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A Note on Erb and Harvey (2005)

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This note is a response to a recent paper by Erb and Harvey (2005). We show that "diversification returns" are mathematical properties of geometric averages of index returns, and not due to rebalancing. We also show how rebalancing affects the performance of the equal-weighted commodity futures index constructed by Gorton and Rouwenhorst (2005). Because rebalancing is an embedded trading strategy, it can be a source of return. Less frequent rebalancing would have increased, rather than lowered the performance of the equally-weighted index.

This note addresses two issues raised in a recent paper by Erb and Harvey (2005) – henceforth E&H – about the estimate of the risk premium on commodity futures in the Gorton and Rouwenhorst (2005) paper. The basic arguments of E&H can be summarized in two points:

1. The risk premium on many commodity futures indices – including the Gorton-Rouwenhorst index – does not stem from a risk premium at the individual commodity level, but rather from a "diversification return." The estimate of the risk premium should be adjusted downward by this diversification return.

2. This diversification return in Gorton and Rouwenhorst is in part driven by a rebalancing strategy, as their index rebalances monthly to equal weights.

While we addressed these issues in the February 2005 version of our paper, we have received several inquiries about the points raised by E&H. This note intends to clarify the issues regarding the role of rebalancing and its relation to the "diversification return" and the presence of risk premium. Before we do so, we reproduce Table I of our paper, which shows that the returns to our index are robust to various rebalancing assumptions. Two conclusions stand out from this table:

1. Collateralized commodity futures have outperformed spot indices and inflation, both in terms of geometric as well as arithmetic average returns.

2. The historical performance of the monthly rebalanced futures index is lower than of an index that is rebalanced less frequently.

Table 1: Average Annualized Returns to Spot Commodities and Collateralized Commodity Futures 1959/7 – 2004/12

		Rebalancing		
Average Return % p.a.	Index	Monthly	Annual	Buy and hold
Arithmetic	Futures	10.69	11.97	11.46
	Spot	8.42	7.51	4.64
	Inflation	4.14		
Geometric	Futures	9.98	11.18	10.31
	Spot	7.66	6.66	3.47
	Inflation	4.13		

What is the "diversification return"?

The "diversification return" is a term coined by Booth and Fama (1992) and arises in the context of a comparison of geometric average returns of assets and portfolios comprised of those same assets. It is well-known that geometric average returns are lower than arithmetic average returns. This is a mathematical property of geometric averages and is true for all assets and all portfolios. Booth and Fama show that a good approximation for the difference between the geometric average (g) and the arithmetic average (m) return on an asset is:

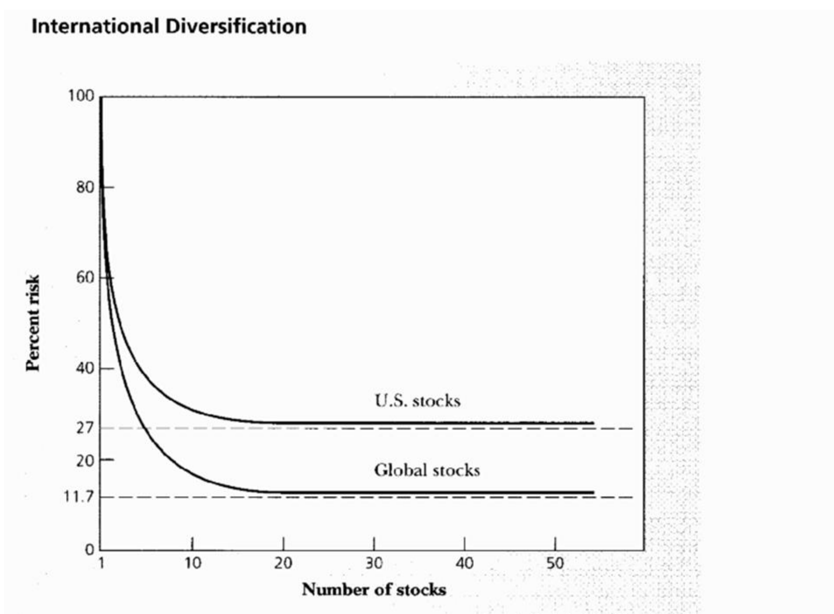
$$g = m - \sigma^2 / 2$$

where σ^2 is the variance of the underlying return time series. The above formula shows that the difference between geometric and arithmetic average returns depends on the variance of the underlying asset. The formula illustrates that:

1. All else equal, more volatile assets will have lower geometric returns compared to arithmetic average returns.

2. It directly suggests that the difference between geometric and arithmetic average returns will be smaller for portfolios than for the most assets included in those portfolios, because portfolios diversify some of the risk that is inherent in individual assets. In other words, in terms of geometric averages portfolios will on average outperform their constituents because they are diversified.

The power of diversification was illustrated by Solnik (1974) in the context of international portfolios. The graph – reproduced below – shows how the portfolio variance drops relative to the variance of its constituents, as a function of the number of assets in the portfolio.



Source: B. Solnik, "Why Not Diversify Internationally Rather Than Domestically," *Financial Analysts Journal,* July 1976. Reprinted

The decrease in the variance of portfolio returns due to diversification, affects the geometric average return of portfolios relative to the geometric average of its constituents. This difference is the "diversification return." All portfolios (and indices) can be expected to have some diversification return: E&H use the commodity futures indices as an example, but the S&P500, the Willshire5000, and the Lehmann Aggregate Bond Index will all share this property. In other words, because the SP500 is a diversified portfolio, the geometric average return of the SP500 will be higher than the geometric average returns of the typical stock in that index. So the concept of a diversification return is not special to the commodity universe. The particular context that Booth and Fama used to illustrate the difference was a balanced portfolio of stocks and bonds.

Diversification Returns and the Risk Premium

If the geometric average return on a portfolio embeds a diversification return, does this have implications for the measurement of a risk premium? We suggest that the reader consider the case of equities for a moment. As an empirical observation, it is quite obvious that the geometric return on the SP500 has exceeded the (weighted) average of the geometric average returns on the individual stocks in the index. This is due to the simple mathematics of geometric averages. It would probably not cause many investors to "downsize" their estimate of the equity risk premium.

As a second example, imagine for a moment a world in which all investors are risk-neutral and require no compensation for bearing risk. In other words all risk premiums will be zero, and the expected return on all assets is equal to the risk free rate. However, in this world, there will still be a "diversification return." Risky assets will have geometric average returns that are below the risk free rate. This illustrates that diversification returns and risk premiums are not different sides of the same coin.

Depending on the length of the investment horizon, an investor may place more weight on the geometric return *of the index* (e.g. long-term investors) than its arithmetic average. But the economics of investing suggests that if a (risky) index historically outperformed a risk free investment – either in terms of geometric or arithmetic average returns – investors in the index have received a compensation for bearing risk. Table 1, shown above, illustrates that over the past 45 years this has been the case for commodity futures.

Note that our discussion of risk premiums and diversification returns does not rely on any assumptions about the method of index construction – in particular whether the weights are constant or not. Diversification reduces risk – and lowers the geometric average return of a portfolio relative to its constituents. We turn to the role of rebalancing on index performance next.

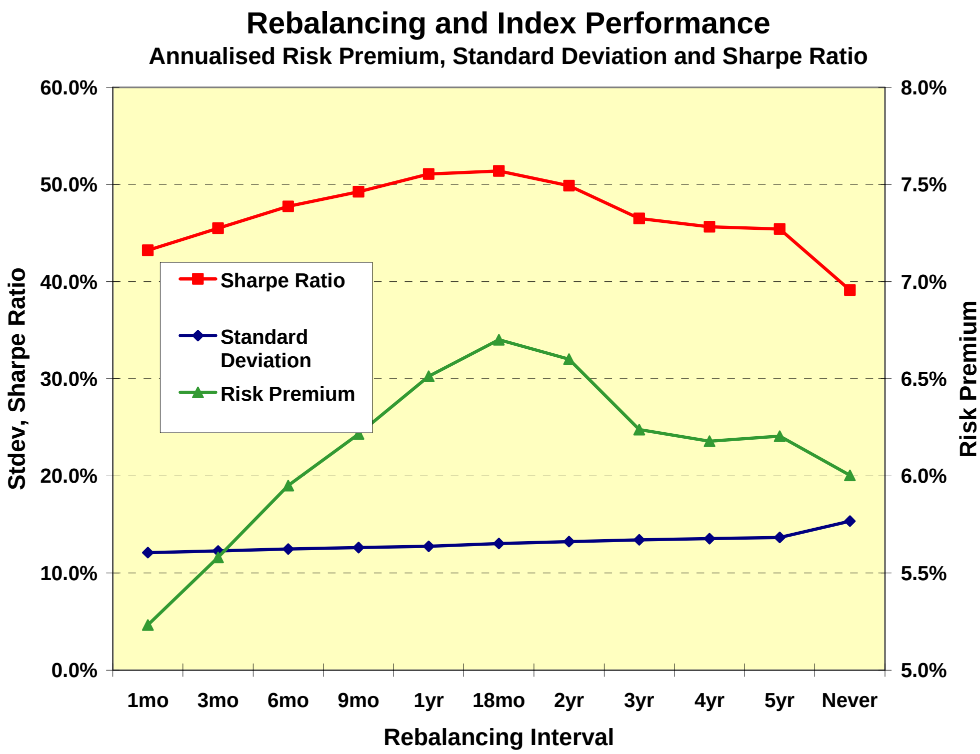
Approximating the diversification return for rebalanced portfolios

At a mechanical level, a "diversification return" calculation involves a comparison between the geometric average returns of individual assets and the geometric average return of a portfolio. The appendix shows that in the special case of a portfolio that rebalances to constant (equal) weights, the diversification return can be approximated by the *difference between the average asset variances and their covariances*. This is the approximation used by E&H. It does not imply that rebalancing is the source of the diversification return (it is called the "diversification" return for a reason), but it allows an indirect calculation of its magnitude.

Does rebalancing matter for index performance?

While rebalancing might affect the variance of a portfolio – and hence the "diversification return" – there is a more important aspect of rebalancing: it is an embedded *trading strategy*. Any embedded trading strategy can potentially affect both the arithmetic as well as the geometric performance of an index. By re-balancing a portfolio to fixed weights, an investor in effect "sells" assets that went up in price and "buys" assets with poor prior performance. If returns are not independent over time, the trading strategy can either lose or make money depending on the time series properties of the underlying assets. For example, E&H show that there is short-term momentum in commodity futures returns. By monthly rebalancing, one would expect to sell some high momentum commodity futures, and buy some of the low-momentum commodity futures. Short-term momentum in commodity futures returns would cause an index that rebalances bi-monthly to outperform an index that rebalances monthly

The figure below illustrates the effect of rebalancing for the Gorton-Rouwenhorst equally-weighted index. The graph shows the excess return (over one month T-Bills), the standard deviation, and the annualized Sharpe ratio as a function of the re-balancing interval of the index.



Several observations stand out:

1. The excess return initially increases if the index is rebalanced less frequently, but gradually drops off at longer rebalancing intervals.

2. The lowest risk premium occurs at monthly rebalancing frequency, which suggests that rebalancing is not the source of the risk premium

3. Rebalancing has little effect on the standard deviation of the index, and does not affect the "diversification return" through the variance.

4. Rebalancing is an embedded trading strategy: its importance is driven primarily by the return on this strategy, but not by its effect on portfolio risk.

5. The initial increase in the excess returns is consistent with the presence of short-term momentum in commodity futures returns.

Conclusions

This note was intended to address two issues raised by E&H about the Gorton and Rouwenhorst paper:

1. *The risk premium of many commodity indices – including the Gorton-Rouwenhorst index – does not stem from a risk premium at the individual commodity level, but rather from a "diversification return."*

Diversification returns are a mathematical property of geometric averages. Unless assets are perfectly positively correlated, the geometric average returns of the constituents of a portfolio will on average be lower than the geometric average return of the portfolio. This is well known. It is not common to subtract this difference from the risk premium estimate. Instead, depending on their investment horizon, investors will compare the geometric or arithmetic average index return to a risk free investment.

2. *The diversification return in Gorton and Rouwenhorst is driven by a rebalancing strategy, because their index rebalances monthly to equal weights.*

Rebalancing has little effect on the variance of our index. But because rebalancing is an embedded trading strategy, it can be a source of return. We show that increasing the rebalancing interval would have increased the performance of our index. This is consistent with the presence of short-term momentum in commodity futures return.

Appendix

The diversification return is the difference between the geometric average portfolio return and the (weighted) geometric average of the individual assets' returns in the portfolio. This appendix shows that an alternative calculation of the diversification return is possible in a situation in which the weights of individual assets in the portfolio are held constant. In this case the diversification returns is approximately equal to the difference between the average variance of the returns on assets in the portfolio and their average covariance.

In the simple case where all assets have equal weight, the variance of the return of a portfolio of N assets is:

$$\sigma_p^2 = \sum \frac{\sigma_i^2}{N^2} + \sum\sum \frac{\sigma_{ij}}{N^2} = \left(\frac{1}{N}\right) \times \overline{\text{var}} + \left(\frac{N-1}{N}\right) \times \overline{\text{cov}} \approx \overline{\text{cov}}\,.$$

For large N, the portfolio return variance approaches the average covariance between the assets' returns.

Portfolio arithmetic returns are by definition simple averages of the asset returns, m_i:

$$m_p = \sum_i \frac{m_i}{N}\,.$$

Substituting in the approximate relationship between geometric and arithmetic returns on both sides we get:

$$g_p + \frac{\sigma_p^2}{2} = \sum_i g_i / N + \frac{\sum_i \sigma_i^2}{2N}\,.$$

Substituting this into the first equation and rearranging terms gives:

$$g_p - \sum_i g_i / N = 0.5 \times \left[\sum_i \sigma_i^2 / N - \sigma_p^2\right] = 0.5\left[\frac{N-1}{N}\right]\left[\overline{\text{var}} - \overline{\text{cov}}\right].$$

The left hand side is called the "diversification return." For large portfolios, it can be approximated by half the difference between the average asset variances and the average asset covariance (as employed in E&H).

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