them what Lewis (1969) calls a convention. Unless their personal favorites are identical, and common knowledge, they would also need to have common knowledge about the order of reasoning their spouse will apply in answering the question from the TV host. For example, suppose, the husband's favorite is restaurant H and the wife's favorite is restaurant W, and these preferences are common knowledge between them. They both could choose to answer the question at level 1 (H and W respectively), or level 2 (W and H respectively), or level 3 (H and W respectively), and so on. In spite of the common knowledge of each others' preferences, they would never win the prize unless the levels at which they choose to answer the question differed by an odd number (1, 3, 5, etc.)

No wonder, even those who have lived together for long, and think that they understand each other very well, find it not so easy to win a prize in this TV game

because they may not have enough common knowledge. Even if they have the common knowledge, they may not have established a convention between them to coordinate their answers to the questions they might be asked.

In his classic work, Strategy of Conflict, Thomas Schelling (1960), discusses the problem of a husband and wife separated (without cell phones, of course) in a department store without a prearranged rule for where they would go if separated. He conjectures that most spouses would have enough common knowledge so they would both go to an “obvious” place where each would think that the other would expect the spouse to go to. He does not cite evidence in support of the conjecture.

Lewis (1969, p. 57-58) identifies three possible ways of arriving at common knowledge. One obvious way is by agreement. If two or more parties negotiate an agreement, unless we assume that some of them misunderstand the agreement, it is not unreasonable to think that the contents of the agreement are common knowledge among the parties.

Salience is a second, though weaker basis for common knowledge. Salient features are more likely to be common knowledge than others. Schelling uses the concept of focal point in a similar vein to suggest that people are more likely to believe that the others will also choose a salient feature.

Precedence or past conformity, even in absence of agreement or salience, can be a basis for common knowledge. If everyone has conformed to a pattern of behavior in the past, it is more likely that everyone knows that it is the case, and that they are also likely to continue to conform in the future. Driving on the right hand side of the road in U.S. is a good example of common knowledge. However, these are only possibilities for how a

group of people may arrive at common knowledge. None of these features, individually or collectively, ensure attainment of common knowledge.

Common knowledge is the basis of language, communication, and social life. If this statement looks too far-fetched, consider the problem of facing the commanders of two army divisions who are trying to coordinate their attack at their common but powerful enemy from two opposite sides. If the two attack simultaneously, they win, else they lose. The only way available for them to coordinate their attack is to through a messenger (again, no cell phones please) who tells one commander when the other will attack. But there is a problem. There is a chance that the messenger might be captured by the enemy and may never reach the other commander to deliver the message.

If Commander 1 sends a messenger saying “I shall attack at 4 AM tomorrow morning; you do the same,” there is chance p that the message will get through and a coordinated attack will take place. But there is chance $(1-p)$ that the messenger will be captured, and the second commander will not attack. Commander 1 may try to solve this problem by altering the message to “I shall attack at 4 AM tomorrow morning; if you get this message, and agree, send me a confirmation” again there is a chance that the messenger bringing the confirmation may get caught by the enemy. Halpern (1986) shows that there is nothing the two commanders can do, with the communications technology available to them, to increase the chances of victory through a coordinated attack from both flanks of the enemy. Sending confirmations of confirmations of confirmations does not help as long as there is a non-zero chance of missing on the confirmation. If this chance is zero, confirmation is unnecessary.

The story carries an important message about common knowledge. As the commanders receive each confirmation, they climb progressively to a higher order of knowledge toward common knowledge. Yet, their optimal action remains unchanged until they actually attain common knowledge. But arriving at common knowledge, in this situation, is not possible. It is possible to create simple examples where a group will, logically fail to arrive at common knowledge. I know of little work so far that addresses the question of how such game theoretic analyses correspond to actual human behavior with respect to common knowledge. I return to this topic later.

3. Theory of Accounting and Control

We have discussed the meaning and significance of common knowledge in general. We can now return to accounting, and speak to the relevance of common knowledge.

In order to discuss the accounting role of common knowledge, I would like to summarize a model of accounting and control developed in my book (Sunder, 1997). We can think of every organization as a set of contracts among the participating individuals or groups of individuals. The provision of shared information among the contracting parties helps design and implement these contracts, and makes it possible for organizations to function.

Organizations as a Set of Contracts

To understand accounting, the firm can be seen as a set of contracts among rational agents. Contracts are mutual understandings, explicit or implicit, short-term or long-term. Both an apartment lease and a lunch date with a friend are contracts. Agents are rational in the sense that they do not knowingly choose what they do not like.

Contracts obligate each agent to contribute resources—capital, skills, or information—to the organization’s pool and, in return, entitle each agent to receive resources from the pool. The form, amount, and timing of resources an agent gives and receives is a matter for bargaining among agents.

Accounting and control systems are designed to serve several essential functions in this system of contracts. They measure resource contributions. They measure out the resources each agent is entitled to. They distribute information about who have or have not fulfilled their contracts. They distribute information to potential participants in the system of contracts. In addition, and this is most important for our present purposes, they provide common knowledge for conflict abatement and resolution.

Shared Facts for Conflict Resolution

Disputes waste resources; provision of shared knowledge helps avert and settle disputes. Unsettled conflicts among agents weaken, or even wreck, the complex fabric of socioeconomic exchanges from which so much of our prosperity is derived. Industrial strikes and lockouts are an example. The practice of carefully collecting and sharing information arises to meet this fundamental demand for a means to preserve our socioeconomic system. Sharing of knowledge and expectations is a large part of acculturation and socialization.

Many conflicts in families, neighborhoods, the workplace, and trade are averted or settled with the help of shared information. The judicial system relies on written documents and the testimony of witnesses—both being forms of shared information. However, only a minuscule proportion of all conflicts ever enter the courts of law. Most

conflicts are promptly and inexpensively resolved through systematic provision of shared information outside any formal system of conflict resolution.

Defining executable contracts among agents also requires common knowledge. When variables that are not common knowledge are used in contracts, contention or deception can arise. Common knowledge is more than the observability of an event by all parties. It also requires that every party be aware of its observability to the others. When everybody knows about the event, but not about others' knowledge of it, some may be tempted to use such information to their own advantage, and create avoidable conflict. Common knowledge helps reduce such conflict and the concomitant losses.

Along with its other functions, accounting and control in organizations produce common knowledge to help define contracts among the agents. When deciding what to do, we may face two kinds of uncertainty. We decide under imperfect information if the rules or structure are common knowledge, only we do not know about events and actions of others. Roulette, for example, is a game of imperfect information because the players do not know where the ball will stop on the wheel. They all know the rules of the game, and the chances of various outcomes. Similarly, when we think of accounting as an information system for decision making, we assume that all parties know the rules of the game and accounting only provides information about various events and the actions of others.

If we do not know the rules or structure of the situation, we decide under more difficult circumstances, called incomplete information. In *The Wizard of Oz*, Dorothy faces a game of incomplete information. She knows neither the rules nor the players in the game who keep popping up to surprise her. Accounting, as a system for implementing

contracts or as an accountability system, must function effectively in an environment of not only imperfect but also incomplete information. In the less certain and more complex environment of incomplete information, accounting informs not only about events and the actions of others, but also about the structure of the game and the relative positions of players in that game. Some parts of accounting and control (e.g., public disclosure of financial statements) may appear to be redundant unless we look at organizations as games of incomplete information.

Common Knowledge for Renegotiation of Contracts

The length of individual contracts in a firm varies in time as well as in the number of transactions covered. A contract to buy or sell could be a one-time deal or a long-term commitment. The same is true of employment and borrowing. The audit contract is usually negotiated each year. With the exception of shareholders, whose commitment is open-ended, all contracts are periodically renegotiated.

An important function of accounting is to provide information in the form of common knowledge in order to facilitate contract renegotiation among current participants. Although agents may also use additional private information, availability of a common verified database helps eliminate certain types of strategic bargaining that may make some participants worse off without improving the lot of any.

The practice of negotiated renewal of contracts is an intermediate solution between two extremes: (1) starting a fresh search for potential participants in the appropriate factor market at the conclusion of each transaction, and (2) entering long-term or permanent, comprehensive contracts. Uncertainty, changing environment, and boundedness of human foresight rule out rigid, long-term comprehensive contracts. The

magnitude of the incremental costs of conducting frequent transactions in many factor markets renders the first option uneconomical. In addition, participants in the firm learn about local conditions, tasks, and techniques from their past experience in their contract slots. Their increased efficiency makes it attractive for other participants to want to retain them in the contract set. However, the special knowledge an agent acquires on the job is not available either to the manager who may negotiate the agent's contract on behalf of the firm, or to the potential replacements of the agent recruited from outside. Existing participants seek to exploit this special knowledge by demanding a larger share of resources. Competition among many such participants reduces their ability to increase their compensation. But contract renewal negotiation can give rise to prolonged conflicts.

The basic theme that the efficiency of economic relations depends on the ability of agents to renew contracts by adjusting them to the changing environment occurs throughout economics. Commons (1934) emphasized the role of organizations in promoting continuity of relationships by reducing actual or potential conflict. Hayek (1945) insisted on the importance of rapid adaptation to changes in "particular circumstances of time and place." Arrow (1969) analyzed the importance of minimizing the cost of bargaining among agents in organizations.

Wiggins and Libecap (1985) provide a dramatic illustration of how large the deadweight losses to social welfare can be when asymmetric distribution of information prevents economic agents from arriving at mutually beneficial arrangements. Owners of leases that cover a single underground pool can extract oil and gas independently, or can form a partnership and operate the field as a single unit. Unitization of oil fields yields large gains, as much as 100 or 200 percent, in the value of extractable hydrocarbons. Yet,

for a majority of oil fields in the United States, lease owners are unable to conclude negotiations for unitization of their leases. Recovery of oil from independently operated leases leads to inefficient utilization of the underground pressure of gas to get the oil out and reduces the extent of secondary recovery. This loss frequently amounts to hundred of millions of dollars. However, since the lease owners and their engineers have superior information about the value of their own leases rather than of the value of other leases, negotiations often break down because the parties fail to agree on the relative shares of the net profits from the unitized operation of the field. It is interesting to note that the same lease owners apparently have no difficulty in sharing the cost of exploratory drilling on neighboring lease tracts, because there is no information asymmetry at that stage of negotiations. Most of the unitization that does take place in the United States occurs during the secondary recovery phase of oil fields. By that time most of the information about the relevant characteristics of various leases has passed into public domain, and it becomes easier to reach an agreement.

Accounting includes some precommitments to reduce information asymmetries among contracting parties by sharing a common base of information in the form of public disclosure. Public financial statements, disclosure of accounting policies and significant details in footnotes, management's analysis of financial statements and results, and even financial forecasts have the effect of reducing surprises at the time of contract renegotiation. The losses to society from such surprises and the confrontational attitudes they engender can be so large and have such significant externalities that securities laws in the United States and in many other countries require public disclosure by publicly-held firms. In the later half of the nineteenth century, state regulators in the United States

used public disclosure as an instrument to reduce confrontation between railroads and a suspicious public.

Private disclosure to those who request information is deemed insufficient. If information were only privately available, many agents may have reason to doubt that others have received the information and, therefore, may be tempted to behave strategically. Public disclosure laws abate such behavior by making financial statements common knowledge. Since contracts of various agents are periodically renegotiated, it provides a pool of common knowledge of verified information to all participants to facilitate negotiation and contract formation.

4. Prospects for common knowledge research

Let me now turn briefly to four areas of accounting and capital markets research in which common knowledge may advance the state of the art. These are accounting standards, human cognition and information processing, financial analysis and mathematical modeling of accounting phenomena.

Accounting Standards

Accounting standards serve many functions in society. They help define template contracts to save transaction costs, propagation of best practices in the corporate economy, and as common knowledge rules of the game. For obvious reasons, I shall dwell on the last of these three functions.

Effective accounting standards define the rules of the game in a corporate economy. For a game to be played well, and fairly, its rules must be common knowledge to all interested parties. When rules pop up as surprises, they lose their legitimacy and

effectiveness. In U.S. and in many other countries the process of setting accounting standards is designed to promote them as common knowledge. Such processes include agenda advisory committees, project task forces, exposure drafts, public hearings, open discussion and debate on merits of accounting standards, and wide dissemination of standards when they are issued.

In addition to the standard setting bodies themselves (e.g., Financial Accounting Standards Board, International Accounting Standards Committee, Securities and Exchange Commission, and the American Institute of CPAs) many accounting periodicals and publications of the bigger accounting firms try to spread the knowledge about accounting standards.

This effort raises many interesting and largely unaddressed questions. How close do we get to the ideal of common knowledge of accounting standards? How far does each layer of knowledge about accounting standards penetrate various segments of accounting public (e.g., CPAs, corporate financial officers, financial analysts, professional investors, casual investors, and students, etc.)? Does the extent of common knowledge vary across standards? Is the degree of common knowledge of a standard linked to the degree of effective compliance with the standard? How effective are various programs to increase the common knowledge of accounting standards? What is the optimal allocation of resources to maximize the common knowledge of accounting standards among the accounting publics?

Addressing questions of this nature may enhance our understanding of accounting standards. It may even help increase the effectiveness of accounting standards, and the standard setting process.

Auditing and Cognition

Certain aspects of auditing can be seen as a strategic game between the auditor and the managers of the firm being audited. As in any other strategic game, common knowledge becomes important in the audit process also. Though auditing research has developed a rich tradition in application of, and contributions to, cognitive psychology, the field of common knowledge has barely been touched. I shall refer briefly to a few related questions.

First, as we discussed earlier, the countless loops of nested knowledge included in the mathematical definition of common knowledge seem well beyond human cognition. Few of us can consciously think beyond the first three or four layers of common knowledge before they dissolve into a vague abstraction. I doubt if many of us are capable of intuitively comprehending the difference between the fifteenth and the sixteenth layers of common knowledge any more than we can visualize a cube in more than three dimensions. Yet, our comprehension of common knowledge may not necessarily depend on our comprehension of each individual layer of knowledge. It is entirely possible that we can intuitively understand common knowledge in its entirety, even without a full grasp of all of its parts. Most cognitive research I know of in accounting has confined itself to the first layer of knowledge. The area of common knowledge remains virtually untouched, and may yield rich rewards to those who venture out to this field.

Second, how much of the behavior and strategies of the auditor and the firm managers are common knowledge between them? What would be the consequences of changing the degree to which the audit plan, for example, is common knowledge between

them? With the introduction of statistical sampling during the past few decades, and the move toward greater emphasis on analytical modeling by auditors, research to understand the common knowledge consequences of such innovations holds rich promise.

Third, cognitive science tradition in accounting has paid much attention to how individuals process information, and develop their beliefs about the state of the world. Extension of this tradition to the domain of common knowledge may yield rich insights into how we process information about information, and how we develop beliefs about the beliefs of others. Jamal and Tan (2000) is an example of an empirical assessment of the extent to which auditors are able to predict the preferences and knowledge of their colleagues within the firm.

Financial Analysis, Trading Volume and Price Bubbles

Valuation of securities supports a great part of accounting as well as accounting research. Virtually all our models of valuation use the fundamental approach rooted in present value of future cash flows (e.g., dividends from equity securities and interest and principal from debt claims). This fundamental model is so popular because it is quite safe. It is really a definition, because it cannot be proved to be wrong. All independent variables of this model—the cash flows—are to be realized in the future, forcing us to work with their current expected values. Since expectations themselves cannot be observed in any reliable fashion, any mismatch between the fundamental valuation model and the data can safely be blamed on the error of measurement in expectations.

Financial analysis and security valuation consist of using currently available data from financial reports and other sources to estimate the future cash flows arising from equity claims. In these models, current and past earnings with a variety of adjustments are

projected into the future to estimate cash flows, and the current market value of claims. It is called fundamental analysis because it is scrupulously confined to the first order of knowledge.

What about the second and the higher orders of knowledge? Do investors think about how others will interpret the information they have received? How confident are the investors that the others' interpretation will be similar to their own? What if an investor's own interpretation differs from his belief about how most others will interpret the same event? Economic theories of price bubbles (e.g., Flood and Hodrick, 1990) rest on such differences that may arise between two or more orders of knowledge about the market phenomena.

Accounting investigations of financial analysis and security valuations have largely stayed clear of such ideas, having hitched our wagon to the efficient markets train almost a generation ago. That track ended, and the train came to a halt. Efficient market theory has no useful economic role for accounting. It is not without some irony that, at the turn of the century, pages of accounting journals are the only remaining shelter for the believers in an extreme form of this theory.

Literature on attempts to explain stock market trading volume is an exception to the uniform stand with respect to identical beliefs among traders. The reason for this exception is, as we discussed earlier, under common knowledge environment no trading is possible. Remember that if I think that the stock is worth \$100, and you bid to buy it at \$105, I should figure, given common knowledge of rationality, that you must know something about the prospects of the stock that I don't, and will promptly adjust my own assessment of its value to \$105, thus eliminating the motivation for trade. Conversely, if I

offer to sell the stock to you for \$100, you will adjust your own assessment of its value to \$100, again eliminating the motivation for trade. This parallels the rationale Will Rogers gave for why he could never join a country club: Any club that would admit people like me to its membership could not be worth joining (Aumann's (1976) theorem on the impossibility of agreeing to disagree under common knowledge).

However, people do trade in stock markets, and the trading volume far exceeds anything that could be explained by the need for new investment and consumption by individuals. Great deal of trading is speculative, which suggests that people must hold diverse beliefs, and trading mechanisms do not entirely eliminate this diversity.

If, for the purpose of explaining trading volume we accept that people hold diverse beliefs about value of stocks, and that this diversity persists in spite of their knowledge of its existence, we have moved well beyond the efficient market model in which all participants in the market must believe that the market value is the unique right value of stocks. This detour from efficient markets opens the door for weakening of the common knowledge assumption to admit formation of price bubbles in the stock market.

Formation of price bubbles in markets—for tulips or silver or shares—has to do with traders' belief that other people may value the object of trading higher than the trader himself does. This same weakening of the common knowledge assumption yields both trading volume as well as price bubbles. Surprisingly, plenty of attention in accounting research has been devoted to the former, but almost none to the latter. Should we hope that a better understanding of the link between the two might help correct this imbalance?

Modeling

Weakening of the common knowledge assumption may open the door to a broad class of models and results in accounting, security valuation, corporate finance and governance. I shall mention four possibilities.

We have already discussed the Amershi and Sunder (1987) model in which allowing for the possibility that corporate managers may believe that the shareholders value shares on the basis of earnings instead of cash flows yields interesting new results. Even in a perfectly efficient market for shares, and even when the welfare of managers is tied to the welfare of the shareholders through stock prices, managers may fail to take the value maximizing decisions from the shareholders' point of view. Moreover, the stock market will not only fail to discipline such wayward managers, it will even provide confirmatory signals that reinforce the incorrect beliefs managers hold about shareholders' valuation model. Here is an example in which the standard solution to the corporate agency problem does not work when the common knowledge assumption is weakened.

Dominance of the fundamental valuation model (net present value of future cash flows, and its derivatives) in accounting and finance is rooted in common knowledge assumption with respect to beliefs about the future cash flows. If we allow for the possibility of diverse beliefs among investors about these cash flows, the value of the fundamental model of valuation drops, unless you commit to hold the security to the maturity (which amounts to holding equity securities forever). If I plan to hold the security for, say, only twelve months, it matters what others believe it to be worth at that time, and not what I believe it to be worth at that time. More attention to common

knowledge may permit more careful development of alternatives to the fundamental model, including even the technical trading models.

As a third example, consider models of financial disclosure by corporations and grade disclosure by students in job interviews. It has been argued, in both instances, that given freedom of choice, best performers will disclose, inducing the second best to disclose, who in turn induce the next best to disclose. This chain of disclosure continues until the very last person for whom disclosure becomes irrelevant. Given this chain of reasoning, it is in the interest of everyone to disclose. Simple Elegance of this unraveling argument fails to match with the evidence from the field. The fact is that given the choice, many corporations and job candidates choose not to disclose. Again the unraveling argument I just gave also depends on the common knowledge assumption. Weakening of the assumption (may be my interviewer does not know the unraveling argument!) might help us build more realistic models of this and many other accounting phenomena.

Fourth, consider the problem of ultimatum games. In these games, two players have to divide a fixed sum of money between them, say ten dollars. Player 1 proposes a division. Player 2 may accept the proposal, in which case it is implemented, or reject it, in which case both players get zero. Subgame perfect game theoretic solution to this game is that the first player should take almost the entire amount, and give the minimum possible amount to the second player. The argument is that the latter should prefer even one penny to nothing. In human experiments, it is found that the player 1 gives away between 25 and 50 percent of the amount on average. This is not consistent with the game theoretic solution.

Lin and Sunder (1999), show that by weakening the common knowledge assumption, it is possible to build models that are much closer to the data than the subgame perfect solution. Specifically, we assume that Player 1 has a probability distribution of Player 2 rejecting the offer as a function of the amount offered, and offers an optimal amount of money conditional on this probability distribution. Thus weakening the common knowledge assumption holds much better prospect of building models that describe actual human behavior more closely.

5. Concluding remarks

Let me summarize. The past half century has seen a great deal of development in thinking about information in the fields of philosophy, game theory, statistics, economics and cognitive science. Some of the most exciting of these developments concern common knowledge or higher orders of information.

These developments have barely found an echo in the field of accounting even though information lies at the heart of accounting. It is easy to show, as I have tried briefly in these remarks, that common knowledge thinking may shed significant new light on virtually all our concerns from financial reporting, analysis, and securities and valuation to managerial control, auditing and information systems.

Thinking about various orders of knowledge will not only yield better understanding of accounting, capital markets and business, it will also make important contributions to the fields where this idea originated and developed. As you may already have noticed, examples from those fields tends to be simplified text book examples lacking real world flavor. In accounting and business we have the advantage of having institutional knowledge of real information systems, and observations of real behavior

from the field. Such data and analysis is the strength of business research. We can conduct our own research, obtain better insights and examples, and feed them back to the basic disciplines to enrich them through our work.

Thank you for this opportunity to speak on this subject. I shall put up my paper (including a bibliography) and these slides on my web page next week. Here is the address.

Table 1

Levels of Analysis in Valuation of Firms

1	Fundamental value of the firm
2	What people believe to be the fundamental value of the firm
3	What people believe about what people believe to be the fundamental value of the firm
4	What people believe about what people believe about what people believe to be the fundamental value of the firm
5	What people believe about what people believe about what people believe about what people believe to be the fundamental value of the firm
6, 7, ...	And so on ...

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Genakopulos (1992) is a good general introduction to common knowledge. Lewis (1969), a highly readable treatise on conventions, also gives the first formal definition of common knowledge and shows how conventions are based on common knowledge. Vanderschraaf (1998) discusses common knowledge from a philosophical perspective. Gamow and Stern (1958) and Gardner (1984) introduce the now famous common knowledge puzzles.

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