Optimal Financial Contracts for a Start-Up with Unlimited Operating Discretion

by

S. Abraham Ravid[†]

and

Matthew Spiegel[‡]

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[†]Rutgers University Graduate School of Management, Newark, NJ 07102. (201) 648-5540.

[‡]U.C. Berkeley, Haas School of Business, S545 Student Services Building #1900, Berkeley, CA 94720-1900. 510-642-3421. spiegel@haas.berkeley.edu. http://haas.berkeley.edu/~spiegel.

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Abstract

This paper presents a model in which asymmetric information and extreme uncertainty lead to the exclusive use of equity and riskless debt for small business financing. The paper derives these results without any restrictions on the available contract space, the distribution function governing a project's payoff, or the risk aversion of most potential entrepreneurs. Linear securities derive from the assumption that small business financing involves more uncertainty than is captured in most financial models. Instead of assuming that business people are faced with a given menu of projects, the model allows entrepreneurs to create (over time) an unlimited number of non-positive net present value projects with any payoff distribution they desire. Also, outside investors cannot observe project choice but only terminal cash flows. As a result, suppliers of funds must design contracts so that in equilibrium entrepreneurs do not wish to undertake undesirable investments. Further analysis of the model shows that in equilibrium entrepreneurs must contribute some of their own capital. The paper also finds that when a firm does not have any collateralizable assets, the equilibrium funding agreements have the property that the investors split the project's proceeds in proportion to their initial investments. We further demonstrate that the existence of an infinite number of non-positive NPV projects can lead in equilibrium to positive abnormal returns earned by outside financiers. The model also produces a pecking order theory of financing. It is shown that highly profitable firms, and/or those companies run by relatively risk neutral individuals will first try to raise funds with riskless debt, and then turn to equity only when the supply of riskless debt has been exhausted. Finally, the paper shows that the linearity of the equilibrium contracts derives from the entrepreneur's discretion with regard to the payoff distribution, rather than the flexibility associated with non-positive NPV projects. As part of the same analysis, we also demonstrate that under some conditions our underpricing results can be derived solely from the infinite supply of non-positive NPV projects.

Most security design and optimal contracting papers assume (explicitly or implicitly) that entrepreneurs are endowed with a menu of projects. Securities are then issued in order to raise money and induce the entrepreneurs to select "better" projects, in a sense which varies with the assumed objective functions. In contrast, this paper focuses on an environment in which the very existence of contracts leads entrepreneurs to create projects which take advantage of the incentive structure presented to them. Furthermore, we assume that outsiders cannot observe project selection. This set-up characterizes a situation of great uncertainty on the part of outside investors, typical of start-up firms, some IPO's and other hard to gauge investments.

One can also view this paper as mapping out the equilibrium financial structure in an environment that forms the polar opposite to the full information setting, where all is essentially anticipated in advance. Yet, despite (and in fact because of) the uncertainty facing investors in our framework, equity and riskless debt emerge as optimal securities within general contract and payoff probability distribution spaces. While the set-up is somewhat extreme, the incentive problems identified in this paper are, we believe, very real and do affect corporate behavior.

The simplest version of the model starts with the following premise. Initially every entrepreneur comes endowed with his own investment opportunity set. Since the economy has a finite value, only a finite subset of these investment opportunities have strictly positive net present values (NPV).[1] However, while finite wealth restricts the endowment of positive NPV projects, there is no reason to bound the endowment of *non-positive NPV* projects. After all, people can easily produce as many of these investments as they wish. Further, the set of *potential* non-positive NPV projects may contain a wide variety of payoff distribution functions. Our primary insights are: (1) under these conditions, linear contracts are the cheapest contracts that outsiders can offer that do not attract applications from *any* non-positive NPV projects and (2) outside investors can expect to earn a strictly positive return on any project they fund. As the final section of the paper shows, the linearity of the equilibrium contract structure derives from the wide variety of potential payoff distribution functions, while the positive returns to the outsiders derive from the assumed supply of non-positive NPV projects.

The empirical motivation for the paper rests on the realization that linear contracts and collateralized loans prevail in a wide variety of settings, including cases of extreme uncertainty. One finds linear contracts in many limited partnership and some venture capital contracts (see in that respect a similar analysis by Admati and Pfleiderer (1994) and a survey by Barry (1994)). Limited partnership agreements are a particularly interesting example. Typically, the general partner makes all the investment decisions, and the limited partners, who provide up to 99% of the capital, must structure the contract so that the general partner's incentives are not distorted. The contracts usually feature a linear sharing rule after the limited partners have recouped their investment. One can find limited partnerships or similar equity like contracts in highly speculative markets such as real estate or the movie industry where studios often served as general partners (see Vogel (1994)). Linear contracts can also appear in the risky business of television programming (barter agreements) in which the station and the program seller-producer share the advertising revenues according to a pre-specified formula.

Small, growing businesses that cannot obtain bank debt financing often resort to "mezzanine loans" (see Levine and Gezon (1993)). Such loans, offered by specialty funds with billions of dollars to invest and by some institutions, involve combinations of risky debt and equity similar to the unlevered equity positions which we describe. Frequently, small businesses also raise money through "Angel Financing," which includes contributions of non-bank and non-corporate (non-professional) investors. This type of financing is very common for small, untested enterprises. The average deal is approximately $250,000, which is insignificant for a large corporation, but could represent a significant amount of money for both the small business owner and the "Angel" involved. As documented in Winninghoff (1993), the securities issued to these investors tend to include exactly the components we describe: collateralized loans and equity.

Economic models usually analyze the impact of contracts on the choice among available projects. The creation of new projects in order to exploit contractual features is rarely discussed. However, casual stories abound. An example, that illustrates our point, comes from Mel Brooks' comedy *The Producers*.

In that movie, a pair of unscrupulous producers manage to obtain money from a large number of investors in return for shares in a play. The producers then endeavor to create the worst musical ever to reach Broadway. The reason for this perverse zeal is that the contracts they sell to the outside investors violate the equilibrium conditions established later on in this paper. First, the producers themselves invest no money in the project (see Corollary 1). Second, they offer a large percentage interest in the project for a small investment (see Propositions 1 and 2). Third, and finally, no collateral is offered (see Proposition 4). However, the outsiders investing in the play have no information regarding project (play) selection. This scenario creates incentives similar to the ones we are discussing. Within the movie, the contracts investors are willing to accept (because of the charms of Zero Mostel and Gene Wilder who play the producers) make it possible for the entrepreneurs to profit most from the worst possible projects. [2] Our paper suggests that this can happen outside the entertainment industry as well if contracts are not properly structured.

I. Related Work

One can think of all security design papers as attempts to understand financial contracting between two parties under a variety of informational and other assumptions. At one end of the spectrum are papers like Townsend (1979), Diamond (1984), and Gale and Hellwig (1985) which assume that while both parties know the distribution function producing the firm's cash flows, the financier cannot costlessly observe the realized profit. This paper, as well as Admati and Pfleiderer (1994), lie at the other end. In both papers the parties may not agree upon the distribution function producing the cash flows, but everyone can observe the realized profit. Somewhere in the middle lie signaling models like those of Ross (1977), Heinkel (1982), Brennan and Kraus (1987), Nachman and Noe (1994), Ravid and Sarig (1991) and many others. In these papers the financier may not know the exact distribution from which the profits are drawn, but has some prior information that limits the set of possible functions. These models naturally apply to different situations and produce different equilibria.

Admati and Pfleiderer (1994) examine an economy populated by risk neutral agents in which a venture capitalist provides funding for a new company and must eventually decide if the firm should remain open. Their analysis is similar to ours in that it does not restrict the set of possible payoff distribution functions or the contracts the parties can choose. However, there are also significant differences between the papers in the restrictions placed upon the risk aversion of the parties involved, and in the forces that drive them to offer and accept particular financial contracts. These similarities and differences will become clearer once the current model is developed. Thus a full comparison of the two frameworks is postponed until section IV.

At the other end of the spectrum lie models where agents know the underlying profit distribution function governing the firm's profits, but cannot necessarily observe the terminal cash flows. Townsend (1979), Diamond (1984), and Gale and Hellwig (1985) show that in this setting the optimal contracts resemble pure risky debt agreements.

A comparison of our framework to the framework and conclusions of security design models based upon the signaling paradigm provides some additional insights. Signaling papers generally assume an exogenous, known ordering of the payoff distribution functions. A typical characterization calls for "better quality" firms to have profit distributions that first order dominate lower quality firms. Without the ordering assumption, proving the existence of separating or pooling equilibria becomes intractable. In this paper no ordering is assumed. Signaling models frequently conclude that if several security classes are permitted, then signaling will involve the use of every security or equivalently a single very complex security to minimize revelation costs. This literature dates back to Brennan and Kraus's (1987) paper and includes Constantinides and Grundy (1989) as well as Nachman and Noe (1994). Our finding that optimal contracts in our context look like equity and secured debt, may thus indicate that increasing the level of uncertainty can force parties to employ less complex financial contracts. Perhaps this is not too surprising. In a world without frictions or asymmetric information, the Modigliani-Miller theorem demonstrates that equilibrium contracts are indeterminate. Signaling models introduce some asymmetric information and find that this restricts the equilibrium contracts to some degree. This paper then adds another level of uncertainty regarding the firm's payoff distribution function and concludes that this leads to even further restrictions on the equilibrium contracts.

Clearly the model presented in this paper does not cover every possible case. For example, many venture capital firms both finance start ups and then take an active managerial role in project development. In these particular instances it seems unreasonable to assume that the financiers cannot deduce with some degree of accuracy the project's payoff distribution. If so, they may very well decide to use nonlinear contracts. For example, a recent study by Gompers (1996) finds that convertible preferred stock is the dominant form of financing in a sample of 50 start-up firms. Other cases, excluded by our model's assumptions, include any situation where investors have some ability to directly verify the ex-ante probability distribution of the firm's payoff.

The rest of the paper is organized as follows. The next section presents a general outline of the model and describes the economic environment. Section III presents the formal model. Section IV characterizes the equilibrium contracts and derives the main results. Section VI concludes.

II. The Economic Environment

In this section, we provide an intuitive description of the economic foundations of our model. In any real world setting, a large number of financial institutions compete with each other to provide business financing for potential entrepreneurs. One can think of this process as taking place in the following stylized environment. Financial institutions advertise that they have money available for new ventures. Specifically, they will contribute up to k dollars for projects that cost C dollars in exchange for a financial contract m. The contract $m(x,k)$ specifies the sharing rule in the event that the project pays x dollars and the outsiders have financed k dollars. Naturally, this implies that the entrepreneur receives x-$m(x,k)$. Since the model is primarily concerned with arm's length financing, the contracts do not include extensive on site monitoring by the bank. Perhaps the projects are too small and too complex to justify monitoring. Thus, while each contract will be carefully crafted and may impose specific covenants or requirements, the institution will not further involve itself with the day-to-day operations. This environment also gives the entrepreneur complete operational discretion.

Each day potential entrepreneurs must decide whether or not they should apply for financing. Initially there will exist some individuals who own projects that will look attractive, from their point of view, given the current financial contracts being offered. These people may seek funding, thereby benefitting themselves and perhaps the financier. Over time others will also discover projects which are profitable from their point of view.

Consider the long run equilibrium implications for the set of financial contracts. Suppose that under some currently offered contract a zero or negative net present valued (NPV) project with a particular probability distribution over its cash flows will earn a positive expected profit for the entrepreneur. If such a project does not exist now, over time one expects that somebody will *eventually* discover a way to produce the required cash flow pattern. Even if the financier has some initial monitoring ability (for example he or she may only lend to people planning to open a flower shop), eventually somebody can be expected to find a non-positive NPV project that both meets the financial institution's criteria and has the necessary distribution.[3] Further, while there are good economic reasons to assume that a project with a positive present value cannot be duplicated ad infinitum, there seems no reason to suppose that a similar restriction should apply to projects with zero or negative values.[4] Losing schemes can usually be found. Thus, once a scheme that meets the above criteria is discovered, financial institutions will find themselves flooded by funding requests. Since the financiers will earn a negative return (on average) from these projects, the financial contract that encourages their creation will be withdrawn. Thus, it seems natural to assume that over the long run outside financiers will not offer contracts that afford people the opportunity to profit via investment in zero or negative valued projects. Our most important requirement for the optimal contract is thus:

Condition 1: *Given the set of available financial contracts, entrepreneurs cannot earn an expected profit* *through any project with a zero or negative net present value.*

To see how this equilibrium condition influences the set of financial contracts, consider the following example. An investor offers to provide \$1 for a project (say a pizza parlor) that costs \$2 to start up. In exchange the investor asks for 1/4 of the firm's equity. Under the notation developed so far C=2, k=1 and $m(x,k)=x/4$. Now suppose somebody discovers a way to create pizza parlors that requires \$2 in capital and pays 0 with probability .6 and 4 with probability .4. This individual will now approach the bank for financing since he expects to earn $.4\times.75\times4-1=.2>0$. The project looks profitable to the entrepreneur given the financial contract. However, the project leaves the bank worse off with an average loss of .6 dollars. Once other entrepreneurs learn how to replicate such projects, one expects them to inundate financial institutions with applications for funds that will then be invested to produce this cash flow pattern.[5] In turn, the losses to the banks will induce them to withdraw the contract. This is precisely the notion of an out of equilibrium contract.

So far the problem has been presented as one where investors simply wish to prevent entrepreneurs from seeking to fund undesirable projects. However, there are many financial institutions competing with each other to lend money. As a result, one expects them to offer prospective borrowers any contract that does not violate Condition 1 (since any contract that does not violate this condition must produce a nonnegative expected profit for the lender). This leads to a second equilibrium condition.

Condition 2: *In a competitive environment any contract that does not violate Condition 1 must be offered in equilibrium.*

At first blush it may appear that Condition 2 does not add anything. However, without it one might imagine an economy where financiers offer to put up only 10% of a project's funds, while demanding 99% of the equity. One can show that such contracts will always profit outsiders, but so will other less draconian agreements. In equilibrium, competition should force financiers to offer the full menu of contracts that do not violate Condition 1.

While the next section provides a formal characterization of the equilibrium contract set created by Conditions 1 and 2, one can establish some of the intuition at this point. Suppose that a financier offers to provide a fraction of the start-up costs (*k/C* in the model's notation) for an equal fraction of the equity. Since the financier puts up *k/C* of the funds, and receives *k/C* of the profits, he must earn a nonnegative expected profit if and only if the entrepreneur does. Notice that this expected profit relationship holds irrespective of the project's payoff distribution. Thus, entrepreneurs do not have an incentive to produce nonpositive NPV projects, and so the contract meets the requirements set forth in Condition 1. Of course, any contract that promises the financial house at least as much in every state of nature will also satisfy Condition 1, and by Condition 2 must also be available. Can contracts that pay less also appear? Yes, if the entrepreneur can use some of his assets as collateral. Collateral allows the entrepreneur to guarantee the financier a minimum payoff, expanding the set of potential equilibrium contracts. Unfortunately, it now become difficult to provide a purely intuitive derivation of the equilibrium contracts. We thus turn to the formal mathematical model.

III. The Formal Model

Initially, entrepreneurs are endowed with a finite supply of projects such that $E(x)>C$. However, they are free to create projects with any payoff distribution such that the expected net payoff is nonpositive, i.e., $E(x)\leq C$. Such projects can also be infinitely replicated. Further assume that, whereas the distribution of returns is not observable, the amount invested in the project is. This excludes cases where the entrepreneur takes the money, ignores any contracts and disappears.[6]

The assumption that outsiders cannot observe the project's payoff distribution function implies that outsiders can only control the behavior of the firm through the financial structure. This assumption places the model at the opposite end of the spectrum from that of Townsend (1979), where the ability to observe the ex-ante distribution coupled with a costly revelation of cash flows conditional on bankruptcy states, results in debt being the optimal security. Much of the literature on contract design assumes that all parties are risk neutral. Here, however, we allow the insiders to be risk averse, with a risk neutral subset. No other restrictions are imposed. An insider's goal is to maximize his expected utility via his selection of projects and security issues. The outsiders are assumed to be diversified investors who price securities in a risk neutral fashion. Since they cannot observe the project's cash flow distribution, they cannot, strictly speaking, maximize expected returns. Therefore their objective can be characterized as a max-min objective, i.e. make sure that no negative NPV projects are presented for financing.[7] **Figure 1** below displays the time line envisioned in our model.

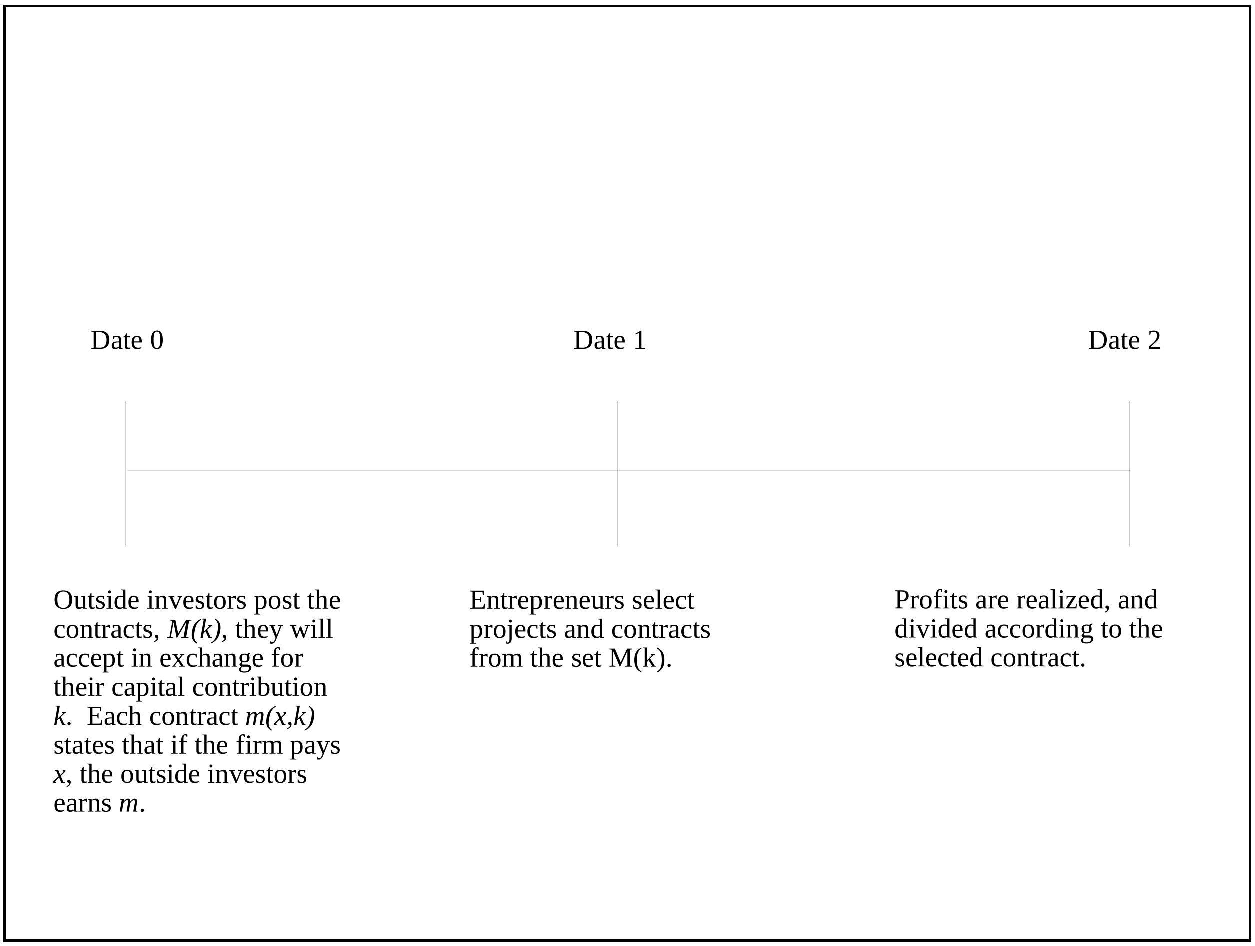

Figure 1: Time Line for the Economy.

Outside investors move first. Since they cannot differentiate "good" from "bad" projects, they announce that they will invest k dollars in any project in exchange for any contract in the set $M(k) \subseteq A$ where A is the set of all available contracts. The *only* restriction we place upon the contract set is that of limited liability, $0 \leq m(x,k) \leq x$ for all $m \in A$.[8] Security payoffs may be nonlinear and even discontinuous in the firm's profits.

After outside investors post the acceptable contracts, entrepreneurs can initiate projects. There are no restrictions upon the number of projects each entrepreneur can select. The model assumes that entrepreneur i is endowed with W_i dollars in capital. He can then contribute $w_{i,j}$ to project j, subject to the restriction that $\sum_j w_{i,j} \leq W_i$. Let $C_{i,j}$ represent the cost of project j selected by entrepreneur i. To begin a project, the entrepreneur selects a contract $m(x,k) \in M(k)$ offered by outside investors, and sets $w_{i,j} = C_{i,j} - k$.[9]

The model assumes that x is bounded below by $\underline{x} \geq 0$. The bound represents the possibility that the project may include collateralizable assets (e.g. machinery, real estate, and inventory) insuring a minimum cash flow when the firm is liquidated at the end of the game.

IV. Equilibrium

The equilibrium contracts must satisfy the following requirements:

Condition 1: No project exists such that $E(x) \leq C$, and $E(x - m(x,k)) > C - k$ for some $m(x,k)$ in $M(k)$.

Condition 2: If $m(x,k)$ does not belong to $M(k)$, then it must be that an outsider offering $m(x,k)$ must expect to earn a strictly negative return.

In addition, the equilibrium actions of the entrepreneurs in the economy must also satisfy:

Condition 3: Among the set of contracts belonging to $M(k)$, an entrepreneur selects those which maximize his expected utility given the projects he has available. Entrepreneurs who do not underwrite a project earn zero profits.[10]

Conditions 1 and 2 restate, in mathematical terms, their counterparts from Section II. As the reader may recall, Condition 1 requires that outsiders refrain from offering financial contracts that make nonpositive NPV projects profitable to potential entrepreneurs.

Condition 1 bears some similarity to Admati and Pfleiderer's (1994) robustness criterion. What differentiate the two is the derivation of the condition, and the restrictions it imposes on the equilibrium. In Admati-Pfleiderer's analysis, a contract is called robust if both a risk neutral financier and the risk neutral entrepreneur will agree to it no matter what underlying distribution function governs the firm's profits. As their paper shows, the only contract which satisfies this robustness criterion is a linear equity-like contract.

Relative to the Admati-Pfleiderer robustness criteria, Condition 1 has a different foundation. Condition 1 rests upon the notion that an outside investor should not provide entrepreneurs with an incentive to create non-positive NPV projects and then seek financing for them. Within the model, any financier who fails to heed this dictum, will lose money, on average, since the proffered contract will bring forth the reserve army of bad projects. Since undesirable projects exist irrespective of any individual's risk aversion, our model produces linear equilibrium contracts even when some entrepreneurs are risk averse (so long as there also exists a subset of risk neutral individuals). In contrast, the robustness criterion depends explicitly upon the type of contracts any two risk neutral individuals may find acceptable.[11] As a result of these underlying differences in the forces driving the contracts, we are also able to produce several additional results including a prediction about the underpricing of accepted projects and an endogenous pecking order of financing.

As noted in the previous section, Condition 2 represents the assumption that competition among the financial firms forces them to offer every contract that earns them a non-negative expected profit. Condition 3 simply states that entrepreneurs are utility maximizers.

Using the above definitions, one can now begin to characterize the equilibrium contracts. The next proposition provides a bound on the set of equilibrium contracts $M(k)$. This bound has the property that it is payoff equivalent to a combination of equity and collateralized debt.

Proposition 1: *Assume that $k \leq C$, and $\underline{x} < C$. Then the contract $m(x,k) \in M(k)$ if and only if*

$$m(x,k) \;\geq\; (x - \underline{x})\frac{k \;-\; m(\underline{x},k)}{C - \underline{x}} \;+\; m(\underline{x},k) \tag{1}$$

for all x.

Proof: Let $\hat{m}(x,k)$ represent a contract such that (1) holds with equality. Now consider a contract $n(x,k)$ such that for some $x=x^*>\underline{x}$, and $\delta>0$ one has $n(x,k) = \hat{m}(x,k)-\delta$. From Condition 1, the contract n must not belong to $M(k)$ if one can find a project with a non-positive present value such that an insider can fund it using n and earn an expected profit.

Consider a project which pays $\underline{x}$ with probability $(x^*-C)/(x^*-\underline{x})$, and x^* with probability $(C-\underline{x})/(x^*-\underline{x})$. As per the first requirement, the project has an expected present value of 0. Secondly, note that any entrepreneur who selects this project earns $\delta(C-\underline{x})/(x-\underline{x})$ which is strictly positive since $C>\underline{x}$ in order to fulfill the requirement $E(x)\leq C$. Therefore, by Condition 1 and the above arguments any contract which violates equation (1) does not belong to $M(k)$.

To complete the proof it is only necessary to show that if a project has a non-positive NPV, then the return to the entrepreneur is non-positive under any contract satisfying equation (1). First, consider the contract $m(x,k)$ such that equation (1) holds with equality. Then one can show by integrating from $\underline{x}$ to infinity, that if $E(x)\leq C$, the entrepreneur earns a non-positive return. Given this result, any contract in which equation (1) weakly holds for some x, must also leave the entrepreneur with a non-positive return. Q.E.D.

Intuitively, the proof uses the fact that investments with two point distributions pose the greatest problems for any contract. By carefully crafting these projects, the entrepreneur can potentially take advantage of even the smallest contractual flaws. For example, suppose a contract offers to give the entrepreneur enough so that at some point (say x^*) equation (1) does not hold. An entrepreneur can then take advantage of the contract by selecting a zero NPV project that pays either $\underline{x}$ or x^*. The proof shows that in this case the entrepreneur will earn a positive expected return, and thus the outside financiers will be left with a negative expected return. Thus, if the contract stops low valued two point projects from seeking financing, one might correctly surmise that poor projects with more general distributions are also avoided.

From equation (1), one can already see that the optimal contracts are combinations of collateralized debt and equity. Set equation (1), to an equality. The term multiplying x-$\underline{x}$ is the outsiders' equity share of the "risky" cash flow. Similarly, the $m(\underline{x},k)$ term provides the outsiders with their share of the project's riskless cash flow. Proposition 2 formalizes the discussion and demonstrates that equity and riskless debt are indeed the equilibrium contracts in our framework.

Proposition 2: *In equilibrium, only contracts such that equation (1) holds with equality are selected. Intuitively, these contracts are combinations of collateralized debt and equity.*

Proof: Suppose that there exists an entrepreneur endowed with a positive NPV project that is undertaken in equilibrium. Further suppose that the support of the project's payoff set includes some realization x. For any such realization x, the expected return to an entrepreneur is strictly declining in the pre-selected value of m. Therefore an entrepreneur prefers the contract in $M(k)$ with the lowest possible value of m for every x. As per Proposition 1, outsiders can break even up to the point where (1) holds with equality. These contracts are therefore the most desirable, among those that are potentially available in equilibrium to the entrepreneurs. Since outsiders are competitive they will offer such contracts in equilibrium. Q.E.D.

One can further characterize the equilibrium contracts described by the above two propositions. Let E and B be defined as follows:.

$$E = \frac{k - m(\underline{x},k)}{C - \underline{x}} \tag{2}$$

and

$$B = m(\underline{x},k). \tag{3}$$

E can be viewed as the fraction of the outside equity held by outside investors. B is then the "riskless debt" held by outsiders.

Notice that C-$\underline{x}$ is a constant and therefore the numerator of (2) determines the sharing rule. We see that the outsider's equity position is linearly increasing in the amount he invests (k) and linearly decreasing in the value of the collateralized assets he retains ($m(\underline{x},k)$). Secondly, note that C-$\underline{x}$ is the uncollateralized investment in the project, and that k-$m(\underline{x},k)$ equals the amount the outsider puts at risk. Thus, the outsider's equity in the firm exactly equals his proportional contribution to the risky portion of the investment. (This intuition is further developed in Corollary 1). The level of riskless debt issued, on the other hand, depends on the risk aversion of the entrepreneur and on the project's value. The more risk averse the entrepreneur, the less equity and the more riskless debt he retains.

Consider the following example: An entrepreneur has a negative exponential utility function $-\exp(-\theta\omega)$, where θ is his risk aversion parameter, and ω his terminal wealth. The project the entrepreneur owns pays $\underline{x}+x$, where x has an exponential distribution $\beta\exp(-\beta x)$. Since the expected value of the exponential distribution equals $1/\beta$, the project has a positive expected return if $\underline{x}+1/\beta>C$. Using Propositions 1 and 2, there is a one to one relationship between E (equity proportion) and B (riskless debt). Therefore, one can characterize the entrepreneur's problem as selecting B subject to the constraint that E satisfies condition (1) with equality.

Some algebra demonstrates that the entrepreneur sets $B = -k + \{1-(\beta+\theta)(C-\underline{x})\}/\theta$. Differentiating with respect to β, one obtains that $\partial B/\partial\beta=-(C-\underline{x})/\theta<0$ if the project's collateral does not cover its setup costs. As the expected value of the project declines, the entrepreneur issues less debt to outsiders, thereby keeping more for himself. From equation (1) and Proposition 2, this also implies that the less valuable the project is, the less equity the entrepreneur keeps, and the more he issues. Next, differentiate B with respect to θ, yielding $\partial B/\partial\theta = \{\beta(C-\underline{x})-1\}/\theta^2$. From Condition 1, only positive NPV projects are selected in equilibrium, and thus $\partial B/\partial\theta<0$ since $\beta(C-\underline{x})-1<0$ (recall that $\underline{x}+1/\beta>C$ if the project has a positive NPV). The conclusion is that the more risk averse the entrepreneur, the greater the amount of riskless debt he keeps, and the more equity he issues. Notice that while the value of the project and the entrepreneur's risk aversion both enter the debt-equity choice, they do not signal either β or θ to the outsiders. In part, the reason is that while this particular entrepreneur has a project whose payoff is exponentially distributed, others may have projects with different payoff distributions. Therefore, outsiders do not know if a low equity level means they are funding a project with a high expected value and an exponential payoff, or a project with relatively low NPV, and some other payoff (for a formal proof that signaling will not occur in a similar environment see Admati and Pfleiderer’s (1994) Proposition 1). Although the quality of the project cannot be signaled, the equilibrium contracts guarantee that all projects that are funded have positive expected NPV's.

Propositions 1 and 2 are related to Myers (1977) work in that entrepreneurs may switch into low valued projects when the design of securities makes such activities profitable. However, there is an important difference between Myers' assumptions and those employed here. In the present paper, entrepreneurs can produce and then seek financing for an infinite number of bad projects. Security holders are therefore unable to avoid the "under investment" problem. Myers' examples assume a scenario where all the feasible projects are known to all investors. The only uncertainty involves the firm’s project choice from among the lot. Here, under the scenario in which bad projects can be manufactured, debt holders can never "price" debt correctly.[12]

As noted previously, risky debt as a sole method of financing does not emerge as an equilibrium contract. However, we can have risky debt as part of a financial bundle. Propositions 1 and 2 only require that outside investors receive a portfolio of securities such that the payoffs are equivalent to a combination of riskless debt and common equity. An acceptable contract, however, could give the outside investors 20 percent of the risky debt and 20 percent of the equity. Their position is then payoff equivalent to holding 20 percent of the firm's unlevered equity and can therefore be an equilibrium contract. Cross holdings, common in many countries, can produce such results, while in the United States strips perform this function. Corporate and personal tax laws may thus partially account for the complex financial instruments which are often payoff equivalent to much simpler securities.

Propositions 1 and 2 also lead to the following intuitive result.

<u>Corollary 1</u>: *In an optimal contract (which specifies a positive share of the cash flows to the entrepreneur), $w>0$, i.e. the entrepreneur must contribute some of his own wealth.*

Proof: To see this set $k=C$ in (1), and set (1) to an equality. This produces an equilibrium contract $(x-\underline{x})[C-m(\underline{x})]/[C-\underline{x}] + m(\underline{x})$. If $m(\underline{x})<\underline{x}$, then for x large enough, $m(x)>x$, which violates the limited liability constraint. If $m(\underline{x})=\underline{x}$ then $m(x)=x$, and the outsiders receive all of the cash flows. Q.E.D.

Corollary 1 formalizes the notion that non-participating entrepreneurs cannot obtain funding.[13] While perfectly consistent with common sense, these results contrast with some models in corporate finance which maintain that an investor can fully finance his project with outside funds. Here one can never credibly demonstrate that a project has enough value to warrant its full funding by outsiders.

Propositions 1 and 2 also imply that if there are any projects with strictly positive present values in the economy, outside investors earn strictly positive expected returns. This is because the equilibrium contract specification does not depend upon the expected value of the projects being financed. In that sense the firm's security issue is "underpriced." The equilibrium holds, because any investor that considers deviating finds himself flooded with zero or negative NPV projects. We can formalize this notion below:

<u>Corollary 2</u>: *Consider an economy in which some entrepreneurs are endowed with potential investment opportunities that have strictly positive present values. Then, in equilibrium, outside investors can expect to earn a strictly positive return.*

We should note that these returns cannot be eliminated by competition. It is the equilibrium nature of the contracts that dictates the positive returns. The benefits must be shared if one wishes to avoid a flood of bad projects.

The source of the underpricing results found in this model differs considerably from those found in the initial public offering (IPO) literature. Models like Rock (1986) and Benveniste and Spindt (1989) conjecture that new issues are underpriced due to the joint participation of both better informed and lesser informed security buyers. In Allen and Faulhaber (1989) insiders know their type, but face a probabilistic implementation which will determine future dividends. They may choose to signal by underpricing the initial offering. (The motive is insiders' sales of stock prior to the revelation of firm quality). In Welch's (1989) model, firms underprice their offers in order to help bolster the price of a seasoned issue they plan to sell in the future. Similarly, Grinblatt and Hwang (1989) show how the fraction of retained shares and the degree of underpricing can act as a joint signal about the project's value and the variance of its cash flows. In our model, all purchasers of the securities are equally informed and there is no secondary offering. Instead, the underpricing derives from the outsider's desire to protect themselves from non-positive NPV projects.

Our framework is also related to the pecking order theory of financing (for the most widely recognized contribution in that area see Myers (1984)). Roughly, casual observation tells us that firms first use internal funds to finance their operations. If this is insufficient, then debt is sought. Finally, when a firm exhausts its low risk debt capacity then equity is issued. Absent taxes, most theories do not provide a compelling reason for this particular ordering. However, the present model produces a financial pecking order endogenously. We formalize it in Proposition 3 below:

Proposition 3: *Suppose that an entrepreneur has one positive NPV project and that he is either sufficiently close to being risk neutral or that E(x) is sufficiently high.*[14] *Then, in equilibrium, projects are funded with a combination of the entrepreneur's own funds and riskless debt whenever possible. If riskless debt and internal financing cannot fully fund the project, then the entrepreneur sets* $w_{i,j}=W_i$*, employs as much riskless debt as possible and finances the rest of the investment with equity.*

Proof: Clearly, if $\underline{x}\geq C$ then the entire project can be financed with riskless debt. This must be at least weakly optimal since setting $m(\underline{x})=C$ allows the project to proceed while giving the outsiders only a normal return. Thus, assume $\underline{x}<C$. Given the results in Proposition 2, the entrepreneur's problem reduces to that of maximizing his expected utility $U(x\text{-}m(x)+W_i\text{-}C+k)$ with k (recall $k=C\text{-}w$) and $m(\underline{x})$ as control variables. Let $f(x)$ represent the distribution function over x. Then differentiating the entrepreneur's utility function over

k and $m(\underline{x})$ produces the following pair of equations:

$$\frac{\partial U}{\partial k} = \int U' \frac{C-x}{C-\underline{x}} f(x)dx, \tag{4}$$

and

$$\frac{\partial U}{\partial m(\underline{x})} = \int U' \frac{x-C}{C-\underline{x}} f(x)dx. \tag{5}$$

Consider the case where U' is constant and equal to one. Then the right hand side of (4) simplifies to $[C\text{-E}(x)]/[C\text{-}\underline{x}]$. From Condition 1, projects with negative present values are not selected in equilibrium, and thus $C\text{-E}(x)\leq 0$. Also by hypothesis, the project cannot be funded entirely with riskless debt and thus, $C\text{-}\underline{x}>0$ implying that (4) is negative. Next use $U'=1$ in (5). The right hand side of this equation then reduces to $[\text{E}(x)\text{-}C]/[C\text{-}\underline{x}]$. From our previous arguments this is positive implying that $m(\underline{x})=\underline{x}$. Now consider a sequence of functions U_n such that they uniformly converge to a risk neutral utility function. Then there exists some n^* such that for all $n>n^*$ such the entrepreneur invests all wealth into the project and uses as much riskless debt as possible.

Finally, similar arguments show that if the expected value of the project is high enough then a risk averse entrepreneur will also invest all of his wealth into the project and use as much riskless debt as possible. Q.E.D.

Proposition 3 provides an endogenous pecking order theory of financing: first use internal financing, and/or riskless debt and then, if necessary, employ equity. It also clarifies some of the intuition behind our security design problem. Sufficiently risk neutral entrepreneurs do not need the help of outsiders in sharing the risk of good projects (however, they will issue riskless debt and then turn to equity if their own capital is insufficient). Similarly, risk averse entrepreneurs, with sufficiently valuable projects, willingly take on more risk by investing their entire wealth in exchange for equity in the project. Accordingly, this “pecking order” will apply mainly to very good projects and to entrepreneurs that are less risk averse.

We now analyze the case where $\underline{x}$=0, so that the firm has no collateralizable assets that can function as a lower bound on the firm's payoffs. For such businesses the model produces a very strong prediction.

Proposition 4: *If the firm does not have any collateralizable assets, then all financing takes place via partnership agreements. Formally, if* $\underline{x}=0$ *then* $m(x,k)=x,k/C$.

Proof: Set $\underline{x}=0$ in (1). Q.E.D.

Proposition 4 implies that when $\underline{x}$=0 each investor receives a percentage of the firm's profits in proportion to his initial investment. This result does not depend upon the project's expected return, the distribution of its payoffs, or the risk aversion of the entrepreneur. Attempts to deviate from the prescribed contract may leave the insiders with an opportunity to earn strictly positive profits via some non-positive NPV project. Proposition 4 may explain why partnership agreements are the rule in such a wide array of seemingly unrelated businesses. Proposition 4 also closely conforms to the Revised Uniform Limited Partnership Act which states that in either a partnership or a limited partnership, distributions will be shared on the basis of each partner's relative capital contribution, in the absence of an agreement to the contrary (Vol., Thrower, and Reiss (1986)). Conforming to our story, one finds limited partnerships in businesses where uncertainty is high and monitoring is very costly, such as movie or TV financing (see Vogel, (1994)). The results may also apply to some extent to partnership agreements in accounting or law firms where each partner runs essentially his own unsupervised business while profits and overhead have to be shared.

V. Generalizations

So far the paper has only explored economies where zero or negative NPV projects are in infinite supply, and where entrepreneurs (given time) can construct every possible payoff distribution with a zero or negative NPV. However, one can drop either of these assumptions and still retain some of the results. In addition to adding generality to the model's conclusions, this analysis will also provide a better understanding of the underpinnings of our model.[15]

To begin, consider an economy where the financier can observe the project's expected value, but the entrepreneur can select among any distribution function. As pointed out in footnote ?, this problem corresponds to a situation where the firm's manager has access to complete capital markets. By buying and selling the appropriate securities, he can then redistribute cash flows any way he wishes. A slight modification of the original proofs, shows that linear securities still emerge as the optimal contracts. Now, however, the entrepreneur may not have to invest his own funds since the NPV of the project can be considered as a payment to the financier. Let N represent the net present value of the project, which is now common knowledge. Then equation (1) becomes

$$m(x,k) \;\geq\; (x-\underline{x})\frac{k \;-\; m(\underline{x},k)}{C+N-\underline{x}} \;+\; m(\underline{x},k). \tag{6}$$

Otherwise, the analysis remains the same. Equation (6) shows that the linearity of the equilibrium contract derives only from the wide discretion the entrepreneur has over the project's payoff probability density, and not from the existence of zero or negative NPV projects. The supply of nonpositive NPV projects only acts to set N=0 in equation (6), and thus produce positive expected returns the financier holding the contract $m(x,k)$.

Setting equation (6) to an equality describes the set of equilibrium contracts in this case which are all linear. The intuition for this result can be found by looking at the problem from the perspective of the outside financiers. If they offer a nonlinear contract, the entrepreneur can always rearrange the probability distribution so that it masses at zero and one other point in order to yield the lowest expected payoff to the selected contract m. In essence then, the outside financier earns the same payoff as if he offered a linear security contract that runs through the point in m where the entrepreneur concentrates the probability mass. Nonlinear contracts are simply redundant from the financier's perspective. This intuition is similar to that found in Nachman and Noe (1995). In their paper, a principal pays an agent to undertake a project. The agent, however, receives some utility from the consumption of perks, and may therefore underinvest in productive assets. Nachman and Noe (1995) then show that when the agent has a sufficiently rich choice set with regard to the project's payoff distribution, then nonmonotonic contracts are redundant. If the contract offers the agent less in states when profits are higher, then the agent will ensure that profits never fall in these states. The same outcome can be obtained with a contract that replaces the appropriate section of the original contract with a flat contract. This flat section pays an amount equal to the original contract's highest value in that area. In the same vein, in the current paper, nonlinear contracts can be replaced with linear contracts that yield the same expected payoff to the outsiders.

To show that the underpricing feature we obtain results from the infinite supply of nonpositive NPV projects, we restore the assumption that the financier cannot observe the project's expected value. However, now we assume that the outsiders know that probability distributions come from a limited set. To take an extreme example, assume that entrepreneurs only have access to two types of projects. A finite number of the projects are "good" (positive NPV) projects. Their payoff is either some positive number x (with probability $p(x)$) or zero with (probability $1-p(x)$). Let $xp(x)=C+N$ where C is the cost of the project, and $N>0$. However, as in the previous sections of the paper, assume that there exists an infinite supply of zero NPV projects that also pay either x or 0 but of course, with a different probability distribution. In that case, one can clearly see that the equilibrium contract must set $m(x,k)=xk/C$. If the financial institutions offer contracts with superior terms, then they will find themselves inundated with project proposals, almost all of which will have zero NPVs. Setting $m(x,k)=xk/C$ discourages people from financing zero NPV projects. The financiers will then only fund positive NPV projects. However, since only positive NPV projects are presented for financing, then clearly the financiers will make a strictly positive expected return on each project they fund. All of this shows that the abnormal return results from the previous section of the paper derive from the reserve army of zero NPV projects and not from the flexibility offered entrepreneurs in terms of the project's payoff distribution.

VI. Conclusions

Our paper describes the security design problem as a constrained solution to an incentive problem within a very general contract space. Outsiders cannot observe project choice, and insiders can create any negative NPV project they like. Since insiders are allowed to create projects in response to contracts offered to them, outside investors must insure that the contracts they issue do not offer an inducement to take on non-positive NPV projects. As a result, only combinations of riskless debt and equity emerge as equilibrium contracts. Variables that affect the exact mix of securities chosen include the availability of collateralizable assets, the quality of the project and the risk aversion of the entrepreneur. The types of securities we propose are broadly consistent with the financing arrangements of start up firms. These contracts include secured bank loans or profit participation contracts. Our results are also consistent with the underpricing of IPO's. One of our propositions offers an endogenous pecking order of financing and demonstrates that only a participating entrepreneur may obtain financing. An important feature of our model is that it highlights the incentive features inherent in securities. The existence of incentives identified in this paper can help explain why firms are not exclusively financed via extreme securities (see Allen and Gale (1988), or via risky debt (as proposed by Townsend (1979) Innes (1990) and Nachman and Noe (1994)).

Perhaps another way to sort out the security design literature is to say that the optimal contracts will depend upon the relative importance of the problems facing a particular firm and its investors. When the parties can easily verify the cash flows from the company but not the distribution function that produced them, as in the model presented here, the firm will issue mostly equity and collateralized debt. On the other hand, if the important problem at hand is costly verification of realized cash flows (Tonwsend (1979)) or unobservable effort (Innes (1990) and Nachman and Noe (1995)) then risky debt may be selected.

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Footnotes

1. Assuming of course that projects with a positive NPV have some minimum value, say one cent.

2. As in our model, only the expected value is negative, and thus, through the magic of Mel Brooks, the play ends up being a smash hit, which presents major problems for the protagonists who have oversold the show many times over.

3. Alternatively, the actual project selection can be taken to be ex-post observable, but ex-ante non-contractible.

4. If financial markets are complete, these arguments become even stronger. Through the purchase and sale of tradable financial instruments, an entrepreneur can remake the probability distribution over the enterprise's cash flows in any manner he wishes. In this case one can show that riskless debt and equity emerge as the equilibrium contracts *even if* the financiers can observe the project's expected value. Section V of the paper explores this issue in greater detail.

5. Once a project has a negative NPV one can imagine any number of ways to duplicate it. In this case the manager may sign a contract with the suppliers, that on average overpays them but allows the payments to vary so that the firm will produce the "right" cash flows with the right probabilities.

6. Some authors have analyzed optimal contracts under the assumption that entrepreneurs can indeed divert proceeds of projects, for an example see Hart and Moore (1994) and other papers.

7. For this interpretation of the objective function we thank Ron Anderson.

8. One can relax the restriction that $m(x,k)$ is non-negative without altering any of the paper's conclusions. This corresponds to allowing the outside investors to contribute additional funds to the entrepreneur's payoff in some states. In equilibrium such contracts are never offered.

9. Throughout most of the remaining analysis, the subscripts are dropped except where needed for expositional clarity.

10. If entrepreneurs have private benefits of control, then they may be tempted to present projects that have negative NPV even if contracts are optimally set. However, in that case we have a complex set of issues that seems to extend beyond the current scope of the paper. Depending upon how one models the problem, non-linear securities may arise in equilibrium. We thank a referee for pointing this out.

11. Admati and Pfleiderer (1994) provide extensions to risk averse settings and linear contracts do not necessarily emerge.

12. Note that our results stem from both risk sharing aspects and incentive design. Each one separately does not work. For example, if there are only risk sharing issues at stake, then, obviously, all risk should be allocated to the risk neutral outside investors. However, in the environment modeled here, any attempt to engage in such extreme risk shifting will yield the outsiders an infinite supply of bad projects.

13. In a *Businessweek* article on LBO's in the 90's Dobrzynski (1991) describes how some institutions restrict their participation in loans. While ready to invest in LBO's, even after the collapse of the junk bond market, investors require that "sponsors boost their equity in a deal to 25% or more". This roughly coincides

with the notions presented here.

14. Here term close refers to types that are “close” in the uniform (L_∞) norm.

15. We thank a referee for pointing these ideas out to us.