Estimating the Dynamics of Mutual Fund Alphas and Betas *

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Abstract

Consider an economy in which the underlying security returns follow a linear factor model with constant coefficients. While portfolios that invest in these securities will, in general, have a linear factor structure, it will be one with time-varying coefficients. However, under certain assumptions regarding the portfolio's investment strategy, it is possible to estimate these time-varying alphas and betas. Importantly, this can be done without direct knowledge of either the portfolio manager's exact investment strategy or of the alphas and betas of the individual securities in which the portfolio invests. This paper develops and estimates a Kalman filter statistical model to track time-varying fund alphas and betas. Several tests indicate that relative to a rolling OLS model the Kalman filter model produces more accurate fund factor loadings both in and out of sample. This appears to be in large part due to the attempts of fund managers to time the market by varying their fund's risk exposure from period to period. Another advantage of the Kalman filter model is that the dynamic parameter estimates can be used to classify funds by their trading strategies and to determine the source of a fund's profits or losses. The tests in this paper indicate that the superior and inferior returns produced by some funds arise almost entirely from attempts at market timing rather than managerial selection ability. However, as other research in the area of mutual fund performance measurement have found, overall there appears to be little evidence that, in aggregate, fund investors earn superior returns.

JEL Classification: G12, G13.

Over the last twenty years the mutual fund industry has grown at an incredible rate, and this has naturally attracted a lot of attention from the academic and financial community. A great deal of attention has gone into both predicting mutual fund returns (see, for example Lehmann and Modest (1987), Grinblatt and Titman (1992), Carhart (1997), Daniel, Grinblatt, Titman, and Wermers (1997), Wermers (2000), Pástor and Stambaugh (2002a) Hendricks, Patel, and Zechhauser (1993), Brown and Goetzmann (1995), and Teo and Woo (2001)) and describing their trading strategies (Ferson and Schadt (1996), Brown and Goetzmann (1997), and Ferson and Khang (2002)). What most studies have in common is the maintained hypothesis that past factor loadings reasonably forecast future factor loadings.[1] While this assumption may or may not be true at an individual security level, it seems rather unlikely to hold for managed portfolios. Investors presumably employ portfolio managers to move assets into and out of various sectors and securities as part of a dynamic strategy.[2] Absent some mathematical coincidence, the simple act of shifting funds across securities will lead to time-varying portfolio loadings on any benchmark.

This paper extends the mutual fund performance literature along the lines of Ferson and Schadt (1996), hereafter FS. In order to estimate time variation in a portfolio's risk loadings FS project the latter onto a set of observable macro variables such as credit spreads. The FS technique is designed to estimate the manager's implicit strategy with respect to these macro variables and then allow for the resulting correlations when judging performance.[3]

However, in contrast to FS, the goal here is to allow for portfolio shifts due to factors unobservable by the econometrician. This is accomplished by assuming that assets are reallocated on the basis of some unobserved factor, and then estimating the system of equations via a Kalman filter. Of course, one can also include the macro economic factors FS use, thereby allowing for both observable and unobservable factors in the specification. Relative to the typical OLS model, this may allow researchers to estimate a portfolio's alpha and betas with less misspecification bias, and thus produce models with better in and out of sample properties.

Using the CRSP mutual fund database, cross referenced to Morningstar's mutual fund classifications, this paper estimates a dynamic model with time-varying parameters for a large subset of all U.S. mutual funds. The resulting alpha and beta time series show that the Kalman filtering approach produces considerably better estimates of their instantaneous values than do standard OLS models. It appears that depending upon the mutual fund category (and thus implicitly the strategy followed) static OLS alphas can be off anywhere from 5 to 87 percent from a fund's time *averaged* alpha and even further off at any one

[1] One exception is Grinblatt and Titman (1994). The methodology they use avoids a direct comparison against a specific portfolio, and instead uses an "endogenous" benchmark. However, their technique requires knowledge of the fund's actual composition, which may not always be available. Ferson and Khang (2002) extend the technique to condition the portfolio betas on exogenous variables such as macro economic data.

[2] See Breen, Glosten, and Jagannathan (1989) for an empirical estimate of the potential value of such actions, and Mamaysky and Spiegel (2002) for a theoretical treatment.

[3] Several recent papers have adopted this technique for performance evaluation. For example, Christopherson, Ferson, and Glassman (1998), and Blake, Lehmann, and Timmermann (2002).

moment in time.[4] These results imply that previous performance estimates may be very sensitive to the security classes a fund trades in. In addition, they show the potential value of explicitly allowing for managerial portfolio reallocation not only on publicly observed variables as in FS, but also on unobserved factors.

Because the Kalman filter model yields dynamic factor estimates it not only provides parameter estimates but also information about a fund's trading style and its impact on returns. Among the questions that can be addressed is how and if managers adjust their portfolio's market risk in reaction to their own or the market's past returns. The estimates derived here indicate that twice as many funds increase than decrease their fund's market risk in response to their own fund's past positive returns. In a sense, managers who have recently seen high market adjusted returns tend to "double down." However, while managers may respond in a variety of ways to their own fund's past returns the data does not indicate that it makes any difference to their expected four factor risk adjusted returns. In contrast, funds are about evenly split in their reaction to high overall market returns between those that increase or decrease their market risk exposure. However, while the response is evenly split the returns are not. Those that act as market return contrarians produce significantly better four factor risk adjusted returns than those using other strategies.

Another style issue examined here is the manner in which funds produce non-zero alphas. This can occur either because a manager is good (or bad) at selecting among stocks or because he is good (or bad) at anticipating market returns. The former is typically called selection ability and the latter timing ability. The Kalman filter's statistical output provides a natural way to look at this issue. Using the model's estimates fund performance is decomposed into various types of skills which the paper refers to as "selection ability," "bull market selection ability," and "market timing ability." For the most part, funds with good in sample selection ability (whether of the standard or bull market variety) do not show similar out of sample performance. By contrast, good timers in sample appear to show some skill in this regard out of sample as well. Papers by Daniel, et al. (1997) and Bollen and Busse (2001) also examine these issues. The former use the underlying stock holdings and concludes fund managers possess little in the way of timing ability. The latter use daily data and come to the opposite conclusion. One advantage of the Kalman filter model is that it provides this decomposition without using each fund's underlying stock holdings and is apparently sensitive enough to detect some degree of market timing ability on monthly data.

While the Kalman filter can adapt itself to time-varying factor loadings does this compensate for the additional computational overhead vis-a-vis a rolling OLS model? As with the FS model the Kalman filter model does a better job of fitting the data in sample, and appears to pick up a number of statistical patterns relative to an OLS model with constant

[4]In contrast, static OLS beta estimates are much more reliable, in that they are never estimated to be off by more than 8% from their time averaged values. However, the dynamic estimates indicate that at any one point in time the OLS betas can lie far from their current values. As with the alphas there is considerable variation across fund types.

coefficients.[5] More importantly, *out of sample tests* show the Kalman filter model does a better job of predicting future alphas and betas than the standard OLS model with constant factor loadings.

The final test in the paper looks at the degree to which conditioning information, as in FS, adds to the model's ability to fit the data within sample. Overall, the conditioning information does not improve the model's fit (as measured by the R^2 statistic). But this is not true of every fund. The number of funds with significant parameter values somewhat exceeds that which would be produced by chance. From an economic point of view, these findings indicate that while some funds condition on the type of macro information tested here, many do not. For those that do not, the Kalman filter picks up the time variation in their betas and alphas via estimates of the unobserved factor's value. The tests in this paper suggest that perhaps 12% of all mutual funds exhibit investment strategies with some dependence on the lagged treasury bill rate, and on the market dividend yield. Of course, the other funds may be conditioning on macro information not included in this paper's tests, a possibility which offers intriguing avenues for future research.

The remainder of the paper proceeds as follows. Section 1 derives our empirical specification for the dynamic alpha–beta model for portfolio returns. Section 2 derives the alphas and betas of an OLS regression for a dynamic coefficient, linear model. Section 3 describes the data used to estimate the model. Section 4 discusses the model's ability to remove intertemporal patterns from the estimated residuals across fund categories. Section 5 examines the dynamic properties of the estimated alphas and betas. Section 6 presents our decomposition of OLS alphas and betas for a large cross-section of mutual funds. Section 7 reports out of sample performance. Section 8 explores the impact of adding macro economic factors like those used in FS to the model. Section 9 concludes. All proofs are in the Appendix.

1 Statistical Model

Portfolio returns and the returns of those securities which constitute them may behave in quite different ways. Therefore a model which appropriately describes the returns of individual securities may poorly describe a portfolio holding those same securities.

If fund managers are to outperform the market on a risk adjusted basis they must receive some sort of private signal that forecasts returns. To accommodate this one needs to start with a general equilibrium model of asset returns with asymmetric information such as Admati (1985). Extending the basic setting to a multiple period framework, from a particular fund manager's perspective the return on asset i can be described by a linear factor model

[5]This empirical result holds whether one estimates the OLS model on the entire data set or that contained within a rolling window. Also, note that the in sample tests presented here offer the OLS model a better chance than would a direct comparison with FS. By its very nature the FS model employs data that the OLS model does not. Here comparisons between the Kalman filter estimates and those of the OLS model use exactly the same predictive data.

with constant factor loadings:

$$r_{it} - r_f = \alpha_{it} + \beta_i{}'(r_{mt} - r_f) + \epsilon_{it}. \tag{1}$$

The risk adjusted abnormal return α_{it} depends upon the current value of the manager's signal. Thus, it should technically include a parameter indicating the signal upon which it is based. However, for notational simplicity it is not displayed here. Under the null hypothesis (which will be formally developed later on) the manager's signal does not forecast stock returns and the α terms are zero. Here β_i is an n by 1 vector of factor loadings, r_m the corresponding per period factor returns, r_f the risk free rate, and ϵ a random shock. Throughout this paper it is assumed that the errors are normally distributed and independent over time. Note that while returns change over time, their loadings on the economy wide risk factor returns (here, the r_m's) remain constant.[6] If the r_m's are known, estimates of a security's loadings on the economy's risk factors can be obtained by regressing security returns on factor returns.

Even when equation (1) accurately describes each individual stock's return it may not extend to a portfolio of such stocks. Consider a fund that holds securities A and B. At any time t the portfolio's return (r_P) equals

$$r_{Pt} \quad = \quad w_{At} r_A + w_{Bt} r_B$$

where the w terms equal the fraction of the portfolio invested in each asset. Using this, and equation (1), it is straightforward to see that portfolio returns are also linear in the factor returns r_{it}'s. However, unless the returns on A and B at time t happen to be the same, the portfolio weights for securities A and B will be different at time $t+1$ than they were at time t. Thus, while time $t+1$ portfolio returns remain linear in the $r_{i,t+1}$'s, the weights attached to each factor's return will have changed from the time t weights. Clearly, even in this simple example, security returns and a portfolio's returns may not be well described by the same model especially a linear factor model with constant coefficients.

Now suppose one wishes to estimate the alphas and betas of the above portfolio, rather than the alphas and betas of its constituent securities. In this case, an OLS estimate of the portfolio's loadings on the r_i's can produce answers that are quite far from the portfolio's true loadings on the factor returns in question.

To address the above problem a statistical model needs to allow explicitly for variation in the fund's portfolio weights over time. A portfolio's time t return equals the weighted

[6]Many studies like those of Ferson and Harvey (1991 and 1993), and Ferson and Korajczyk (1995) question whether or not individual security loadings are constant. However, this will not qualitatively alter this paper's conclusion that fund loadings change over time. If anything such underlying intertemporal variation in the underlying securities will only add to the importance of allowing for time variation in the mutual funds themselves.

average of the returns from the underlying I assets:

$$\begin{aligned} r_{Pt} - r_{ft} &= w'_{t-1}\Big(\alpha_t + \beta'(r_{mt} - r_{ft}) + \epsilon_t\Big) - k_t \\ &= \alpha_{Pt} + \beta'_{Pt}\Big(r_{mt} - r_{ft}\Big) + \epsilon_{Pt}, \end{aligned} \tag{2}$$

where the variables α_P, β_P, and ϵ_P are defined by

$$\alpha_{Pt} \equiv w'_{t-1}\alpha_t - k_t, \tag{3}$$

$$\beta_{Pt} \equiv \beta w_{t-1}, \tag{4}$$

$$\epsilon_{Pt} \equiv w'_{t-1}\epsilon_t, \tag{5}$$

with w, α, and ϵ, the I by 1 vectors containing their corresponding firm specific elements w_i, α_i, and ϵ_i. The β term represents a matrix with I columns containing the vectors β_i. Finally, k equals the transactions costs incurred by the portfolio, which for mathematical tractability are assumed to be proportional to the funds under management. In (2), if the CAPM or APT holds period by period, then α_t equals a vector of zeros for all t and all managers. If a model such as Admati's (1985) holds then individual managers may use their information to produce non-zero alphas. Thus, again, one should keep in mind that the α terms are manager and signal dependent.

Equation (2) is the main focus of the econometric analysis in this paper, and as such, deserves some discussion. Thus far two important assumptions have been employed:

1. The evolution of portfolio wealth must satisfy an intertemporal budget constraint.

2. All stocks have constant betas.

These two assumptions together imply that portfolio returns will satisfy a linear factor model, but with time-varying coefficients, and with a heteroscedastic innovation term. This suggests that linear-factor, constant-coefficient models for portfolio returns, a common paradigm for empirical work in asset pricing, are misspecified.

Absent information about a fund's holdings and the alphas and betas of the underlying assets, the empirical system in (2) through (5) cannot be estimated. However, these problems can be overcome by adding some additional assumptions. As will be shown, with the proper specification of the dynamics governing a fund's portfolio weights, knowledge of the individual weights, alphas and betas is not necessary.

Let F_t represent some signal (normalized to have an unconditional mean of zero) that the fund uses to trade. Once again, for notational simplicity, the subscript identifying the signal's recipient is suppressed. Assume that it follows the AR(1) process (though more general specifications are possible)

$$F_t = \gamma F_{t-1} + \eta_t \tag{6}$$

through time. The $\gamma \in [0, 1)$ coefficient measures the degree to which the signal's value persists over time, and η_t represents an i.i.d. innovation.

If the signal F has value then one expects it to influence both the fund's present holdings and future expected stock returns. Statistically, these dual impacts can be represented by assuming that the portfolio weights follow:

$$w_{it} = \bar{w}_i + l_i F_t, \tag{7}$$

and that stock alphas equal

$$\alpha_{it} = \bar{\alpha}_i F_t. \tag{8}$$

Here $\bar{w}_i$ represents the steady-state fraction of the strategy invested in a given security. Alternatively, $\bar{w}_i$ can depend upon any set of observable variables, in which case it may be time dependent. The variable l_i is stock i's loading on a common unobservable factor $\tilde{F}_t$ which shifts the portfolio weights from their steady-state values. This formulation holds exactly under Admati's (1985) model and is generally consistent with Blake, Lehmann, and Timmermann's (1999) empirical finding of mean reversion in fund weightings across securities among UK pension funds. Finally, $\bar{\alpha}_i$ represents the degree to which a stock's expected return is predictable by the signal F. If the signal has no value then all of the $\bar{\alpha}_i$ terms equal zero. Also, the present specification insures that the steady state alpha values equal zero.[7]

Now use (3), (4) and (8) in the above formulation. Also, define $\bar{w}$, l, and $\bar{\alpha}$ as the I by 1 vectors with elements $\bar{w}_i$, l_i, and $\bar{\alpha}_i$ respectively, and one finds that

$$\begin{aligned} \alpha_{Pt} &= \bar{w}'\bar{\alpha}F_{t-1} + l'\bar{\alpha}F_{t-1}^2 - k_t \\ &= \bar{\alpha}_P F_{t-1} + b_P F_{t-1}^2 - k_t, \end{aligned} \tag{9}$$

for the appropriately defined $\bar{\alpha}_P$ and b_P. Similarly, one has

$$\begin{aligned} \beta_{Pt} &= \beta\bar{w} + \beta l F_{t-1} \\ &= \bar{\beta}_P + c_P F_{t-1}, \end{aligned} \tag{10}$$

for the appropriately defined $\bar{\beta}_P$ and c_P.

The $\bar{\alpha}_i$, $\bar{\alpha}_P$, and b_P each play a unique economic role in the analysis. In equation (8), $\bar{\alpha}_i \neq 0$ implies that a given fund's signal has a systematic relationship with the instantaneous excess returns of individual stocks in an economy. Therefore, one can add an indicator

[7] Beyond the asset allocation case outlined above, the modeled interaction between the signal F_t and security alphas can also accommodate market timing strategies. Imagine a fund manager that uses macroeconomic information to move in and out of the market index. In this case F_t equals the current value of the macroeconomic variable, $\bar{\alpha}_1$ its impact on next period's market return, and l_1 the fraction of the fund the manager invests in the market (with 1-l_1 invested in the risk free asset). Within this setting a high value of F_t implies an expected period $t+1$ market return that the manager's information indicates will be higher than the overall market expects. While testing for this type of timing behavior is possible within the model's framework such tests are not conducted here.

variable to the $\bar{\alpha}_i$ that indicates the coefficient is both stock *and* fund dependent. The point, though, of having non-zero $\bar{\alpha}_i$'s is to allow the fund's α_P to systematically depend on the fund's trading strategy F. This dependence comes about through a linear term, the $\bar{\alpha}_P$ and a quadratic term b_P. There is no constant alpha term in α_P because in the long-run all alphas are assumed to be zero (their unconditional value). The linear term $\bar{\alpha}_P$ simply measures the degree to which a given fund's strategy is actually related to the instantaneous alphas of individual stocks. Since F can be positive or negative, a non-zero α_P does not indicate either under- or overperformance. The quadratic term b_P, on the other hand, does indicate exactly this – it measures the degree to which a fund is able to systematically go long (short) positive (negative) alpha stocks.[8] Note that this is a sufficient, though not necessary, condition for a given fund to exhibit occasional (as opposed to systematic) risk-adjusted outperformance. A weaker and necessary condition is that a fund's α_P is persistent and occasionally positive (which obtains when $\bar{\alpha}_P \neq 0$ and when $\gamma > 0$).

The empirical model derived above is very flexible. For example, if one assumes that η_t has a variance of zero, or that γ equals zero, the FS specification can be reproduced. Importantly, however, even absent these assumptions the model can still be estimated. Also note that nowhere does the econometrician need data on the actual portfolio weights used to produce the observed returns.[9]

Equations (2), (6), (9), and (10) can be estimated via extended Kalman filtering. To obtain the observation equation, use (9), and (10) in (2) to eliminate α_{pt} and β_{pt} and produce:

$$r_{Pt} - r_{ft} = b_P F_{t-1}^2 - k_t + \bar{\beta}_P\Big(r_{mt} - r_{ft}\Big) + \Big(\bar{a}_P + c_P\Big(r_{mt} - r_{ft}\Big)\Big)F_{t-1} + \epsilon_{Pt} \tag{11}$$

after some minor algebra. Due to the F_{t-1}^2 term standard Kalman filtering techniques will fail, as the conditional variance of $r_{Pt} - r_t$ will no longer be independent of the estimated values of F_{t-1}. The standard solution is to use a first-order Taylor expansion around the conditional expectation of F_{t-1}, or

$$\begin{aligned} F_{t-1}^2 \quad \approx \quad & 2\,\mathbb{E}\Big[F_{t-1}\Big|r_{P,t-1} - r_{f,t-1}, F_{t-2}\Big]F_{t-1} \\ & -\mathbb{E}\Big[F_{t-1}\Big|r_{P,t-1} - r_{f,t-1}, F_{t-2}\Big]^2 \end{aligned} \tag{12}$$

to replace the F_{t-1}^2 term in equation (11) where $\mathbb{E}$ is the expectations operator.[10] Equation (6) then forms the state equation.[11] Note that the vector c_P has n elements (one for each

[8]Intuitively, b_P can be thought of as the covariance between a fund's security weights (w_t) and the underlying security alphas.

[9]Of course, other modeling choices are possible, and this is an interesting area for future research. For example, some portfolio strategies lead to known security weightings. In such cases the portfolio alpha and beta in (3) and (4) may be calculated directly, as long as alphas and betas of individual stocks are known.

[10]For details about extended Kalman filtering see Harvey (1989).

[11]The estimated dynamic Kalman filter model bears some philosophical resemblance to the Bayesian approaches found in Baks, Metrick and Wachter (2001), and Pástor and Stambaugh (2002b). In those papers, the authors wish to investigate optimal fund holdings across investors with different priors regarding

risk factor) but only n-1 degrees of freedom. Thus, in the scalar case (as in the CAPM) it can be normalized to one when estimating the model. In the case where n is greater than one, at least one element's value must be fixed or some other normalization must be applied. The other fact needed for estimation is that the variance of ϵ_{pt}, conditional on time $t-1$ information, is given by

$$\mathrm{Var}_{t-1}\Big(\epsilon_{Pt}\Big) = \sum_{i=1}^{I} w_{i,t-1}^2 \mathrm{Var}_{t-1}\Big(\epsilon_{it}\Big).$$

This follows from (5), and from the fact that all ϵ_{it}'s are independent.

The system specified in equations (6) and (12) imbeds an important timing convention. The alphas and betas which determine time t returns are known at time $t-1$ (assuming that k_t is deterministic). Therefore any covariance between a portfolio's time t alphas and time t market returns indicates that the portfolio manager makes investment decisions at time $t-1$ which successfully anticipate market returns at time t. The same is true for time t betas and time t market returns. Whether a portfolio manager has such ability or not will affect the interpretation of our results in later on.

2 Problems with Constant Coefficient Models

If funds dynamically adjust their portfolio holdings in response to changes in the economy then estimates from a constant coefficient model will generally be systematically biased. As it turns out these biases are readily quantifiable. Roughly, the estimated OLS coefficients can be decomposed into a number of elements which can themselves be estimated. Thus, it is possible to determine just how biased a particular OLS coefficient may be, and what part of the dynamic structure is responsible. The analysis that follows is similar to that in both FS, and Grinblatt and Titman (1989a) but is reproduced here to accommodate this paper's particular setting and notation.

Assume that the return generating model for a given strategy is the following

$$r_{Pt} - r_{ft} = \alpha_t + \beta_t \left(r_{mt} - r_{ft}\right) + \epsilon_t. \tag{13}$$

One example of a structural derivation of such a specification is in the previous section of this paper. However, for the analysis which follows, no assumptions about the dynamics of the above coefficients and error term are necessary, other than the usual regularity conditions needed for the law of large numbers.

Now, assume that for data generated using equation (13), one estimates a single factor, constant coefficient, linear model as follows

$$r_{Pt} - r_{ft} = \hat{\alpha} + \hat{\beta}\, x_t + \eta_t, \tag{14}$$

managerial ability. As with this model, past data is used to form forecasts of future performance. However, the focus of the present model is on inferring the dynamics of mutual fund holdings, rather than on identifying skilled or unskilled managers.

where $x_t \equiv r_{mt} - r_{ft}$. The following proposition shows that the above coefficient estimates converge asymptotically to expressions which depend on the co-dynamics of α_t, β_t, and $(r_{mt} - r_{ft})$ in (13).

Proposition 1 *Using data originating from equation (13), ordinary least squares estimates of the regression in (14) converge in probability to the following limits:*

$$\begin{aligned}\text{plim}(\hat{\alpha}) &= \mathbb{E}[\alpha_t] - \frac{\mathbb{E}[x_t]}{\text{Var}(x_t)}\Big(\text{Cov}(\alpha_t, x_t) + \text{Cov}(\beta_t, x_t^2)\Big) \\ &\quad + \left(1 + \frac{(\mathbb{E}[x_t])^2}{\text{Var}(x_t)}\right)\text{Cov}(\beta_t, x_t), \end{aligned} \tag{15}$$

and

$$\begin{aligned}\text{plim}(\hat{\beta}) &= \mathbb{E}[\beta_t] + \frac{1}{\text{Var}(x_t)}\Big(\text{Cov}(\alpha(t), x_t) + \text{Cov}(\beta_t, x_t^2)\Big) \\ &\quad - \frac{\mathbb{E}[x_t]}{\text{Var}(x_t)}\text{Cov}(\beta_t, x_t). \end{aligned} \tag{16}$$

The proof is in the Appendix. Note as well that the proof easily generalizes to the multi-factor case.

2.1 Interpreting the Covariance Terms

The covariance terms that make up the OLS alphas and betas have natural economic interpretations. A fund's "selection ability" enters through the $\mathbb{E}[\alpha_t]$ term. One can think of this as the static part of the fund's alpha (denoted as α_{static} from here on). The α_{static} variable represents the alpha parameter typically sought in an OLS regression that attempts to decompose a fund's returns into factor loadings and an ability to outperform (or underperform) the market on a systematic basis. Another aspect of a fund's selection ability comes in via the $cov(\alpha_t, x_t)$ term. Formally, it represents the covariance of a fund's alpha and the market return. This covariance, however, comes about even though the fund does not necessarily alter its market risk exposure (which is captured in the $cov(\beta_t, x_t)$ and $cov(\beta_t, x_t^2)$ terms). Thus, these funds seem to be good at selecting stocks that do particularly well in up markets. As such, for expository purposes, this term will be referred to as "bull market selection ability." Finally, the $cov(\beta_t, x_t)$ and $cov(\beta_t, x_t^2)$ terms represent a fund's ability to time the market by increasing (decreasing) its beta in anticipation of high (low) market returns. Intuitively, if there exists a function relating a fund's beta to the market return these two values represent a second order Taylor series approximation of that function. From the point of view of the fund's actual operations, a high covariance estimate implies that it increases its market risk exposure when the market return is unusually high. These two terms thus represent a fund manager's "market timing ability." Later sections of the paper decompose the estimated OLS alphas and betas into their constituent parts and make use of these economic interpretations to understand their sources.

3 Data Description and Model Estimation

Monthly mutual fund data from 1970 to 2000, as supplied by CRSP, is used to estimate the model. A fund is only included if it has more than 48 months of return data. Some of the tests in the paper use data from Morningstar. For those tests, a fund must also have a Morningstar assignment into one of nine categories as of the end of 1999. The categories used in this study (the set of domestic equity funds) can be found in Table 1. These criteria leave a total of 572 funds with which to conduct the estimation. Other data includes the market factor returns, T-bill returns from Ken French's web site, [12] and the CRSP stock decile returns.

The empirical model also uses the dividend yield on the market which is constructed using a three step process. First, the dividends from the previous twelve months of the CRSP value weighted index is divided by the "with dividends" index level. Second, the same is done using the "without dividends" index level as the divisor. Third, the result from the second step is subtracted from the first to get the dividend yield.

Most of the tables and graphs presented here derive from estimating the dynamic model discussed in Section 1 within a single factor structure. Unless otherwise stated, estimates are conducted under the assumption that the $\bar{w}_i$ are constants.

A note is in order at this point about the use of Morningstar data. Since requiring that a Morningstar assignment for a given fund should exist as of 1999 introduces survivorship bias into the sample, care must be taken as to the tests that use this classification and those that do not. For analyses that seek only to characterize mutual fund alphas and betas, or look at model comparisons (but not from a performance point of view), and hence are not sensitive to survivorship issues, the Morningstar classification is used in order to provide further insights into the results. For those tests where statements about performance of a given strategy are made, no classification into Morningstar categories is done. Hence these tests use the entire CRSP mutual fund sample, thereby maintaining to the greatest possible degree unbiasedness of the data, and rendering the results comparable with those of other studies.[13]

4 Detecting and Tracking Dynamic Factors

Table 1 breaks down the funds by Morningstar category. For each category the last column displays the number of funds for which the Kalman filter estimates diverge from the static OLS estimates. In what follows these are referred to as "dynamic funds" in that they appear to employ strategies that produce time-varying alphas and betas. Note that within each category the vast majority of funds fall within the set of dynamic funds. This should not be too surprising. Fund managers are generally active traders, and as the discussion in Section 1

[12] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[13] See Elton, Gruber and Blake (2001) for a discussion of biases in the CRSP mutual fund database.

shows such activity will produce time-varying return parameters.

Table 2 reports the results of a CUSUMSQ (see Harvey (1989)) test on the residuals of each portfolio. Under the OLS specification the test has little difficulty detecting time series patterns within the residuals. Fully 97% of all funds fail the test. Relatively, the Kalman filter model does a much better job. For those cases in which the dynamic model does not converge to the OLS model the test cannot reject the hypothesis that the errors for about 31% of the funds have been purged of their time series patterns. In total this means that after using the Kalman filter 29% of the funds have had their residuals sufficiently washed of their intertemporal patterns that the test can no longer detect anything.

Looking across categories the model's ability to purge the errors of any time pattern varies somewhat. From a low of 18% in category 18 (large value) to a high of 45% in category 16 (large blend). A chi-squared test rejects the null hypothesis that the percentage differences across categories are due to chance. This rejection indicates that a fund's investment objectives will affect the model's statistical performance. However, there does not appear to be a pattern across the market capitalizations of the portfolio's target firms. Rather it is those funds that invest in large and mid-cap value stocks that seem to give the model the greatest problems. Across the other size and objective categories the results are fairly uniform.

5 Alpha and Beta Dynamics

In many studies such as Gruber (1996), Carhart (1997), and FS the estimated alphas tend to be negative. However, those alphas include the fund's expenses and thus represent what might be called the "investor's alpha." Here, as in Grinblatt and Titman (1989b) fund expenses and performance alphas are estimated separately. Under the model, a fund incurs expenses at an estimated rate k. In exchange, the fund manager generates an informative signal F that produces occasional excess returns by allowing trades based upon a stock's sensitivity to the signal via the parameter $\bar{\alpha}$. Table 3 displays the model's average parameter estimates across Morningstar categories. For most categories the estimated expenses are about 1.5% per annum (a monthly k of 0.0013). Given industry filings this seems to be about right, since expenses in this case include both management fees and transactions costs.

The fact that the estimated $\bar{\alpha}_P$'s are non-zero suggests that stocks in the economy have non-zero $\bar{\alpha}_i$'s. This indicates that, in general, funds choose trading strategies which are related to the instantaneous alphas of stocks in the economy. This, together with the fact that the γ_F's are non-zero, suggests that there is some hope of finding funds that are currently in an "outperformance" period (recall the discussion of Section 1). From equation (9), note that b_P measures the degree to which funds choose trading strategies that systematically profit from variation in security alphas over time. Table 3 suggests that the average fund within each category has no ability in this area. The two exceptions to this are small growth funds (category 39), which overall have some ability to spot high alpha stocks, and small value funds (category 40) which seem to have the unfortunate ability to systematically go long negative alpha stocks.

Table 3 also provides an estimate of the degree to which fund betas vary over time.[14] Some algebra shows that the estimated standard deviation of beta equals $\sqrt{\sigma_F^2/(1-\gamma_F^2)}$. These values range from a low of 0.18 to a high of 0.41 per month, and average .27. For a typical fund with an intertemporal average beta of one, this implies that in any one period the 95% confidence interval for its beta lies within .5 and 1.5. Empirically then, trading appears to induce economically significant time variation in mutual fund betas. If anything, one's intuition might indicate that the variation is too large, but consider that almost half of all funds have documented records of moving at least 20% of their assets (over the time period from 1991–1999) from stocks into bonds, and vice versa (Mamaysky and Spiegel (2002)).[15] Also, note that the drift in the fund's beta is not a random walk, as the signal is assumed to mean revert. The estimated persistence parameter (γ_F) takes on values between 0.12 and 0.35 in the data. These rather low estimates indicate that funds deviate from their baseline portfolio betas for only a few months at a time.

Along with the Morningstar categories a natural division across funds is by their response to either their own portfolio's or the market's past returns. Those employing a "momentum style" can be expected to increase their market risk exposure in response to high past returns while "contrarians" should decrease their market risk exposure. Because the Kalman filter model estimates each fund's beta over time it is particularly well suited to the task of identifying such reactions. Table 4 reports the results from regressing each fund's current market beta on either the market or the fund's own lagged return. It shows that the model can identify with 95% confidence that just under 10% of all funds act as market contrarians and just under 8% as market momentum traders. These figures are significantly different from those one expects via chance at any reasonable level.[16] Not too surprisingly, given the relatively similar number of funds identified as either market momentum or contrarian traders, the hypothesis that a fund is equally likely to follow either strategy or that the distribution across categories is identical cannot be rejected.[17] However, while funds are about equally likely to fall into either category their out of sample returns differ significantly. The market momentum traders earn a statistically insignificant negative four factor risk adjusted out of sample return. In contrast, the contrarians earn a statistically significant

[14]Since this is a one factor model, the value of c_P has been set to one as a normalization.

[15]Such behavior seems consistent with an attempt to implement something like Breen, Glosten, and Jagannathan's (1989) algorithm for optimally shifting between treasury bills and stocks. When done properly they show that such a strategy can potentially add as much as 2% to a fund's annual returns.

[16]Here market momentum and contrarian trading strategies refer only to a how a fund adjusts its exposure to market risk in the current period in response to the previous period's market return. They have no other connotations. Thus, it is possible that a market momentum fund reacts to a high previous period market return by reducing its cash position and then purchasing a set of stocks it did not previously hold. This is not a strategy that would take advantage of the well known "momentum anomaly' that seems to influence individual stock returns.

[17]The χ^2 statistic with 8 degrees of freedom from the Kruskal-Wallis test equals 13 which is not significant at even the 10% level.

12bps per month. This seems to indicate that at least some funds can time the market to some degree and offers some insight into the strategy they use to do so.

Table 4 also shows that for many funds their own past returns influence their current market risk exposure. However, unlike the results discussed above the funds are no longer split evenly between momentum and contrarian strategies. With 95% confidence the model identifies about 23% of all funds as using a momentum strategy with regard to their own returns, and about 12% as contrarians. Both of these figures are statistically different from what one might expect to get by chance and they are also statistically different in size from each other. Essentially, more fund managers increase their market exposure in response to high returns in their own fund than decrease it. However, while funds may vary in their response to their own past returns no group seems to provide its investors with any overall risk adjusted outperformance.[18]

6 Decomposing Alphas and Betas

As discussed in Section 2 if the dynamic model accurately describes mutual fund return dynamics then the OLS parameter estimates are actually an agglomeration of several covariance terms. If so, then this provides a mechanism for checking the model. By using the equations in Proposition 1 one can create "synthetic" OLS regression estimates by properly summing up the covariance of the dynamic alpha and beta estimates with the market portfolio. The resulting values can then be compared to what one obtains by actually running an OLS model on the data. If the dynamic model properly describes the data, then the synthetic and actual values should be fairly close to each other. Table 5 reports the results from this experiment. Column six shows that in no category does the average absolute percentage difference between the synthetic and the actual OLS parameter estimate for alpha exceed 6% and it is generally under 1%. For beta every category displays an absolute average difference under 1% (column 8).

Another test of the Kalman filter model is the degree to which it can better explain the data relative to both a standard and rolling OLS model. The standard model produces parameter estimates based upon a fund's entire history. The rolling OLS model uses only the 48 months of data prior to any particular date. To calculate an R^2 statistic for the rolling OLS model only the final period's error term is used. This gives the OLS model a natural edge since the squared errors come from finding the set of parameter estimates that best fit only the last four years of data, while the Kalman filter parameter estimates are forced to fit the entire time series.

[18] The propensity of funds to act as momentum rather than contrarian traders with regard to their own past returns shows statistically significant variation across categories. The Kruskal-Wallis test produces a χ^2 statistic of 22 with eight degrees of freedom. This is significant at the 1/2% level. However, looking across the categories it is difficult to find any strong economic interpretation for this result. For example, the two most likely groups to produce own-return momentum traders are the large capitalization value fund and the mid-capitalization growth funds.

Columns four and five from Table 5 show that the dynamic model generates a higher R^2 relative to both the ordinary and rolling OLS model by 0.09 to 0.30 depending upon the category. Thus, in every case the Kalman filter model, with its time-varying alphas and betas, does a better job of tracking fund returns. While this is clearly not conclusive, since the Kalman model may be over fitting the data, later tables provide out of sample tests (where over fitting is obviously penalized) with similar results.

Table 6 breaks down the OLS alphas into their constituent parts based upon equations (15) and (16), and as discussed in Section 2.1. Table 7 shows the percentage contributions of each component to the OLS alpha. Recall from (15) that the OLS alpha is composed of four components. The percent contribution for component i is given by

$$\text{Percent Contribution} \equiv \frac{|c_i|}{|c_1| + |c_2| + |c_3| + |c_4|}.$$

where c_i is the i^{th} component. Panel A reports results from first aggregating the cross-section of funds to compute each component, and then computing the percent contribution using the absolute value of the cross-sectional means of each component. Panel B computes the percent contribution at an individual fund level, and then takes an average of these for the category level numbers.

Note from Panel A that the selection ability measure (the static part of the dynamic alpha) can account for as little as 13% of the estimated OLS value. For categories 38 (small blend), 39 (small growth), and 40 (small value) selection ability accounts for under 50% of the estimated OLS alpha. This implies that OLS estimates for small capitalization fund alphas may be very misleading. Along the same lines the OLS model's parameter estimates become unstable as the fraction of the dynamic alpha explained by selection ability declines. The Spearman rank correlation between the absolute value of the OLS t-statistics in column one of Table 6 and column two and the α_{static} percentage in Table 7 equals .82 (t-statistic of 3.74). This implies that the OLS alphas become more reliable (have higher t-statistics) when funds use strategies that produce stable alphas (have high α_{static} percentages). Using the standard deviation of the OLS alpha estimates instead of the t-statistics produces a similar result; a rank correlation of -.77 and a t-statistic of -3.16. Thus, as the α_{static} percentage increases the standard deviation of the estimated OLS alpha goes towards zero. Overall this indicates that the poor intertemporal stability of OLS alpha estimates may derive in part from the fact that the funds in question use dynamic strategies and this leads to misspecification errors when a static statistical model is used to produce estimates.

What seems to drive the difference between the OLS and dynamic alphas is each fund's market timing ability (the covariance in each fund's beta with the market) and only to a much smaller degree bull market selection ability (the covariance between the fund's alpha and the market). The market timing ability accounts for 21% or more of the estimated OLS alpha in half the categories. In contrast, the median contribution of bull market selection ability is only 5.5%. Within the static OLS model it thus appears that because of the dynamic strategies employed by funds the induced time variation in beta leads to erroneous conclusions about performance (alpha).

Table 7's Panel B produces similar conclusions to those of Panel A. The primary change is that looking at the component contributions this way (i.e. first computing the percent contribution at the fund level, and then aggregating into categories) increases the importance of the fund beta with the squared market return (the second order term in the Taylor series approximation). This increase generally comes at the expense of the contribution made by the covariance between a fund's beta and the market (the first order term). Overall though, it is the time variation in beta that seems to induce the discrepancy between the estimated static OLS alphas and the steady-state values of the dynamic Kalman filter alphas.

Table 8 decomposes the OLS beta estimates into their constituent parts. These tables show that the OLS betas are quite close to the expected value of the dynamic beta. In no case does the OLS beta differ from the expected value of the dynamic beta by even 10%. However, this does not mean that month by month the OLS beta equals the fund's actual beta, only that the long run averages are the same. As shown in Table 3 column ten, month by month fund betas exhibit considerable volatility and any long run average value is likely to be far from the current mark.

Because the Kalman filter model simultaneously estimates the alpha and beta dynamics it is possible to determine the degree to which one influences the other. Table 9 examines this issue by sorting funds according to their static alphas (selection ability) by thirds designated as low, medium, and high and their market timing ability (again in thirds).[19] Out of sample the static alphas do not help forecast future fund returns. More interesting though, is the comparison across the market timing ability groups. In sample, market timing seems to help for the low and medium selection ability funds, but not the high selection ability funds. Out of sample, however, there is evidence that funds with high market timing ability produce higher returns than those with low market timing ability. The t-statistics range from 1.74 to 1.90 and are all significant at the 10% level.

Table 10, like Table 9, breaks funds down by their ability estimates. This time the comparison is with each fund's bull market selection ability $cov(\alpha_t, x_t)$ and market timing ability $cov(\beta_t, x_t^2)$ and $cov(\beta_t, x_t^2)$. Remarkably, in sample there is little variation in the estimated returns across categories except for those funds with low scores on both ability measures. Out of sample, however, it again appears that funds with high market timing ability estimates do well while other funds do not. Bull market selection ability appears to play no role in a fund's future performance. This reinforces the conclusions from earlier tables. Some funds apparently adjust their beta in anticipation of future market returns; those that are successful produce higher returns out of sample than their counterparts and as was shown earlier also generate less accurate in sample OLS alpha estimates.

[19] Note that the Morningstar categories are not used in this table and thus the out of sample tests do not suffer from survivorship bias. The out of sample data set is further described in Section 7.

7 Out of Sample Tests

The tests presented so far have primarily been within sample. As Ghysels (1998) shows even if a model with time-varying risk factors performs well in sample, it may not outperform simpler models out of sample. Since both the simple time invariant and more complex dynamic models are likely to be misspecified it is an empirical question as to which will work better. The out of sample tests in this section are designed to answer this question, and as will be shown the dynamic Kalman filter model often produces superior out of sample predictions.

For a fund to be included in these out of sample tests, that fund must included in the CRSP mutual fund database and have an ICDI objective of AG, BL, GI, IN, LG, PM, SF or UT. A fund then stays in the sample until the end of the sample period in 2002, or until it ceases to exist. Hence, all tests in this section control for survivorship bias. Furthermore, note that none of the results in this section use Morningstar classification data, and therefore are free of the survivorship bias that such classifications can introduce (see the discussion in Section 3).

In the out of sample tests each model is asked to forecast each fund's upcoming alpha and beta in each period. A portfolio for the period with a predicted zero alpha and zero beta is then formed by going long the fund, taking countervailing positions in the underlying factors, and then subtracting the predicted alpha value. By repeating the above procedure a time series of returns from 1970 to 2002 is produced. This return sequence is then regressed against the appropriate factor model. A model without any forecasting error should produce portfolios that yield excess returns (alphas) and factor loadings (betas) of exactly zero. Positive regression parameters indicate that a model has *underestimated* a value, while negative regression parameters imply the opposite. Table 11 reports the resulting distributions from bootstrapping the regression results 1,000 times. Each bootstrap samples the pool of funds with replacement prior to conducting the calculations.

Table 11 Panel A displays the distribution of the resulting excess returns (alphas). Ideally, each model should produce a nearly mean zero error and encompass zero within as small a confidence interval as possible. Of the four models tested only the one factor Kalman model meets the latter criteria. Thus, if the goal is to produce an unbiased forecast of a fund's alpha this model appears to provide the best performance. By contrast, the one factor OLS model produces portfolio returns with a 95% confidence of 3.72bps to 7.29bps (basis points) per month; indicating that it underpredicts fund alphas. For both the four factor OLS and Kalman filter models the results are reversed. In these two cases the 95% confidence intervals are entirely negative, implying that they tend to overpredict fund alphas. Between the four factor Kalman and OLS models, the four factor Kalman model has a median return closer to zero and a 90% confidence interval that is also closer to zero. The only place in the distribution where the OLS model appears to be superior is at the fifth percentile. Here its return is somewhat closer to zero: -5.00bps versus -5.28bps for the Kalman filter model. This indicates that the four factor Kalman filter model is somewhat more likely to produce a large overestimate of a fund's alpha than is the four factor OLS model. Perhaps one should expect this from a relatively nonlinear model but the error is not symmetric. At the ninety-fifth percentile the Kalman filter's return is much closer to zero than the OLS model's return. Here the OLS model yields an error of -2.59bps versus only -1.25bps for the Kalman filter model. The implication is that the Kalman filter model is much *more* likely than the OLS model to produce a *very small* overestimate of a fund's alpha. Thus, the four factor Kalman filter model appears to be the more accurate prediction tool from the tenth percentile and up in the return distribution.

Table 11 Panel B reports the bootstrapped distribution of the "return weighted beta error." This is constructed by multiplying the factors estimated on the out of sample predicted zero-alpha zero-beta returns by each factor's average value over the sample period and adding the products together:

$$\text{return weighted beta error} = \sum_i \hat{\beta}_i \bar{r}_i. \tag{17}$$

In this equation $\hat{\beta}_i$ is the estimated value of factor i from the regression, and $\bar{r}_i$ the factor's average return during the sample period. This metric is designed to give greater weight to those factors which if misestimated will yield the largest systematic errors regarding a fund's predicted performance. The closer a model comes to producing return weighted beta errors of zero the better it is at predicting a fund's overall future factor risks and returns.

Table 11 Panel B shows that each of the single factor models produces a nearly identical return weighted beta error distribution. Both, however, are biased towards negative values and neither has a 95% confidence interval that covers zero. This implies that in both cases the market betas are overestimated and thus the "zero beta" portfolios are in fact over hedged. In contrast to the single factor models, the four factor OLS model has the opposite problem as evidenced by the fact that its 95% confidence interval is entirely positive. Thus, this model appears to systematically underpredict a fund's factor loadings, leading to portfolios that are under hedged. The only model that encompasses zero within the 95% confidence interval is the four factor Kalman model. If one looks at the median values in absolute terms the best performing models are the two Kalman models and the one factor OLS model. These three models yield median errors with absolute values between 0.62bps and 0.65bps. The four factor OLS model's performance on this metric is relatively poor. Its median value of 1.64bps is worse (in absolute value terms) than the errors from any of the other three models at either the fifth or ninety-fifth percentiles. Thus, if one is interested in producing accurate out of sample beta forecasts it appears that in the one factor case both the OLS and Kalman models will perform about equally well; however, in the four factor case the evidence indicates that the Kalman model will lead to smaller prediction errors than the OLS model.

One might think that the relative out of sample performance of each model might depend on the variability of a fund's predicted betas. To test this the bootstrap was repeated on subsamples divided into groups based upon the variance of each fund's market beta as estimated by the Kalman filter model. Intuitively, one would expect the OLS model to produce better predictions for the low variance funds and the Kalman filter model for high variance funds. Overall there appears to be no systematic patterns that are not evident in Table 11. To conserve space the table from testing this hypothesis is not included here since the results are primarily negative. As in the whole sample, the Kalman filter model is more likely to cover zero in its 95% confidence interval for both the out of sample alphas and return weighted betas in the subsamples that were tested.

8 Comparison with FS Model

As noted earlier the dynamic model developed here has as a special case the FS model. However, so far all tests have been conducted under the restriction that the fund betas depend only upon some unobservable factor. This section examines the impact on the estimated model when observable conditioning information is added. The tests conducted here use the lagged treasury bill rate and the dividend yield on the CRSP value weighted index. Thus, the equation for $\beta_{P,t}$ becomes

$$\beta_{P,t} = \beta + F_t + k_1 z_{1,t-1} + k_2 z_{2,t-1}. \tag{18}$$

If the observable information improves the model's predictive ability then k_1 and k_2 should differ from zero. To test this the model was run with and without the conditioning variables on the monthly returns of 437 mutual funds during the period of 1994 to 1998. Asymptotically, the likelihood ratio under the null should follow a chi-square distribution with two degrees of freedom. In Figure 1, the bars represent the cross-sectional distribution of the likelihood ratio while the dashed line traces out a chi-square distribution with two degrees of freedom.

Overall, the null hypothesis that k_1 and k_2 are zero cannot be rejected at the traditional 1%, 5% or 10% levels. However, for individual funds, the fraction that reject the null hypothesis at the 1%, 5% or 10% level is 10.8%, 15.7% and 20.3%, respectively. These numbers are somewhat higher than might be expected by chance, which implies that for some funds the conditioning information appears to improve the model's fit.

Table 12 provides further evidence about the richness of the dynamic coefficient model used in this paper. This table shows the R^2's of fund return regressions on a market index using OLS, the FS two factor conditional beta model, and the Kalman model of this paper.[20] As can be seen, across all fund categories and for the entire sample, the FS model provides an improved fit relative to the OLS model. Consider however how the R^2 statistic changes as one moves across models. The increase when one goes from the OLS to FS model is approximately a tenth as large as the increase obtained when moving from the FS to the Kalman model.

[20] The Kalman model used in these tests does not use the FS conditioning variables and looks only at the return series of funds and of the market index.

Based upon this, it appears that the Kalman model can account for a considerably larger portion of fund return fluctuations than either the OLS or the FS models.

Between this paper, FS, and Grinblatt and Titman (1989a) there is now considerable evidence that mutual fund managers produce portfolios with time-varying betas, and possibly alphas, too. Thus, it is clear that portfolio managers are altering their portfolios in response to some set of economic variables. Why then are k_1 and k_2 statistically indistinguishable from zero for most funds? The model has two ways of fitting a fund's alphas and betas. One way is to use the observable conditioning variables in some manner. Another is to use the estimated lagged values of alpha and beta, and then let them change according to an estimated relationship with an unobserved factor following an AR(1) process. Figure 1 indicates that the latter prediction method often dominates, at least when using the lagged treasury bill rate and dividend yield on the CRSP value weighted index. One conclusion may be that a few funds use treasury bill rates and the market dividend yield to help manage their assets, while most do not. More practically, one can use the model to identify both those funds with a more macro based approach to asset allocation and the variables they concentrate on.

9 Conclusion

Even if security returns are well described by a factor model with time invariant loadings, the same will not be true of an actively traded portfolio holding these securities. This point goes at least back to Admati and Ross (1985), and Dybvig and Ross (1985). This paper develops an empirical specification of which the FS setting can be thought of as a special case. In FS, time-varying factor loadings are estimated via the use of observable macro economic factors. By contrast, this paper assumes that the portfolio holdings may also vary in response to some unobservable variable that follows an AR(1) process.

In terms of both fitting the historical data and making out of sample predictions the empirical generalizations presented here can provide many potential advantages. By assuming an unobservable variable with a known stochastic process drives portfolio holdings, the econometrician can effectively use past changes in a portfolio's alpha and betas to predict future changes. If the underlying assumptions are even approximately true, then one expects the model to better fit the data. In fact, a number of tests presented here show that it does.

Looking at the historical data, the empirical results show that the dynamic model developed here does a much better job of capturing mutual fund portfolio returns and factor loadings than does a static OLS model. This opens up the possibility that managerial over performance, even if it exists, may be difficult to detect with a static model. In addition, because the model produces dynamic estimates it is possible to explore what trading strategies seem to be associated with either historical or predictive out performance. One strategy examined here is whether or not some funds attempt to time the market. The data indicates that a substantial number of funds do, although with varying success.

Out of sample tests also indicate that the Kalman filter model has a number of desirable properties. Bootstrap tests lay out the relative predication errors from the one and four factor OLS and Kalman filter models tested here. For out of sample alphas the four factor Kalman filter model has a generally tighter distribution around zero than its four factor OLS counterpart. When it comes to the out of sample betas the four factor OLS model systematically underpredicts them. In contrast, the four factor Kalman filter model encompasses a zero prediction error within the standard confidence intervals.

Finally, the empirical work examines the degree to which observing macro economic variables such as treasury bill rates and the dividend yield on the CRSP equally weighted index helps to fit the data. Indeed, if one assumes that a fund's trading strategy is driven by some unobservable variable following an AR(1) process, does observing these two variables help at all? Statistically, adding these observable variables to the model does not improve its overall fit, and the estimated coefficients are statistically indistinguishable from zero. However, for some funds the conditional information is helpful, and this may indicate that while some fund managers trade on the macro economic variables included in the estimation process, most do not.

Appendix

A Proof of Proposition 1

The data generating process is given by equation (13), reproduced here for convenience

$$y(t) = \alpha(t) + \beta(t)\, x(t) + \epsilon(t),$$

where $y(t) \equiv r_P(t) - r(t)$, $x(t) \equiv r_m(t) - r(t)$, and where we assume that $\mathbb{E}[\epsilon(t)] = 0$ and $\mathbb{E}[\epsilon(t)|x(t)] = 0$. Consider the OLS estimate of the following constant-coefficient equation:

$$y(t) = \hat{\alpha} + \hat{\beta}\, x(t) + \hat{\eta}(t).$$

Define $X \in \mathbb{R}^{T\times 2}$ and $Y \in \mathbb{R}^{T}$ as

$$X \equiv \begin{bmatrix} 1 & x(1) \\ \vdots & \vdots \\ 1 & x(T) \end{bmatrix} \qquad Y \equiv \begin{bmatrix} y(1) \\ \vdots \\ y(T) \end{bmatrix}.$$

Then the OLS estimates $\hat{\alpha}$ and $\hat{\beta}$ are given by

$$\begin{aligned} \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} &= \left(\frac{1}{T}X'X\right)^{-1}\left(\frac{1}{T}X'Y\right) \\ &= \frac{1}{v}\begin{bmatrix} \frac{1}{T}\sum x(t)^2 & -\frac{1}{T}\sum x(t) \\ -\frac{1}{T}\sum x(t) & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{T}\Big(\sum \alpha(t) + \sum \beta(t)x(t) + \sum \epsilon(t)\Big) \\ \frac{1}{T}\Big(\sum x(t)\alpha(t) + \sum \beta(t)x(t)^2 + \sum x(t)\epsilon(t)\Big) \end{bmatrix}, \end{aligned}$$

where

$$v \equiv \frac{1}{T}\sum x(t)^2 - \left(\frac{1}{T}\sum x(t)\right)^2.$$

Note all the summations are over $t = 1, \ldots, T$. We then find that

$$\begin{aligned} \hat{\alpha} &= \frac{1}{v}\Big[\frac{1}{T}\sum x(t)^2 \frac{1}{T}\Big(\sum \alpha(t) + \sum \beta(t)x(t) + \sum \epsilon(t)\Big) \\ &\qquad -\frac{1}{T}\sum x(t)\frac{1}{T}\Big(\sum x(t)\alpha(t) + \sum \beta(t)x(t)^2 + \sum x(t)\epsilon(t)\Big)\Big] \\ &\to \frac{1}{\mathrm{Var}(x(t))}\Big[\mathbb{E}[x(t)^2]\Big(\mathbb{E}[\alpha(t)] + \mathbb{E}[\beta(t)x(t)]\Big) - \mathbb{E}[x(t)]\Big(\mathbb{E}[\alpha(t)x(t)] + \mathbb{E}[\beta(t)x(t)^2]\Big)\Big] \\ &= \frac{1}{\mathrm{Var}(x(t))}\Big[\mathbb{E}[\alpha(t)]\mathrm{Var}(x(t)) - \mathbb{E}[x(t)](\mathrm{Cov}(\alpha(t), x(t)) + \mathrm{Cov}(\beta(t), x(t)^2)) \\ &\qquad + \mathbb{E}[x(t)^2]\mathrm{Cov}(\beta(t), x(t))\Big]. \end{aligned}$$

where the limit is in probability, and follows from an application of the law of large numbers. Also we have that

$$\begin{aligned}
\hat{\beta} &= \frac{1}{v}\left[-\frac{1}{T}\sum x(t)\frac{1}{T}\Big(\sum\alpha(t)+\sum\beta(t)x(t)+\sum\epsilon(t)\Big)\right.\\
&\qquad\left.+\frac{1}{T}\Big(\sum x(t)\alpha(t)+\sum\beta(t)x(t)^2+\sum x(t)\epsilon(t)\Big)\right]\\
&\to \frac{1}{\mathrm{Var}(x(t))}\Big[-\mathbb{E}[x(t)]\Big(\mathbb{E}[\alpha(t)]+\mathbb{E}[\beta(t)x(t)]\Big)+\mathbb{E}[\alpha(t)x(t)]+\mathbb{E}[\beta(t)x(t)^2]\Big]\\
&= \frac{1}{\mathrm{Var}(x(t))}\Big[\mathbb{E}[\beta(t)]\mathrm{Var}(x(t))+\mathrm{Cov}(\alpha(t),x(t))+\mathrm{Cov}(\beta(t),x(t)^2)\\
&\qquad -\mathbb{E}[x(t)]\mathrm{Cov}(\beta(t),x(t))\Big].
\end{aligned}$$

Again the limit is in probability, and follows from an application of the law of large numbers. Q.E.D.

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Table 1: **Descriptive statistics.** The statistical analysis uses CRSP mutual fund monthly return data from 1970 to 2000. To be included in this table a fund must have more than 48 months of valid return data, and located in one of nine Morningstar categories as of 1999. Reported below are the 9 Morningstar categories, the total number of funds within each category, the mean excess return and Sharp ratio for funds within each category, and the number of funds estimated to be "dynamic" in each category. A fund is said to be dynamic if the Kalman filter estimates do not converge to the static OLS estimates.

Category	Category Name	Total Funds	Return	Sharp Ratio	Dynamic Funds
16	Large Blend	112	0.0096	0.2334	96
17	Large Growth	107	0.0129	0.2559	85
18	Large Value	105	0.0077	0.1961	93
22	Mid-Cap Blend	66	0.009	0.1959	57
23	Mid-Cap Growth	48	0.0133	0.2137	28
24	Mid-Cap Value	36	0.0069	0.1589	31
38	Small Blend	29	0.0085	0.1734	28
39	Small Growth	50	0.0136	0.1938	36
40	Small Value	19	0.0075	0.1662	19
Summary		572			473

Table 2: **Convergence of Kalman filter and the CUSUMSQ residual test.** This table reports the convergence properties of the Kalman filter estimates and the CUSUMSQ residual test. Each entry lists the number of funds. For funds in the OLS columns the Kalman filter estimates converge to the OLS parameters. For funds under the DYN column the Kalman filter estimates diverge from the OLS estimates. The last column lists the total number of dynamic funds within each category (the sum of the two DYN columns). A "Q=0" indicates that the CUSUMSQ test accepts the null that the residual has no time structure, at the 5% level. A "Q=1" indicates that the CUSUMSQ test rejects the null that the residual has no time structure, at the 5% level.

Cate	funds	OLS,$Q = 0$	OLS,Q=1	DYN,Q=0	DYN,Q=1	Dynamic Funds
16	112	2	14	43	53	96
17	107	3	19	33	52	85
18	105	0	12	17	76	93
22	66	4	5	21	36	57
23	49	2	19	12	16	28
24	36	2	3	6	25	31
38	29	0	1	6	22	28
39	50	3	11	8	28	36
40	19	0	0	4	15	19
SUM	573	16	84	150	323	473
Percentage	100	3	15	26	56	83

Table 3: **Estimation of a one-factor model.** This table reports the parameters estimated from the following model: $r_P(t) - r(t) = -k + b_P F(t-1)^2 + \bar{\beta}_P(r_m(t) - r(t)) + (r_m(t) - r(t) + \bar{\alpha}_P)F(t-1) + \epsilon_P(t)$, where $F(t) = \gamma_F F(t-1) + \eta_F(t)$. Here $r_m(t)$ is the market portfolio's return. $\bar{\beta}_P$ is the static exposure of the portfolio to the market excess returns. σ_ϵ^2 is the variance for $\epsilon_P(t)$, and σ_F^2 is the variance for $\eta_F(t)$. In the last column, σ_β is the standard deviation of beta ($\sigma_\beta = \sqrt{\sigma_F^2/(1-\gamma_F^2)}$). For each category of funds, the first line reports the cross sectional mean for the estimated parameters while the second line reports the cross sectional T-ratio for the mean. The whole system is estimated using an extended Kalman filter.

Category		$-k$	$\bar{\beta}_P$	γ_F	σ_ϵ^2	σ_F^2	$\bar{\alpha}_P$	b_P	σ_β
16	mean	-0.0008	0.9332	0.3446	0.0001	0.0348	0.0327	0.0746	0.20
	T ratio	-3.2913	67.6583	8.7938	7.9700	5.1765	3.8792	0.7357	
17	mean	-0.0008	1.0909	0.3251	0.0002	0.0295	0.0778	-0.0236	0.18
	T ratio	-3.2203	65.0044	7.9177	7.8602	7.2951	8.9236	-0.1902	
18	mean	-0.0013	0.8467	0.3475	0.0001	0.0316	0.0696	-0.2073	0.19
	T ratio	-3.9877	63.9289	8.8629	8.7819	7.6943	7.8703	-1.8951	
22	mean	-0.0022	0.9279	0.3486	0.0003	0.0750	-0.0026	-0.0155	0.29
	T ratio	-5.1357	40.6962	6.4198	11.7703	3.7868	-0.1993	-0.1168	
23	mean	0.0004	1.1303	0.3294	0.0004	0.0702	0.0538	0.2342	0.28
	T ratio	0.7450	23.4796	5.1599	6.6534	4.7233	3.3583	1.2607	
24	mean	-0.0016	0.8574	0.4041	0.0004	0.0517	0.0399	-0.2657	0.25
	T ratio	-2.6351	31.5863	4.9125	5.5452	3.4231	2.7608	-0.8768	
38	mean	-0.0013	0.8516	0.2099	0.0006	0.1194	-0.0279	-0.1612	0.35
	T ratio	-1.7792	28.8316	3.2067	13.1113	3.2952	-1.1432	-0.9159	
39	mean	0.0002	1.1997	0.1272	0.0008	0.1640	0.0817	0.3336	0.41
	T ratio	0.2154	23.6649	2.3737	4.9963	4.5008	3.2419	2.1897	
40	mean	-0.0013	0.7885	0.3351	0.0006	0.0897	-0.2185	-0.2708	0.32
	T ratio	-0.9082	18.1309	3.7387	6.4321	4.2417	-0.8279	-2.8581	

Table 4: **Beta strategies in response to market or fund realized returns (1970-2002).** This table explores how mutual funds change their betas in response to past market or fund returns. A mutual fund is defined as adopting a momentum, invariant, or contrarian strategy in response to the market return, if the parameter b in the following regression: $\beta_t = a + b \times \bar{r}^{MKT}_{t-1,t-LAG} + \epsilon$ is positively significant at the 5% level (1-side test), insignificant, or negatively significant at the 5% level, respectively. Here β_t is estimated by the Kalman filter, $\bar{r}^{MKT}_{t-1,t-LAG}$ represents the average market excess return realized in month $t-1$ to $t-LAG$, and LAG is 6 months in the current case. Similarly, a mutual fund is defined as adopting a momentum, invariant, or contrarian strategy in response to its own return, when $\bar{r}^{MKT}_{t-1,t-LAG}$ is replaced by the average risk-adjusted return realized in the previous 6 months. (The risk adjusted return is defined as the fund's return minus the product of its estimated beta from the Kalman filter and the market return) Using the dynamic funds from Table 1 Panel A sorts the funds by their Morningstar category and within each category reports the number of funds for which the model can detect a particular dynamic beta strategy. Panel B forms an equal-weighted (EW) portfolio for funds adopting a same dynamic beta strategy. Reported is each portfolio's 4-factor adjusted return and corresponding t-statistic.

	Market Return			Fund Return		
	Momentum	Invariant	Contrarian	Momentum	Invariant	Contrarian
A. Number of Funds in each category						
Large- Blend	9	80	7	20	67	9
Large-Growth	9	68	8	23	58	4
Large- Value	12	77	4	30	58	5
Mid - Blend	4	45	8	12	36	9
Mid -Growth	2	25	1	10	16	2
Mid - Value	2	28	1	5	25	1
Small- Blend	1	24	3	1	19	8
Small-Growth	3	32	1	5	31	0
Small- Value	0	14	5	4	13	2
Sum	42	393	38	110	323	40
B. 4-factor adjusted return for each strategy						
Alpha	-0.0003	-0.0001	0.0012	0.0004	-0.0002	-0.0003
T-stat	-0.6006	-0.1873	2.0458	0.8734	-0.4413	-0.5098

Table 5: R^2 **increment and decomposition error of the OLS Alpha and Beta.** Kalman filter estimates are used to calculate predicted OLS alphas and betas according to the following formula $\hat{\alpha} = E(\alpha_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)}cov(\alpha_t, x_t) + (1 + \frac{\mathbb{E}(x_t)^2}{Var(x_t)})cov(\beta_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)}cov(\beta_t, x_t^2)$ and $\hat{\beta} = E(\beta_t) + \frac{1}{Var(x_t)}cov(\alpha_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)}cov(\beta_t, x_t) + \frac{1}{Var(x_t)}cov(\beta_t, x_t^2)$, where the α_t and β_t are dynamic portfolio alphas and betas, and x_t the market excess return. Asymptotically $\hat{\alpha}$ and $\hat{\beta}$ should converge to the OLS estimates. The table also lists the R^2_{OLS} from the OLS regression, and the R^2 improvement from estimating the dynamic Kalman filter model. Consistent with the OLS model, the R^2_{Kal} for the Kalman filter equals $1 - \mathbb{E}(\epsilon_P(t)^2)/\mathbb{E}((y_t - \bar{y})^2)$, where y_t is the excess portfolio return and $\epsilon_P(t)$ is the residual from the dynamic model. The variable $\bar{y}$ equals the mean value of y_t, and $\triangle R^2 = R^2_{Kal} - R^2_{OLS}$. The $\triangle R^2_2$ measure compares a rolling OLS model with the Kalman filter. Here the rolling OLS model estimates employ data from the previous 48 months, with the current month as the 48th. A residual from the 48th month is then calculated and used to compute the R^2 statistic, labeled R^2_{ROLS}. Finally, the time period is incremented by one and the process repeated until the end of the data set has been reached. The $\triangle R^2_2$ variable equals $R^2_{Kal} - R^2_{ROLS}$. The numbers in the corresponding columns report the absolute and average percentage errors between $\hat{\alpha}$ and α_{OLS}, and the corresponding statistics regarding the fund betas.

category		R^2_{OLS}	$\triangle R^2$	$\triangle R^2_2$	$\lvert\%err\alpha\rvert$	$\%err\alpha$	$\lvert\%err\beta\rvert$	$\%err\beta$
16	mean	0.8502	0.0980	0.1046	1.8375	-0.7134	0.1491	0.0115
	T ratio		8.3080	7.0693	4.3327	-1.5392	3.9289	0.2793
17	mean	0.7900	0.1590	0.1687	1.9367	-1.0208	0.1472	-0.0400
	T ratio		13.3937	10.6376	3.4011	-1.6993	3.3324	-0.8493
18	mean	0.7523	0.1971	0.2531	3.2518	-1.2980	0.2660	-0.0834
	T ratio		14.4164	11.1611	3.2965	-1.2474	4.0752	-1.1742
22	mean	0.7262	0.1612	0.1606	1.2465	-0.1311	0.1924	0.0234
	T ratio		7.8809	7.9059	3.4214	-0.3236	2.5423	0.2912
23	mean	0.6909	0.2240	0.2233	1.7470	-0.9109	0.2501	-0.0195
	T ratio		7.6710	7.1438	2.2648	-1.1014	3.2108	-0.2105
24	mean	0.6749	0.1859	0.1792	1.9382	0.9353	0.2505	0.0245
	T ratio		8.2108	6.3536	2.7805	1.2063	3.2131	0.2651
38	mean	0.5717	0.2170	0.2185	2.4482	1.7071	0.5077	0.3839
	T ratio		8.2048	7.5761	2.3324	1.5357	1.9069	1.3956
39	mean	0.5525	0.2859	0.3018	0.8575	-0.1833	0.0257	0.0068
	T ratio		7.7169	7.5502	2.3994	-0.4544	1.6708	0.4149
40	mean	0.5640	0.2095	0.1878	5.5118	3.4770	0.6355	0.4031
	T ratio		6.0037	6.0649	1.8530	1.1037	1.9330	1.1526

Table 6: **Decomposition of the OLS Alpha.** This table shows each of the four components for the following decomposition of the OLS alpha $\hat{\alpha} = E(\alpha_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\alpha_t, x) + (1 + \frac{\mathbb{E}(x_t)^2}{Var(x_t)}) cov(\beta_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\beta_t, x_t^2)$, where α_t and b_t are dynamic alpha and beta, and x_t denotes the market excess return. For each category of funds, the first line reports the mean while the second line reports the cross sectional T-ratio for the null hypothesis that the component equals zero.

Category		α_{OLS}	$E(\alpha_t)$	cov(α_t,x_t)	cov(β_t,x_t)	cov(β_t,x_t^2)
16	mean	-0.0008	-0.0008	-0.0000	-0.0000	-0.0000
	T ratio	-4.0256	-3.7697	-0.8837	-0.2670	-0.8418
17	mean	-0.0002	-0.0004	-0.0001	0.0002	0.0001
	T ratio	-0.8277	-1.6248	-1.4517	3.4613	2.2654
18	mean	-0.0018	-0.0017	-0.0000	-0.0001	-0.0000
	T ratio	-6.3687	-6.3907	-0.0196	-0.9528	-0.3409
22	mean	-0.0017	-0.0017	0.0000	0.0001	-0.0001
	T ratio	-4.9560	-5.1258	0.0981	0.6472	-1.4891
23	mean	0.0010	0.0012	0.0003	-0.0004	-0.0000
	T ratio	1.9970	2.2095	3.3088	-2.7784	-0.3545
24	mean	-0.0025	-0.0020	-0.0001	-0.0003	-0.0001
	T ratio	-4.4783	-3.6393	-1.3809	-2.0696	-1.0917
38	mean	-0.0025	-0.0011	-0.0001	-0.0010	-0.0004
	T ratio	-3.9895	-1.6027	-1.4530	-6.9325	-4.9655
39	mean	-0.0001	0.0002	0.0004	-0.0008	0.0001
	T ratio	-0.1151	0.2178	2.0645	-4.5043	0.8668
40	mean	-0.0018	-0.0005	-0.0001	-0.0008	-0.0004
	T ratio	-2.9564	-0.8172	-1.6062	-5.2917	-3.6075

Table 7: **Components of OLS Alpha: absolute percentage.** This table reports the results for the decomposition, $\hat{\alpha} = E(\alpha_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\alpha_t, x_t) + (1 + \frac{\mathbb{E}(x_t)^2}{Var(x_t)}) cov(\beta_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)} cov(\beta_t, x_t^2)$, where x_t denotes the market excess return. Panel A columns three through six report the percentage contribution of each component to the value of $\hat{\alpha}$. To arrive at these values the average value of each component across all funds is computed. Then the absolute value of the average is calculated. The resulting numbers are then added together to produce column seven, which then serves as the denominator for the percentage contributions reported in columns three through six. Panel B repeats the analysis in Panel A, except that absolute values are taken fund by fund prior to calculating the mean value of each component.

Category	α_{OLS}	α_{static}	cov(α_t,x)	cov(β_t,x)	cov(β_t,x_t^2)	Sum($\lvert\alpha(components)\rvert$)
A						
16	-0.0008	90.9497	2.4515	2.5525	4.0464	0.0008
17	-0.0002	53.8183	10.2323	27.8133	8.1361	0.0007
18	-0.0018	95.4011	0.0650	3.7367	0.7972	0.0018
22	-0.0017	88.6364	0.1510	5.1362	6.0765	0.0019
23	0.0010	62.1663	14.8368	21.4123	1.5846	0.0019
24	-0.0025	80.0956	3.8869	12.4355	3.5819	0.0025
38	-0.0025	42.0451	5.5257	38.6425	13.7868	0.0025
39	-0.0001	12.7093	27.2640	53.5571	6.4696	0.0014
40	-0.0018	28.9745	6.1534	44.3250	20.5472	0.0019
B						
16	-0.0008	62.8109	7.3571	20.0651	9.7668	
17	-0.0002	62.1060	10.3851	17.5283	9.9806	
18	-0.0018	65.6543	7.6162	15.7693	10.9602	
22	-0.0017	67.6874	4.1612	17.3352	10.8162	
23	0.0010	59.2356	9.4512	20.7004	10.6128	
24	-0.0025	60.9653	5.8977	22.8405	10.2965	
38	-0.0025	55.5984	5.0968	27.3056	11.9992	
39	-0.0001	53.9725	10.4055	21.8664	13.7556	
40	-0.0018	46.3407	6.8832	31.6218	15.1543	

Table 8: **Decomposition of OLS Beta.** This table shows each of the four components for the following decomposition of OLS beta, $\hat{\beta} = E(\beta_t) + \frac{1}{Var(x_t)}cov(\alpha_t, x_t) - \frac{\mathbb{E}(x_t)}{Var(x_t)}cov(\beta_t, x_t) + \frac{1}{Var(x_t)}cov(\beta_t, x_t^2)$ where α_t and b_t are dynamic part of alpha and beta, and x_t denotes the market excess return. For each category of funds, the first line reports the mean while the second line reports the cross sectional T-ratio for the null hypothesis that the component is zero.

Category		β_{OLS}	$E(\beta_t)$	cov(α_t,x_t)	cov(β_t,x_t)	cov(β_t,x_t^2)
16	mean	0.9373	0.9361	0.0016	0.0001	-0.0003
	T ratio	64.0177	70.2978	0.9286	0.1841	-0.0777
17	mean	1.0897	1.0923	0.0061	-0.0012	-0.0081
	T ratio	65.9604	65.9432	1.7305	-2.9941	-3.1629
18	mean	0.8496	0.8488	0.0010	0.0006	-0.0015
	T ratio	65.7914	67.5370	0.2499	1.1252	-0.4364
22	mean	0.9339	0.9276	0.0006	-0.0004	0.0066
	T ratio	35.1753	39.7614	0.2191	-0.4607	1.0119
23	mean	1.1105	1.1327	-0.0238	0.0025	-0.0003
	T ratio	26.3939	23.9411	-3.4784	2.4814	-0.0416
24	mean	0.8807	0.8647	0.0083	0.0012	0.0067
	T ratio	31.6878	32.4088	1.3961	1.4033	0.7859
38	mean	0.8997	0.8606	0.0104	0.0070	0.0253
	T ratio	37.6198	30.7002	1.4812	6.2558	4.4721
39	mean	1.1669	1.2001	-0.0270	0.0056	-0.0117
	T ratio	26.0820	23.7304	-2.2857	4.1762	-1.4276
40	mean	0.8392	0.7925	0.0103	0.0048	0.0345
	T ratio	20.8500	18.7460	1.6194	5.7729	3.7118

Table 9: **Four-factor adjusted return across estimated skill sets (1970-2002).** This table reports four-factor risk adjusted returns for nine equally weighted portfolios holding mutual funds with particular trading ability measures. Panel A ranks each mutual fund by selection and market timing ability. Funds with high ability measures generate higher returns than funds with low ability measures. Panel B repeats the analysis but reports the out of sample risk adjusted returns. Portfolios are formed at the beginning of each year. Mutual fund ability measures are estimated using the previous 60 months of data.

A. In sample alphas.

	Four-factor adjusted return				T-ratio for four-factor adjusted return			
Market Timing Ability	Low	Media	High	H-L	Low	Media	High	H-L
Selection Ability								
Low	-0.0052	-0.0028	-0.0015	0.0037	-3.5061	-4.2783	-2.2765	2.5451
Media	-0.0010	-0.0006	0.0002	0.0011	-1.5933	-1.4819	0.5193	2.1718
High	0.0015	0.0017	0.0014	-0.0000	2.5715	3.8858	3.0022	-0.0693
H-L	0.0067	0.0046	0.0030	0.0067	4.7294	6.9625	4.6059	4.5436

B. Out of sample alphas.

	Four-factor adjusted return				T-ratio for four-factor adjusted return			
Market Timing Ability	Low	Media	High	H-L	Low	Media	High	H-L
Selection Ability								
Low	-0.0007	0.0006	0.0002	0.0009	-1.1867	1.0762	0.4167	1.7422
Media	-0.0006	-0.0004	0.0004	0.0009	-1.1978	-1.3108	0.7121	1.8648
High	-0.0009	0.0004	0.0002	0.0011	-1.6543	0.8693	0.2822	1.9008
H-L	-0.0002	-0.0001	-0.0000	0.0009	-0.3398	-0.1819	-0.0564	1.0629

Table 10: **Four-factor adjusted return across estimated skill sets II (1970-2002).** This table reports four-factor risk adjusted returns for nine equally weighted portfolios holding mutual funds with particular trading ability measures. Panel A ranks each mutual fund by bull market selection and market timing ability. Funds with high ability measures generate higher returns than funds with low ability measures. Panel B repeats the analysis but reports the out of sample risk adjusted returns. Portfolios are formed at the beginning of each year. Mutual fund ability measures are estimated using the previous 60 months of data.

A. In sample alphas.

	Four-factor adjusted return				T-ratio for four-factor adjusted return			
Market Timing Ability	Low	Media	High	H-L	Low	Media	High	H-L
Bull Market Selection Ability								
Low	-0.0039	0.0005	-0.0006	0.0033	-2.2848	0.9126	-0.8407	1.9535
Media	0.0004	-0.0005	0.0004	0.0000	0.6708	-1.2707	1.1650	0.0405
High	-0.0002	-0.0002	0.0006	0.0008	-0.2598	-0.3086	0.8034	0.9800
H-L	0.0037	-0.0007	0.0012	0.0045	2.2893	-0.9047	1.4412	2.6001

B. Dynamic Style, out of sample.

	Four-factor adjusted return				T-ratio for four-factor adjusted return			
Market Timing Ability	Low	Media	High	H-L	Low	Media	High	H-L
Bull Market Selection Ability								
Low	-0.0007	0.0001	0.0003	0.0011	-1.3897	0.1514	0.6007	1.6464
Media	-0.0006	-0.0000	-0.0002	0.0004	-1.2158	-0.0618	-0.3903	0.8178
High	-0.0010	0.0005	0.0002	0.0013	-2.0124	1.1339	0.4811	2.5275
H-L	-0.0003	0.0004	-0.0001	0.0010	-0.6216	0.6663	-0.1917	1.7145

Table 11: **Out of Sample Returns for Zero-Alpha and Zero-Beta Portfolios.** For all domestic equity mutual funds (ICDI-OBJ: AG, BL, GI, IN, LG, PM, SF and UT) that had at least 5 years of monthly return data, both 1-factor and 4-factor (the Fama-French three factors plus the momentum factor) Kalman and OLS models are used to forecast a fund's alpha and beta at the beginning of each year from 1970 to 2000 . These forecasts are then used to construct zero-alpha and zero-beta portfolios, one for each fund. At the beginning of each available year, a fund's predicted risks (the market risk for 1-factor models and the 4 factors for 4-factor models) are hedged out and the model's predicted alpha is subtracted from the realized monthly returns. Next, the resulting monthly time series for the zero-alpha and zero-beta portfolio is regressed against the market factor (for 1-factor models) and the 4 factors (for 4-factor models). This process results in risk-adjusted returns and factor loadings for each zero-alpha zero-beta portfolio. The parameter distributions are then calculated by bootstrapping with replacement the above procedure 1000 times. Panel A reports the risk-adjusted returns (alphas), and Panel B the "return weighted beta errors." The return weighted beta error is defined as: $return\ weighted\ beta\ error \equiv \sum_i \hat{\beta}_i \bar{r}_i$, where $\hat{\beta}_i$ is the estimated value of factor i, and $\bar{r}_i$ the factor's average return during the sample period. The Kolmogorov-Smirnov test rejects the hypothesis that the same probability distribution produced any pair of distributions generated by the different models (all P-values virtually zero).

	mean	Std Dev	5%	10%	50%	90%	95%
A. Risk adjusted return (alpha) 1970-2000							
OLS_1	0.000547	0.000107	0.000372	0.000408	0.000545	0.000691	0.000729
KAL_1	-0.000142	0.000135	-0.000363	-0.000321	-0.000142	0.000030	0.000084
OLS_4	-0.000378	0.000074	-0.000500	-0.000475	-0.000380	-0.000286	-0.000259
KAL_4	-0.000317	0.000119	-0.000528	-0.000470	-0.000307	-0.000172	-0.000125
B. Return weighted beta error 1970-2000							
OLS_1	-0.000063	0.000011	-0.000082	-0.000077	-0.000063	-0.000048	-0.000044
KAL_1	-0.000064	0.000013	-0.000086	-0.000082	-0.000065	-0.000047	-0.000041
OLS_4	0.000164	0.000033	0.000109	0.000123	0.000164	0.000206	0.000219
KAL_4	0.000061	0.000040	-0.000008	0.000007	0.000062	0.000112	0.000126

Table 12: **A Comparison with Conditional Model.** The sample used for this table includes all available funds in the CRSP database that contain 60 months of monthly return data from 1994 to 1998. Model parameters are estimated for the unconditional CAPM model, the conditional beta model, and the Kalman filter model. The variables R^2_{CAPM}, R^2_{Cond} and R^2_{Kal} represent R^2 statistics for each of the models respectively. Consistent with the OLS model, R^2_{Kal} is defined as $1 - \mathbb{E}(\epsilon_P(t)^2)/\mathbb{E}((y_t - \bar{y})^2)$, where y_t is the excess portfolio return and $\epsilon_P(t)$ is the residual from the Kalman filter model. The variable $\bar{y}$ equals the mean value of the y_t. Also reported are the cross-sectional means and T-ratios for the improvements of R^2, $\triangle R^2_1 = R^2_{cond} - R^2_{CAPM}$ and $\triangle R^2_2 = R^2_{Kal} - R^2_{cond}$.

Category		fund No.	R^2_{CAPM}	R^2_{Cond}	R^2_{Kal}	$\triangle R^2_1$	$\triangle R^2_2$
16	mean	92	0.8994	0.9013	0.9382	0.0031	0.0369
	T ratio					7.07	3.17
17	mean	81	0.861	0.8673	0.9146	0.0063	0.0473
	T ratio					8.74	6.97
18	mean	75	0.8676	0.8734	0.9287	0.0059	0.0553
	T ratio					7.52	6.67
22	mean	46	0.7618	0.7709	0.9095	0.0090	0.1386
	T ratio					5.56	6.51
23	mean	36	0.7501	0.7601	0.8638	0.0100	0.1036
	T ratio					2.94	4.97
24	mean	31	0.7408	0.7477	0.8484	0.0069	0.1007
	T ratio					7.11	5.65
38	mean	23	0.6479	0.6536	0.8614	0.0056	0.2079
	T ratio					3.37	4.76
39	mean	35	0.6413	0.6448	0.7011	0.0035	0.0563
	T ratio					5.12	3.15
40	mean	18	0.6527	0.6581	0.8007	0.0055	0.1426
	T ratio					5.44	6.35
All	mean	437	0.8047	0.8098	0.8877	0.0059	0.0779
	T ratio					14.03	13.80

Figure 1: **Likelihood ratio tests.** This figure shows the results from a likelihood ratio test in which the Kalman filter model includes conditional information similar to that of Ferson and Schadt (1996). As in their application, lagged macroeconomic information drives the portfolio weights. However, here portfolio weights are also assumed to vary from some unobserved factor following an AR(1) process. As a first order approximation, the portfolio weights become $w_{it} = \bar{w}_i + l_i F_t + D_i Z_{t-1}$, where Z_{t-1} is the lagged information. This model is estimated via extended Kalman filter. For two macro instruments, the estimated system of equations is given by $r_{Pt} - r_{ft} = \alpha_{Pt} + \beta_{Pt}(r_{mt} - r_{ft}) + \varepsilon_{Pt}$, where $\beta_{Pt} = \bar{\beta}_P + F_{t-1} + k_1 z_{1,t-1} + k_2 z_{2,t-1}$ and $\alpha_{Pt} = -k_t + \bar{a}_P F_{t-1} + b_P F_{t-1}^2$ and $F_t = \gamma_F F_{t-1} + \eta_t$. Here z_1 and z_2 are instruments for the lagged T-bill rate and the CRSP value weighted index's dividend yield. The null hypothesis is that $k_1 = k_2 = 0$. The constrained and unconstrained models are estimated using monthly returns from 437 mutual funds during the period of 1994 to 1998. Asymptotically, the likelihood ratio test under the null should follow a chi-square distribution with two degrees of freedom. In Figure 1, the bars represent the cross-sectional distribution of the likelihood ratio while the dashed line displays the mathematical values for a chi-square distribution with two degrees of freedom. The fraction of funds that reject the null hypothesis at the 1%, 5% or 10% levels are 10.8%, 15.7% and 20.3%, respectively.