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“Bundling”

Barry Nalebuff
School of Management

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BUNDLING

BARRY NALEBUFF
School of Management
Yale University
Box 208200
New Haven, CT 06520-8200
barry.nalebuff@yale.edu
(203) 432-5968

Abstract: In this paper, we look at the case for bundling in an oligopolistic environment. We show that bundling is a particularly effective entry-deterrent strategy. A company that has market power in two goods, A and B , can, by bundling them together, make it harder for a rival with only one of these goods to enter the market. Bundling allows an incumbent to defend both products without having to price low in each. The traditional explanation for bundling that economists have given is that it serves as an effective tool of price discrimination by a monopolist. Although price discrimination provides a reason to bundle, the gains are small compared to the gains from the entry-deterrent effect.

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1. INTRODUCTION

In this paper, we look at the case for bundling in an oligopolistic environment. We show that bundling is a particularly effective entry-deterrent strategy. A company that has market power in two goods, A and B , can, by bundling them together, make it harder for a rival with only one of these goods to enter the market. Bundling allows an incumbent to defend both products without having to price low in each. While it is still possible to compete by offering a rival bundle, a monopolist can significantly lower the potential profits of a one-product entrant without having to engage in limit pricing prior to entry.

We also show that bundling continues to be an effective pricing tool even if entry deterrence fails (or if there is already an existing one-product rival). A company with a monopoly in product A and a duopoly in product B makes higher profits by selling an A - B bundle than by selling A and B independently. Leveraging market power from A into B and accepting some one-product competition against the bundle is better than using the monopoly power in good A all by itself. Since bundling mitigates the impact of competition on the incumbent, an entrant can expect the bundling strategy to persist, even without any commitment.

The traditional explanation for bundling that economists have given is that it serves as an effective tool of price discrimination by a monopolist [see Stigler (1968), Adams and Yellen (1976), Schmalensee (1982, 1984), McAfee, McMillan, and Whinston (1989), and Bakos and Brynjolfsson (1999a)]. Typically, a firm has to charge one price to all consumers. In these cases, variability in customer valuations frustrates the seller's ability to capture consumer surplus. Thus, a tool that helps reduce heterogeneity in valuations will help a monopolist earn greater profits. This advantage of bundling is especially apparent when the values of A and B are perfectly negatively correlated: offering an A - B bundle leads to homogeneous valuations among consumers and thus the monopolist can capture 100 percent of the consumer surplus. Even if A and B have independent valuations, McAfee, McMillan and Whinston (1989) show that a monopolist still does better by selling A and B as a bundle rather than independently.

Although price discrimination provides a reason to bundle, the gains are small compared to the gains from the entry-deterrent effect. Our baseline model has two goods, where consumer valuations are independent and uniformly distributed over $[0,1]$. In this environment, price discrimination through bundling raises a monopolist's profits from 0.50 to approximately 0.544, a gain of about nine percent. Certainly worthwhile. But the same act of bundling cuts an entrant's potential profits by 60

percent. If this deters entry, profits are more than doubled. Even if this is not enough to deter entry, bundling is still of great value—post-entry profits to the incumbent are more than 50 percent higher with a bundle offering compared to when the goods are sold independently.

The literature on bundling as a price discrimination tool emphasizes that it works best when the bundled goods have a negative correlation in value. This is when bundling most reduces the dispersion in valuations and allows the monopolist to capture the lion's share of consumer surplus. Bundling still works when the valuations are independent, but the gain from bundling disappears with perfect positive correlation.

The opposite is true when bundling is used as an entry deterrent or monopoly extension strategy. It is most effective when the bundled goods are positively correlated in value. Even with independent valuations, bundling is still an effective tool, but it loses its effectiveness when the goods are perfectly negatively correlated in value. The reason is that a one-product entrant has everything its consumers want when the valuations for A and B are negatively correlated. The markets for A and B are essentially different groups of consumers. In contrast, when A and B are positively correlated, the same group of consumers are buying both A and B and, thus, a one-product entrant can't satisfy its customers.

The plan of the paper is to first present the main results in as simple a model as possible. Thus we begin with the case of independent valuations, a neutral ground on which to compare the price-discrimination effect with the entry deterrent or market-power effect. Conveniently, this is also the most mathematically tractable case. Then, in an extensions section, we show that the results are quite general. The extensions include three or more good bundles, non-uniform distributions, non-zero production costs, positive and negative correlation in valuations, complementarity amongst the products, and alternative pricing environments. The basic intuition for all the results comes from the two-good uniform case. The extensions help demonstrate to the naturally skeptical reader that there's no rabbit in the hat. Given the enormous power of the two-good bundle, perhaps the one surprise we find is the limited incremental gain from including additional goods in the bundle.

Price discrimination and entry deterrence are only two of many reasons to offer a bundled product. Creating cost savings is another [see Salinger (1995)] as is creating a more valuable product.¹ In a larger sense, almost everything is a bundled product.

¹ Salinger (1995) recognizes that cost synergies from bundling are most valuable when consumer valuations are positively correlated. Thus, if most consumers would buy both (or neither) A and B when sold separately, then any cost savings from selling them together will create an incentive for a monopolist to sell bundled products when valuations are positively correlated.

A car is a bundle of seats, engine, steering wheel, gas pedal, cup holders, and much more. An obvious explanation for many bundles is that the company can integrate the products better than its customers can. For simplicity, we start with the case in which there is no complementarity in creating the bundle, either in consumption or in production. On the consumption side, that means $\text{Value}(A+B \text{ together}) = \text{Value}(A \text{ alone}) + \text{Value}(B \text{ alone})$. Similarly, there is no complementarity in producing the bundle: $\text{Cost}(A+B \text{ together}) = \text{Cost}(A \text{ alone}) + \text{Cost}(B \text{ alone})$.²

The motivating example for this paper is Microsoft Office, in which Word, Excel, PowerPoint, and Exchange are bundled together into a software suite. The consumer could just as well assemble his or her own bundle. In fact, many consumers may feel that they could do an even better job of assembling a bundle using “best-of-breed” components, perhaps combining Microsoft’s PowerPoint with Corel’s Word Perfect, IBM’s Lotus 123, and Qualcomm’s Eudora for email. Since not all of its products are best-of-breed, how does Microsoft gain an advantage by selling its office products as a bundle?

One could well argue that there are synergies between the software applications in Microsoft Office. For example, the commonality of commands and the ability to create links between applications make the products easier to use. A single telephone number to call for help also makes the package more attractive. On the supply side, it is cheaper to include multiple products in a single CD disc than to package each one individually.³ Although synergies no doubt exist, we will show that even absent these gains, a monopolist concerned about competition would have a strong incentive to sell these products as a bundle rather than individually.

While bundled products are common in many sectors of the economy, the software suite is a particularly good example of the power of bundling because the marginal cost of software is zero. As marginal costs rise, bundling becomes less attractive. Bundling creates an inefficiency in that some consumers are “forced” to buy the bundle even though they value one of its components at below production cost (see Adams and Yellen (1976) and extensions section below). Thus it is not surprising that software is especially conducive to bundling since this countervailing force does not arise.

² This assumption rules out such topical bundles as Microsoft Windows 98 and Explorer as $\text{Value}(\text{Windows} + \text{Explorer}) > \text{Value}(\text{Windows}) + \text{Value}(\text{Explorer})$. This is because the value of Explorer by itself is essentially zero since it needs an operating system on which to run. The case of bundling complements is considered in Matutes and Regibeau (1992), Economides (1998), Heeb (1998), Spiller and Zelner (1997), and later in the extensions section.

³ We believe that the valuation of software, especially office software components, are positively correlated. Thus, Salinger’s (1995) argument regarding cost savings is particularly relevant to this case.

The potential for very large scale bundling of information goods is demonstrated in Bakos and Brynjolfsson (1999a) who consider the limiting case of bundling together an infinite number of goods.

Literature Review

Remarkably few papers examine the role of bundling as an entry deterrent device or as a competitive tool to use against a one-product rival. One explanation for this is that the Chicago School has largely succeeded in discrediting the idea of leveraging monopoly power [see, for example, Director and Levi (1956) and Schmalensee (1982) for a more formal argument]. A company with a monopoly in good A gains no advantage by only selling A as part of a bundle with a competitively supplied product B .⁴

The reason is that good B is freely available at its marginal cost. Thus, consumers evaluate an A - B bundle by whether or not A is worth more than the incremental cost of the bundle over B alone. Anyone who buys the bundle would also be willing to buy A alone, at the same profit margin for the monopolist. In fact, the monopolist would typically do even better by selling A alone as some customers who would buy A alone would not choose to buy the bundle.⁵

Whinston (1990) was the first to reexamine and resurrect the role of tying as an entry deterrent. He recognized that the Chicago School's criticism of leveraging monopoly power from market A to market B applies only if market B is perfectly competitive. Whinston demonstrates the advantages of tying when one firm has a monopoly in A and faces a competitor in B with a differentiated product. In his model, tying commits the monopolist to being more aggressive against an entrant, and this commitment discourages entry. Of course, if entry occurs, the incumbent would then prefer to abandon his bundling strategy. We, too, find a reason to leverage monopoly power, although the model and the mechanism of action are quite different. For example, we find that bundling reduces the entrant's potential profits while mitigating the profit loss to an incumbent if entry occurs. Thus bundling is credible even without any commitment device. Following the presentation of the model, we offer an extended literature review and compare our approaches in more detail.

A commitment to bundling also changes an incumbent's incentives to innovate, as shown by Choi (1998). Bundling gives the monopolist a greater incentive to engage in cost-cutting R&D and thus helps preserve and extend its advantageous position.

⁴ In this paper we will focus on creating an A - B bundle of fixed proportions. More generally, as the Chicago School has recognized, tying the sale of A to B but allowing for variable proportions creates the possibility of profitable price discrimination or metering.

⁵ A customer who has a small surplus for A but who values B below its production cost might prefer to purchase nothing than purchase an A - B bundle.

In a recent series of papers, Bakos and Brynjolfsson (1999a,b) examine the effect of bundling at a very large scale. Their first paper (1999a) demonstrates that bundling an infinite number of goods together allows a monopolist to achieve perfect price discrimination. Along with being a super-effective price-discrimination tool, bundling also creates a significant barrier to entry (1999b). Because the monopolist is able to sell its bundle to 100% of consumers, an entrant can sell its individual products only for their incremental value. When all the entrant's potential customers have already purchased the incumbent's B product, the entrant's price is thus limited by the extent to which its product is differentiated and viewed as superior by a subset of consumers.

Our focus lies at the other end of the spectrum, namely small bundles, mostly two goods. Although two-good bundles are not that effective at price discrimination, they are surprisingly effective at entry deterrence. We explain the difference in our approaches and results when we discuss multi-good bundling in the extensions section.

Whinston, Choi, Bakos and Brynjolfsson all emphasize the exclusionary power of bundling. In contrast, results by Carbajo, De Meza, and Seidman (1990) and Chen (1997) demonstrate the potential for bundling to be used as a facilitating device. In both these papers, the intuition is that bundling becomes a way for the two competing firms to better differentiate themselves. For example, in Carbajo et. al. one company sells both A and B , while the rival firm only sells B . If the two goods are sold separately, then the profits in B are competed away and thus the first firm simply earns the monopoly profits on A . If, in contrast, the first firm only sells A and B as a bundle then it can go after the high value customers and leave the rival firm to pick up the low value customers who go unserved.⁶

Chen's model achieves a similar result through different means. Two companies can each produce products A and B . The two firms are duopolists in the A market, but the B market is competitive. Instead of both firms selling A (and competing away all profits), one sells just A and the other sells just an A - B bundle. In essence, the two firms commit to dividing up the A market. Firm 1 gets the A customers who don't care for B , while firm 2 takes the A customers who also like B .

There is a sense in which the present paper is able to capture the best of both approaches. Bundling is a facilitating device in that it mitigates the impact of entry on the incumbent. But it is a facilitating device only for the incumbent. For the entrant, bundling makes it much harder to enter the market. Again, there is more to say about these approaches and we continue our discussion of the literature following the presentation of our model.

⁶ The consumer value of A and B are perfectly correlated. This is done so as to eliminate any price discrimination gain from bundling and thus concentrate of the competition effect.

2. A MODEL

Consider a market with two goods, labeled A and B . The two strategic players in the game are an incumbent and a potential challenger. The incumbent produces both goods A and B , each at zero marginal cost. The challenger is assumed to have a perfect substitute product for one of A or B , also produced at zero marginal cost. Whether the challenger will have a rival A product or a rival B product is random, and each outcome is equally likely. The incumbent's challenge is to prepare against possible entry in either A or B without knowing which flank the entrant will attack.

Even though a challenger has a rival product, it need not enter the game. The entry decision will be based on whether the expected profits in the game cover its costs of entry. The entry costs are determined by the environment and thus known to all players.

We assume that the incumbent sets its prices prior to the challenger's entry decision. The incumbent's prices are then fixed for the rest of the game. We follow this approach for several reasons. First, it is the most favorable toward an entrant. If an incumbent can deter entry without being able to lower prices post entry, then even a myopic entrant would be deterred from coming into the market.⁷

This is a decidedly old-fashioned approach, one that has typically been supplanted by having the entrant anticipate the post-entry price equilibrium. In the modern approach, the pre-entry price is irrelevant, except, perhaps, as an indication of the incumbent's costs.⁸ But as Bain (1949) persuasively argued "the supposition that the potential entrant's judgment of industry demand and rivalry he will meet is entirely unrelated to current price or profit in the industry, however, probably goes too far. Even if he does not believe the observed price will remain there for him to exploit, he may nevertheless regard this price as an indicator both of the character of industry demand and of the probable character of rival policy after his entry."⁹ Thus we don't imagine that an entrant expects the incumbent not to react at all to entry—rather that the post-entry price will be somewhere between the incumbent's pre-entry price and

⁷ In contrast, if the firms played a Bertrand-Nash pricing game post entry, pricing and profits would both fall to zero and entry would be costlessly deterred. Since this is a game of perfect information, the incumbent could then earn monopoly profits.

⁸ The status quo price of the incumbent is relevant to an entrant when it can be taken as a signal of the incumbent's cost structure. Here, all the costs are known and there is no private information and, hence, nothing to signal.

⁹ Bain noted that this perspective was most applicable to oligopolies without significant product differentiation. In the present model, the entrant's product is a perfect substitute so that there is zero product differentiation.

perfect competition. If an entrant can't justify entry costs at the prevailing pre-entry prices then this is a persuasive argument not to enter the market.

Another reason to fix the incumbent's price is that the entrant might be able to approach some customers and offer them a better deal before the incumbent can react. Once the incumbent reacts, the ensuing price war might destroy subsequent profitability; however, the short-term profits that arise during the period prior to discovery could be enough to cover entry costs.¹⁰ In a similar vein, the entrant may employ a judo strategy by entering into the market with a limited capacity. If the incumbent is constrained to charge one price to all its customers (perhaps by a most-favored customer clause), it may not be worthwhile to reduce price in order to regain the limited number of customers that were stolen away.¹¹

Consumers are interested in purchasing exactly one unit of A and/or one unit of B . A consumer of type α values A at α_a and B at α_b . The distribution of consumers is given by $f(\alpha)$ where we normalize the total market to be of size 1. For simplicity, we assume that $f(\alpha)$ is uniform over the unit square and that budget constraints are not an issue. This implies that the valuations of A and B are independent and uniform over $[0, 1]$.

Independent Pricing

As a baseline, we consider the case in which A and B are only sold separately. If the incumbent were alone in the market and priced the two goods independently, profits would be maximized at $p_a = p_b = 0.5$. The incumbent's profits would be $0.25 + 0.25 = 0.50$.

This pricing makes it particularly easy for a challenger to enter the market. The challenger comes in with a price of $0.5 - \epsilon$ and steals all the market in whichever product he has. If this happens, the entrant earns 0.25 and the incumbent's profits are reduced by half to 0.25.

The incumbent can also engage in limit pricing. For example, if the incumbent lowers the price of A , this would reduce a challenger's potential profits. Symmetrically, if it made sense for the incumbent to lower the price of A , it would also make sense for it to lower the price of B . Since the challenger can make essentially all of the profits of an incumbent in whichever market it enters, if an incumbent has pre-entry profits

¹⁰ Here, we are not considering the repeated game and the incentives this might create to engage in a price war to deter future entry.

¹¹ Although in our model there are no capacity constraints on the entrant, this could be one of several geographic markets, and the incumbent might be constrained to charge the same price across its markets.

of Z , the entrant can make $Z/2$. Thus, instead of choosing a price directly, it is easier to think of the incumbent's strategy as choosing a profit level, which translates back into a price.

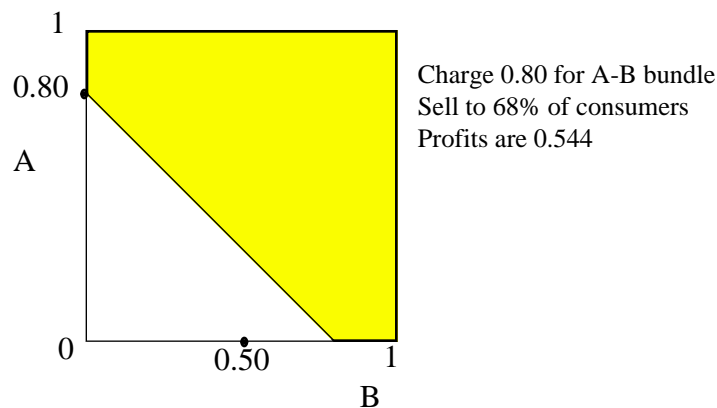
To deter a challenger with entry costs of E requires the incumbent to price at a point p such that its profits are no more than $2E$. This is because the entrant can take half of the incumbent's market and profits. If the incumbent chooses not to deter entry, it should charge $(1/2, 1/2)$, the optimal price absent entry. This is because the firm will lose one of two markets. In the remaining market, the firm does best to maximize profits, which occurs when price is $1/2$.

Thus, the incumbent's choice is between deterring entry and earning profits of $2E$ or accepting entry and earning profits of $1/4$. The incumbent will deter entry if $E > 1/8$ and accept entry otherwise.

Pure Bundling

Here we assume that the incumbent can sell A and B only as a bundle. If the incumbent prices the bundle at 1, the sum of the monopoly prices for A and B , it would sell to half the market and its profits would remain unchanged at $1/2$. However, there are now relatively more marginal consumers and this creates an incentive to cut price.¹² Absent entry, the profits of an incumbent with bundle price x are $x*(1-x^2/2)$, for $x \leq 1$, which is maximized at $x = \sqrt{2/3}$. (Since the optimal monopoly bundled price is below 1, henceforth we will restrict our attention to the case of $x \leq 1$.)

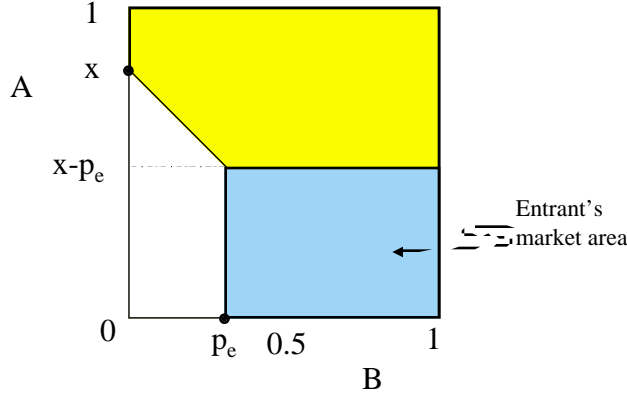
Figure 1



¹² The intuition of bundling as a more cost-effective way to discount comes from McAfee, McMillan, and Whinston (1989).

If an entrant comes into the market at price p_e , it captures the set of consumers who value its item at more than p_e , and the other item at less than $(x - p_e)$. Thus, the entrant's market area is the shaded region below.

Figure 2



The entrant's profits are $\Pi_e = p_e * (1 - p_e) * (x - p_e)$.

Proposition 1: The entrant maximizes profits at

$$p_e^* = \frac{1+x}{3} - \frac{1}{3}\sqrt{1-x+x^2}.$$

This follows from differentiating the profit function. The first-order condition is $(1-2p)(x-p) - p(1-p) = 0$. Rearranging and applying the quadratic formula leads to the result in proposition 1.

Corollary 1: $\frac{x}{3} \leq p_e^* < \frac{x}{3} + 0.045$.

The first inequality, $p_e^* \geq x/3$, follows from the fact that $(1-x+x^2) \geq 1$ for $0 \leq x \leq 1$; the second inequality, $p_e^* < x/3 + 0.045$, follows as $1-x+x^2$ is minimized at $x = 0.5$.

Corollary 1 implies that the elasticity of the entrant's profits with respect to the incumbent's price is at least 150 percent.

Corollary 2: $\frac{\partial \log(\Pi_e)}{\partial \log(x)} \geq \frac{3}{2}$.

This follows as $\frac{\partial \log(\Pi_e)}{\partial \log(x)} = \frac{x}{x-p_e}$ and $x/(x-p_e) \geq 3/2$ as $p_e \geq x/3$.

Intuition

Having presented the basic mathematics, we are now ready to explain how bundling makes entry much less profitable without lowering the profits of the incumbent if entry

is deterred. The intuition for this result comes from two channels, what we will call (i) the pure bundling effect and (ii) the bundle discount effect.

To see the pure bundle effect, we look at the case in which the incumbent and challenger simply translate their independent pricing strategy into the bundled case without re-optimizing. Call this “equivalent prices.” If the bundle is priced at 1, the incumbent makes the exact same amount as it would selling the two products independently, assuming no entry. The entrant considers coming into the market with a price of 0.50, essentially the same price he would charge against an incumbent selling A and B each for 0.50.

Of course, neither the incumbent’s bundled price of 1 nor the entrant’s contemplation of 0.50 is an optimal strategy. The point of these simple translations from the independent case is to demonstrate the “pure bundling effect.”

Pure Bundling Effect: At equivalent prices, the act of putting two independent products into a bundle reduces the entrant’s profits by 50%.

The reason is straightforward. Assume the entrant has product B . An entrant who comes in to the B market with independent pricing can steal the entire market with a marginal discount and thereby earn 0.25. Consider, in contrast, what happens when an entrant comes into a market against an incumbent with a bundle priced at 1. If the entrant prices at 0.50, it sells only to people who value B at above 0.50 and A at less than 0.50. (A consumer who values A at more than 0.50 would be better served by buying the bundle.) Consequently, the entrant only sells to 25 percent of the market and its profits are cut by 50 percent. Figure 2 illustrates this effect. Instead of capturing all of the area to the right of p_e , the entrant is limited to a box with height $x - p_e$. Thus, if $x = 1$ and $p = 0.5$, the entrant’s market is reduced by 50 percent—the entrant only gets the bottom half of its potential market.

The second channel that provides additional entry deterrence is what we call the “bundle discount effect.” As the bundle price is reduced and the bundle is sold at a discount relative to the original component prices, entry becomes less and less profitable. This is low-cost or even costless deterrence, as even absent considerations of entry an incumbent would choose to price its bundle at below 1.

Bundling Discount Effect: Selling the bundle at a discount to the optimal independent pricing provides an opportunity to raise the incumbent’s profits absent entry, while making entry even less profitable. Further reductions in the bundled price have a second-order loss to the incumbent and a first-order loss to the entrant.

Recall that the optimal bundled price for an uncontested monopoly is $\sqrt{2/3} \approx 0.8$. Lowering the incumbent’s bundle price from 1 to 0.8 reduces the potential profits of

an entrant, while raising profits if entry is deterred. Reducing the bundled price below 0.8 further reduces the potential profits of an entrant, while also lowering profits if entry is deterred. However, the incumbent's price is near an optimum, and so profits fall slowly while the entrant's profits continue to fall rapidly.

Table 1 shows the incumbent's profits alongside the opportunities presented to an entrant. These calculations are all based on the optimal price response by an entrant.

Incumbent charges

1.0 =>	$\Pi_{\text{inc}} = 0.5;$	$\Pi_e = 0.148$
0.8 =>	$\Pi_{\text{inc}} = \mathbf{0.54};$	$\Pi_e = 0.10$
0.66 =>	$\Pi_{\text{inc}} = 0.52;$	$\Pi_e = 0.077$
0.42=>	$\Pi_{\text{inc}} = 0.38;$	$\Pi_e = 0.032$

If the incumbent charges 0.8, its profits absent entry rise from 0.50 to 0.54. From corollary 2, the 20 percent reduction in the incumbent's bundle price should lead to at least a 30 percent reduction in the entrant's profits. In fact, the entrant's profits fall by 33 percent, from 0.148 to 0.10.¹³

The incumbent can push things even further at little cost. For example, a move down to a bundled price of 0.66 reduces the incumbent's profits absent entry to 0.52, or by about 2 percent. Meanwhile, the entrant's profits are reduced by another 33 percent, down to 0.077.

Further price cuts begin to impose first-order costs on the incumbent. For example, if the bundle price falls to 0.42, the incumbent's profits fall to 0.38, about a 25 percent reduction. The entrant's profits fall almost 60 percent, all the way down to 0.032.

The cumulative effect of these price cuts is impressive. Taking the bundled price down from 1 to 0.66 results in an inconsequential loss to the incumbent, while reducing the entrant's profits by 50 percent. Going all the way down to a bundled price of 0.42 reduces the incumbent's profits by 25 percent, but leaves the entrant with potential profits of only 0.032, a 78 percent reduction from its profits when $x = 1$.

¹³ Applying Proposition 1, in response to an incumbent price of 1, the optimal entry price is $1/3$; in response to an incumbent price of 0.8, the optimal entry price is approximately 0.27. Thus the entrant's potential profits fall from $(1/3)(2/3)(2/3) = 0.148$ at $x = 1$ to $(0.27)(0.73)(0.53) = 0.10$ at $x = 0.8$ or a further 33 percent at no cost, and even some gain, to the incumbent. Note: one has to be careful in applying Corollary 2 to price cuts that are more than marginal as the original profits are too large to use as the baseline in calculating percentages.

Putting together the pure-bundling effect with the bundle-price-discount effect leads to even more powerful effects. If the incumbent charges 0.66 for a bundle versus selling each good independently at 0.50, its no-entry profits rise by 4 percent from 0.50 to 0.52. Meanwhile, the entrant’s potential profits fall from 0.25 to 0.077, or 69 percent. Moving to a bundled price of 0.42 reduces the incumbent’s profits by 25 percent, yet reduces the entrants potential profits from 0.25 to 0.032 or a full 87 percent!¹⁴

Post-Entry Protection

Up until this point, we have focused on the reduction in potential profits to an entrant. A further benefit to the incumbent is that, should entry occur, the loss to the incumbent is significantly reduced relative to the independent pricing case. Going back to the pure bundling effect helps illustrate the point.

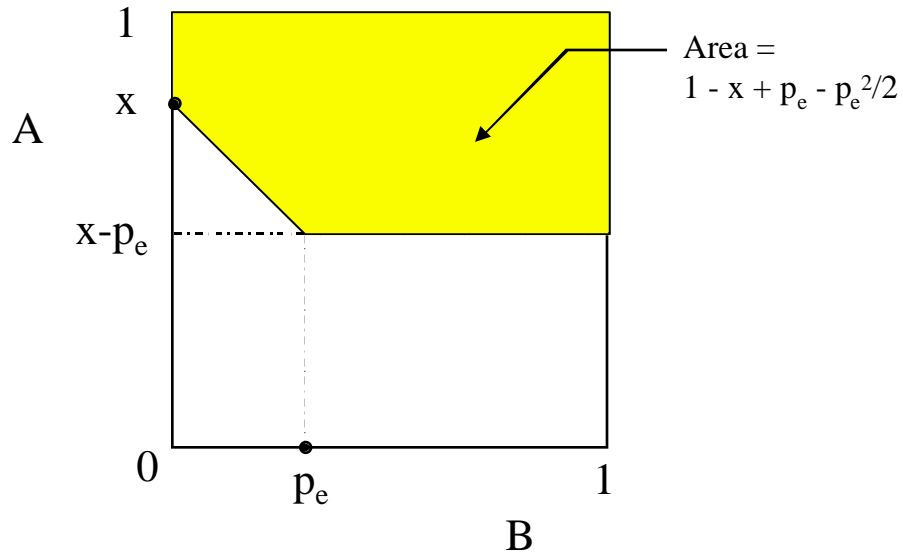
Take the case in which the incumbent charges $x = 1$ for the bundle and the entrant comes in selling B at $p_e = 0.5$. Instead of losing all of its B sales, the incumbent loses sales only to its customers who value the bundle at above 1, but value A at less than 0.50. In fact, this is only one quarter of the incumbent’s market—but, since it loses the bundled sale, this is like losing half the sales on one of the products.

This, of course, understates the cost of entry. Against a bundle price of 1, the entrant would charge $1/3$, not $1/2$, and capture $(2/3)(2/3) = 4/9$ of the market. When $x = 1$ (and only when $x = 1$), it is always the case that the incumbent loses half of what the entrant captures. Thus, the incumbent loses $2/9$ ths of the market. Since the incumbent was charging a price of 1 for the bundle, this also represents its lost profits. In comparison, when the challenger came into the market with independent pricing, the incumbent lost all of the B market and half of its profits, or 0.25.

Anticipating entry, the incumbent can do much better by charging a price below 1. Figure 3 below illustrates the profits to the incumbent following entry.

¹⁴ At $x = 0.66$, $p_e = 0.26$, and $\Pi_e = 0.26 * (0.74)(0.4) = 0.077$. At $x = 0.4$, $p_e = 0.176$, and $\Pi_e = 0.176 * (0.824)(0.224) = 0.032$ a fall of 87%.

Figure 3



$$\Pi_I|entry = x * (1 - x + p_e - p_e^2/2).$$

Numerical calculations show that $\Pi_I|entry$ is maximized at $x \approx 0.66$, which leads to profits of 0.38. (In response to this price, the entrant would charge roughly 0.26 and earn 0.08.)

Incumbent charges

$$1.0 \Rightarrow \Pi_{inc} = 0.50; \quad \Pi_{inc} | e = 0.277$$

$$0.80 \Rightarrow \Pi_{inc} = 0.53; \quad \Pi_{inc} | e = 0.36$$

$$0.66 \Rightarrow \Pi_{inc} = 0.52; \quad \Pi_{inc} | e = \mathbf{0.38}$$

$$0.42 \Rightarrow \Pi_{inc} = 0.38; \quad \Pi_e = 0.032$$

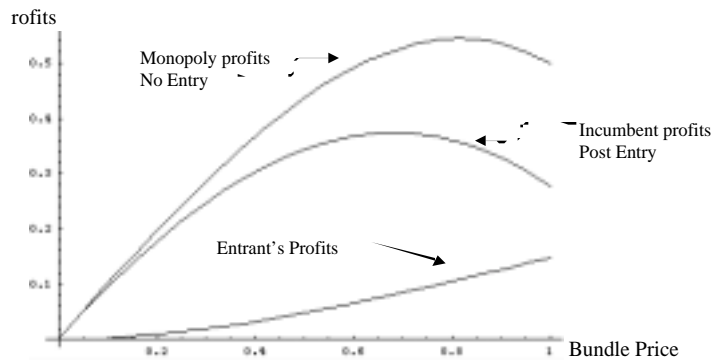
Since the incumbent can earn 0.38 by allowing entry, the incumbent should attempt to deter entry only if doing so leads to profits above 0.38. Recall that the incumbent's profits without any entry are given by $x(1 - x^2/2)$. A bundle price of 0.42 translates to incumbent profits of 0.38, absent entry. But a bundle price of 0.42 would allow an entrant to earn only 0.032.

Putting this all together, the incumbent should charge 0.66 and accept entry if the rival's entry costs are below 0.032. Otherwise, the incumbent should engage in "limit pricing" and set a bundled price just low enough to deter the rival from entering. The

incumbent follows this policy all the way up to a price of $\sqrt{2/3} \approx 0.8$, at which point there is no further gain from raising price.

The results about bundling can all be summarized in the following two diagrams: The top curve shows the incumbent's profits as a monopolist. The middle curve traces the incumbent's profits if entry occurs, while the bottom curve illustrates the entrant's potential profits for each bundle price. The incumbent can always achieve the maximum of its profits-with-entry curve (profits of 0.38). Therefore, it deters entry only when the top curve lies above 0.38.

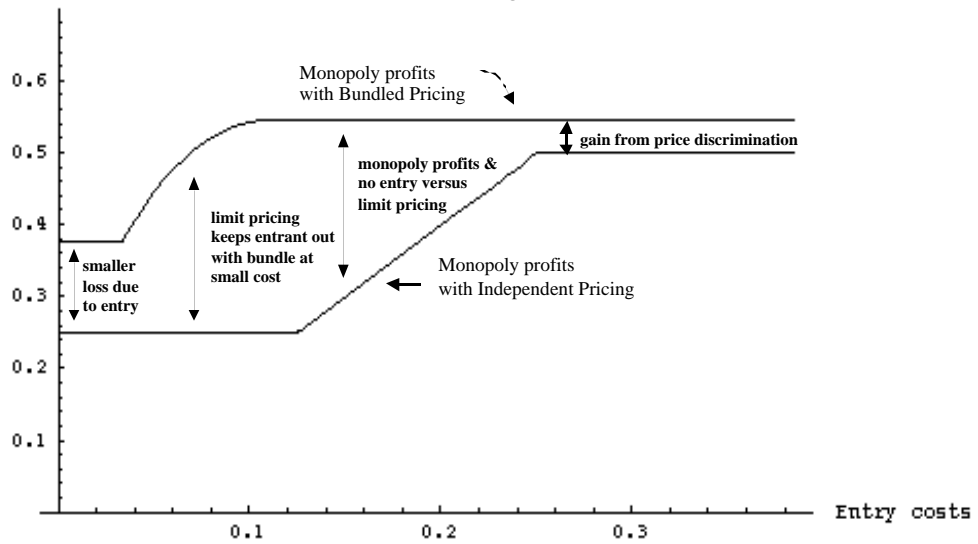
Figure 4



The results of this graph can be incorporated into a new graph, in which the horizontal axis now reflects entry costs.

Inc. Profits

Figure 5



The price-discrimination effect on which previous authors have focussed is evidenced by the fact that the bundle line reaches profits of 0.54 as opposed to 0.50. The ability of bundling to deter entry is reflected in the fact that entry occurs only when entry costs are below 0.032, compared to 0.25 in the case with independent pricing. It is remarkable that once entry costs are above 0.10, the incumbent can pick its unconstrained monopoly price and not be concerned about entry.

The result that entry is less costly to the incumbent is reflected in the fact that when entry occurs, the incumbent's profits are 0.38 versus 0.25, a 52% improvement. Thus, bundling mitigates the cost of entry. For this reason, it is in the incumbent's self-interest to maintain the bundle post-entry. There is no issue of credibility or commitment.¹⁵

The rapidly rising profits in the curved section, where entry costs are between 0.032 to 0.1, demonstrate that bundled pricing is an efficient way to deter entry. When entry costs are 0.1, an incumbent that bundles earns 0.54, more than double the profits of an incumbent that sells its products independently. The take-away message demonstrated by Figure 5 is that the price discrimination effect offered by bundling is valuable, but the really big gains come from the entry-mitigation effect and the efficient entry-deterrence.

There is a side benefit to bundling that is hard to quantify. This benefit arises when entry costs are so low (below 0.032) that entry is not profitably deterred. When the entrant comes into the market, its profits are only 0.08, 68 percent less than when it enters against unbundled pricing.¹⁶ Once the decision to accommodate entry has been made, there is no reason in our model for the incumbent to be concerned with a competitor's profits. Even so, we recognize that most firms would feel less threatened when their rivals are less profitable.

3. EXTENSIONS

In this section we develop the basic model along several lines. We begin with an exploration of the results to include the case where there is uncertainty regarding the potential challenger's entry costs. Bundling continues to be attractive even when

¹⁵ In contrast, in Whinston (1990) bundling is used as a way to commit to *greater* post-entry competition. This lowers the entrant's profits along with the incumbent's. Thus the credibility of this commitment will be an issue.

¹⁶ In the case of independent pricing, firm 1 charges 0.5 for both A and B. Firm 2 comes in at a penny less for its product, say B, and earns $1/4$ (minus ϵ). When firm 1 employs bundling, the low-cost entrant faces a bundle price of 0.67 against which it would charge roughly 0.26 and earn 0.08.

entry costs are unknown. We then explore the incentive to create more complicated bundles, both through mixed bundling and by bundling more than two items together. The potentially surprising result is that the gains from expanding the number of products in the bundle beyond two are relatively small.

We next turn to consider how the results change if the distribution of valuations is other than uniform. For all symmetric quasi-concave (single-peaked) densities, the advantages to bundling are even greater. We also look at the effect of positive and negative correlations in value. Positive correlation amplifies the gains to bundling while negative correlation makes it less valuable. The intuition for positive and negative correlation is often better captured by the existence of complementarity between the bundled goods, and we turn to this topic next. When the bundled products are complements, bundling is an even more powerful tool. The value does not disappear with substitutes, although it is diminished. In cases where the products are costly to produce, bundling is less valuable. Finally, we consider the issue of commitment. We show that the incumbent has an incentive to persist with a bundled offering post entry. Furthermore, even if the incumbent cannot commit to holding prices fixed post entry, we demonstrate there are still significant advantages to competing with a bundled offering. This can be thought of as the incentive to bundle against an existing one-product rival.

Uncertain Entry Costs

Our analysis was made under the assumption that the incumbent knows the precise entry cost of the challenger and thus can fine-tune its entry deterrent strategy. The same curves can be used to illustrate the advantages of bundling when the potential challenger's entry costs are uncertain. As before, we assume there will be exactly one potential challenger, equally likely to have good A or B . Now, its entry costs, c , are distributed according to cumulative density function $G(c)$. We solve the case where entry costs are uniformly distributed on $[0, 1]$.

With independent pricing, let the incumbent charge an amount p such that $p(1 - p) = z$. The entrant will come in if $c < z$ in which case the incumbent will make z ; otherwise the incumbent will make $2z$. Thus expected profits are $2z(1 - z) + z^2 = 2z - z^2$, which is maximized at $z = 0.25$, the highest possible profits which correspond to $p_a = p_b = 0.5$.¹⁷ The incumbent does not attempt to deter entry and its expected profits are 0.4375.

¹⁷ If distribution of entry costs is $G(c)$, then maximize $[1 - G(z)]2z + G(z)z = 2z - zG(z)$. Expected profits reach a maximum at $2 - G(z) = zg(z)$.

With bundled pricing, if the incumbent charges an amount x , then, if the challenger enters, it will come in at price $p_e^* = \frac{1+x}{3} - \frac{1}{3}\sqrt{1-x+x^2}$ and earn $\Pi_e = p_e^* (1-p_e^*) (x-p_e^*)$. The incumbent's expected profits are thus

$$E\Pi_I = x*(1-x^2/2)*[1-G(p_e^*(1-p_e^)*(x-p_e^*))]+x*(1-x+p_e-p_e^2/2)*G(p_e^*(1-p_e^)*(x-p_e^*)).$$

In the case of the uniform distribution, this is maximized at $x = 0.79$, just a bit below 0.80, and expected profits are 0.525 rather than the maximal 0.544 without entry.¹⁸ This is well above the expected profits with independent pricing.

Mixed Bundling:

An uncontested monopolist, selling a mixed bundle, A , B , and an $A-B$ bundle, could always achieve higher profits. However, mixed bundling is less effective in the presence of a rival than in the pure monopoly model. The reason is that the incumbent has to be concerned that a rival with one product, say B , will use the incumbent's other product, A , to create a rival bundle and thereby steal all of the incumbent's bundle sales. Thus, the individual items need to be priced very high relative to the bundle, and so the individual items in the mixed bundle generate relatively few additional sales. Given the limited potential for this approach to increase profits, we do not pursue it further.

Bundling Three or More Goods:

It might be tempting to extrapolate and predict that if bundling two items together is a good idea, then putting three together would be even better. It turns out that there are advantages of adding more goods to the bundle, but they are smaller than one might expect given our preceding analysis. The discrete gains that come from the act of bundling two goods do not continue to grow when more goods are added to the bundle. Eventually there is a gain from creating a very large bundle but that gain is due to a third effect, the law of large numbers.

More precisely, Proposition 2 below shows that the pure bundling effect remains constant for any number of products in the bundle. Assume that a one-product entrant prices at 1/2. Its sales will be 1/2 against a monopolist with independent pricing, but only 1/4 against a two-product bundle priced at 1. In fact, the entrant's sales *remain* at 1/4 against a three-product bundle priced at 1.5, a four-product bundle priced at 2, a one-hundred-product bundle priced at 50, and so on.

¹⁸ Note that the chance of entry is only 0.10, in which case the incumbent's profits fall down only to 0.36.

Proposition 2: For all $n \geq 2$, with $x = n/2$ and $p_e = 1/2$, the challenger has sales of $1/4$ units and revenue of $1/8$.

Proof: In Appendix

The intuition for this result is as follows: if a one-product entrant offers its product at $1/2$ against a 100-product bundle priced at 50, whether consumers will buy the single product or the bundle depends on whether they value the other 99 products above or below 49.50. Half do and half don't. Thus the entrant's demand is cut by 50 percent, just as in the two-product bundle.

The advantage of adding more products to the bundle is the incremental gain from the bundle discount effect. In the Appendix, we calculate the uncontested monopolist's optimal bundle price for multiple values of n . For example, the optimal price for a 3-good bundle is 1.16 and the corresponding profits are 0.86, absent entry. A potential entrant can earn 0.078 and if this is profitable, entry cuts the incumbent's profits down to 0.67.

In general we find that the price-discount effect gets stronger as the dimensionality of the bundle increases. By the time there are ten goods in the bundle, the entrant gets only one-third as many customers (at its optimal entry price) compared to its optimal entry against a two-good bundle.

As the number of goods in a bundle becomes very large, the price-discrimination effect of a bundling becomes dominant. This is the insight of Bakos and Brynjolfsson (1999a). Intuitively, the standard deviation of valuations becomes small relative to the average valuation. Thus if the average valuation of a product is \$0.50 and a monopolist charges \$4,900 for a 10,000 item bundle this is roughly 3.4 standard deviations below the mean bundle valuation and thus 99.9% of consumers would buy this bundle. The incumbent has to give up only 1 cent per item to capture almost all the demand.

Given that almost all consumers will choose to buy the bundle, almost no one will choose to buy the offering of a single entrant rival, even without any response from the incumbent. Imagine that the rival gave away its product. The maximum surplus from buying the single good is 1. In contrast, the mean surplus from buying the bundle is 100 and so still somewhere around 99.9% of consumers would have a surplus greater than 1. Thus a rival might be able attract only 0.1% of consumers, even when its product is free.

In the present model, these super-large bundles would be the ultimately effective entry deterrent devices. Bakos and Brynjolfsson (1999b) make the case more interesting by allowing the entrant to have some product differentiation. Their approach to differentiation is very appealing. Allow the incumbent to have A and B , just as

before. The rival has a good C , the value of which is distributed independently of B . Customers can only consume either B or C . The rival's product is a substitute for B , but the products are differentiated. Thus, even if all consumers are buying the bundle, individuals who particularly like C over B will buy the C good in addition to the bundle and this makes it possible for a one-product entrant to sell against a mega-bundle.¹⁹

When it is feasible to create super-large bundles, this will clearly be an effective strategy, both for price-discrimination and entry deterrence reasons. There are several constraints that may prevent this. First, as the size of the bundle grows, the i.i.d. assumption implies that the bundle will also become increasingly valuable (and expensive). Although AOL might bundle thousands of information goods together in its package, the value of the bundle doesn't seem to grow proportionately. Nor does the distribution in valuations appear to converge. One likely reason is that valuations across goods are typically positively correlated, so that dispersion remains. (This would be the case if valuations depend on some common factors, such as personal or business use.) As we will observe later in the extensions section, bundling as an entry deterrent continues to work – is even enhanced – with positive correlation, while the price discrimination effect is diminished. Even if the additional goods are not identically distributed, they must have some value to the average consumer.²⁰ And, all the goods in the bundle must be provided at zero marginal cost. Even with information goods, if the consumer faces a time cost of downloading based then there is no free disposal and this has the same effect as a positive marginal cost. Freeware illustrates the point. The supply is essentially infinite, but most products have essentially zero value or what little value they have is often exceeded by the download time and risk of software incompatibilities.

With a relatively modest number of goods in the bundle the price discrimination effect is only modest. For example, even with 10 goods the monopoly profits are \$3.45

¹⁹ This element of product differentiation makes entry much less costly to an incumbent, even with only two goods. For example, if the incumbent doesn't bundle and charges $1/2$ and then the rival comes in at $1/2$ minus ϵ , the incumbent and entrant in effect each have demand of $3/8$, versus 0 and $1/2$. If an entrant were to come in against a bundle at price 1, its demand would fall only to $3/8 - 1/48$, about a 5% reduction. It is worth noting that this is a very large degree of differentiation. For example, it would be unusual for a consumer's value of two word processing programs to be completely independent. More common might be a 10% range in valuations. Of course, this makes the Bakos and Brynjolfsson (1999b) result even more impressive because in spite of all this differentiation the rival is unable to attract more than 23% of the market. Sales will fall from $1/2$ to $2/9$ even while price falls from $1/2$ to $1/3$ and thus profits fall from $1/4$ to $2/27$ or 70%.

²⁰ One could imagine the average component having a value of $1/n$ so that the total value is bounded. But, that would lead to complete convergence in consumers' valuations for the bundle. This seems counterfactual for existing large information good bundles such as AOL, Lexis-Nexis, or Dow Jones online services. A common factor or factors is likely in these cases.

compared to an ideal of \$5.00. Profit are certainly better than the \$2.50 without any bundling, but there are many consumers who remain unserved by the bundle and even more who have only a small surplus and who might be attracted away by a well-priced single product. Thus the emphasis of this paper is how effective bundling is as an entry deterrent even when the number of components is small and bundling is far from achieving perfect price discrimination.

Upon reflection, when the scope of bundling is modest, the strongest argument for adding more goods to a bundle is to keep one step ahead of a potential entrant. An incumbent with a three-good bundle has less to fear from a potential entrant who can put together a two-good package. The third good prevents the incumbent from competing head-to-head and makes it harder for the entrant to gain share.

Non-zero marginal cost of production

Selling two or more goods together will create inefficiencies when production costs are no longer zero at the margin. Even so, significant gains from bundling persist, even for moderate costs.

In the case of independent pricing, the optimal prices are $p_a = p_b = (1 + c)/2$ and profits are $2 \left(\frac{1-c}{2}\right)^2$.

In the case of bundling, the optimization problem becomes: Maximize $(x - c)(1 - x^2/2)$ which implies

$$1 - 3/2x^2 + cx = 0 \quad \text{or} \quad x^* = [c + \sqrt{c^2 + 6}]/3.$$

If we consider the bundle price of 1 and the entrant price of $1/2$, we should do this for costs such that the optimal bundle price chosen by a monopolist is still below 1. That implies $c + \sqrt{c^2 + 6} < 3$ or that $c < 1/2$.

We first examine what happens when the monopolist charges 1 for the bundle. If the entrant comes in at $p_e = 1/2$, profits are $(1/2 - c)/4$ compared to $(1 - c)^2/4$. The ratio of profits is thus $(1/2 - c)/(1 - c)^2$. If $c = 0$, this is $1/2$, the original pure bundling result. If $c = 0.1$ this ratio is $0.4/0.81$, just barely less than $1/2$. If $c = 0.25$, then the ratio is $0.25/(9/16) = 4/9$ or 0.44 .

If the entrant comes in at $(1 + c)/2$, its profits are $[(1 - c)/2]^3$. If we compare this to the profits it would get coming in against independent pricing the ratio of profits is $(1 - c)/2$. Thus the pure bundling effect remains at $1/2$ if $c = 0$. But if $c = 0.1$, then profits fall by 55%. If $c = 0.25$, then profits fall by $5/8$.

Next we consider the comparison when the monopolist charges $1 + c$ for the bundle and the entrant comes in at $(1 + c)/2$. Entrant profits are now $(1 - c)/2 * (1 - c)/2 * (1 +$

$c)/2$ and so the ratio of profits is $(1 + c)/2$. The pure bundling effect is diminished by the realization of production costs. But the effect is still large. If $c = 0.1$, the ratio is 0.55, so that entrant's profits fall by 45% as a result of the bundling. Even if $c = 0.25$, the ratio is $5/8$, so the entrant's profits fall by $3/8$ as a result of the bundling.

Non-uniform distribution of values

In this section we maintain the assumption that the valuation of the two goods are independent and consider the extent to which the results rely on a uniform distribution of valuations. We show that over the entire class of symmetric, quasi-concave densities, the uniform density is the least favorable to bundling. Thus all other single-peaked densities (such as the multi-variate normal) make bundling even more desirable.

To make a fair comparison, we choose a density function which is symmetric around $1/2$. Thus with independent pricing, monopoly profits at $p_a = p_b = 1/2$ are constant at 0.50. By holding the monopoly profits constant, this allows us to consider the differential benefits of bundling.

Lemma 1: Assume $f(\alpha)$ is single-peaked and symmetric. Then the optimal single good price, \hat{p} , is below $1/2$.

Proof: Imagine that the monopolist chooses a price, \hat{p} , other than $1/2$. Since the density is symmetric on $[0, 1]$, we know that at $p = 1/2$, the first-order condition is $1/2 - 1/2f(1/2)$. Since the density is maximized at $1/2$, it must be the case that $f(1/2) \geq 1$ (as otherwise the cumulative density could not integrate up to 1). Thus the first-order condition is weakly negative at $p = 1/2$. Moreover for $p \geq 1/2$ the second-order condition is $-2f(p) + pf'(p)$. Since $f(p)$ is single-peaked and maximal at $p = 1/2$, the profit function is clearly concave for $p \geq 1/2$. Thus the maximum must arise for some $\hat{p} \leq 1/2$.

If the monopolist chooses a bundle price of 1, its profits are $1/2$, as before. This follows from the symmetry of the distribution. Similarly, against a price of 1, the area for an entrant who comes in at $1/2$ is $F(1/2)[1 - F(1/2)] = 1/4$.

Pure Bundling Effect with non-uniform densities: With $f(\alpha)$ unimodal and symmetric, at equivalent prices, the act of putting two independent products into a bundle reduces the entrant's profits by $[1 - \frac{1}{4\hat{p}}]$, where by Lemma 2, $\hat{p} \leq 1/2$.

To calculate the pure bundle effect, the bundle price is twice the optimal independent price, or $2\hat{p}$. If the entrant were to come in with a single good at \hat{p} , its market area would be $F(\hat{p})[1 - F(\hat{p})]$ compared to $F(\hat{p})$ with independent pricing. An entrant with product B only captures those customers who value A at below \hat{p} . Thus the reduction is entrant profits is $F(\hat{p})$, which we can bound based on profit maximization.

If the monopolist chooses any price other than $1/2$ it must be that

$$\hat{p}[1 - F(\hat{p})] \geq 1/4 \rightarrow F(\hat{p}) \leq 1 - 1/(4\hat{p}).$$

Positive and Negative Correlation in Values

In the introduction, we foreshadowed the result that unlike the price discrimination literature, bundling works best as an entry deterrent when the two goods are positively correlated in value. We can now be more precise about this result.

Positive correlation in values lowers the monopolist's profits *absent* entry since it makes price discrimination more difficult. It also makes entry significantly less profitable. However, should entry still occur, the entrant is forced to come in with a very low price ($x/4$ in the case with $\rho = 1$) and thus profits are correspondingly very low. Thus positive correlation in values is helpful to the incumbent to the extent that it allows the incumbent to deter entry when it would otherwise not succeed. But if entry can not be prevented, then positive correlation in values ends up hurting the incumbent. Similarly, a negative correlation in values makes entry easier, but also less costly to the incumbent.

To illustrate these results further we examine the two extreme cases, a correlation of $+1$ and -1 . But first, we consider the pure bundling effect for the entire range of cases between $\rho \in [-1, 1]$. For comparison purposes we want to keep the density for each individual good unchanged (and thus optimal independent price at $1/2$).

When the two values are perfectly correlated, all of the individuals lie along the 45-degree line, $\alpha_a = \alpha_b$. Similarly, when the two values are perfectly negatively correlated, all of the individuals lie along the opposite diagonal, $\alpha_a + \alpha_b = 1$.

The way we model intermediate levels of correlation is to take a weighted average of the uniform density and either the diagonal density or the off-diagonal density.

Pure Bundling Effect with correlation: At equivalent prices, the act of putting two independent products into a bundle reduces the entrant's profits by $(1 + \rho)/2$.

Thus at perfect negative correlation there is no pure bundling effect, while at perfect positive correlation the pure bundling effect reduces the entrant's profits to zero. The argument is still straightforward. With independent pricing, the entrant can come in with a marginal discount and sell to half the customers and thereby earn 0.25. Consider, in contrast, what happens when an entrant comes into a market against an incumbent with a bundle priced at 1. If the entrant prices at 0.50, it sells only to people who value B at above 0.50 and A at less than 0.50. Consequently, the entrant only sells to 25 percent of the uniformly distributed market. There is no

overlap with the 45-degree line so none of the perfectly positively correlated consumers will go to the entrant. Conversely, half of the off-diagonal consumers lie in the lower right quadrant. Thus with $\rho = 1$, the entrant's sales are zero while with $\rho = -1$ sales are 0.50. Intermediate cases lie linearly in between.

As becomes clear from the picture, it is very difficult for the entrant to gain customers. For example, if the monopolist charges 1 for the bundle and the entrant comes in at $1/2$, he makes zero sales! This is because all customers lie on the 45-degree line which does not overlap with the right-hand corner of the square. In contrast, when the valuations are perfectly negatively correlated, all the consumers lie along the other diagonal of the box. Now if the monopolist charges 1 and the entrant comes in with one good at price $1/2$, it would take fully half of the customers. In this sense, the pure bundling effect disappears with perfect negative correlation.

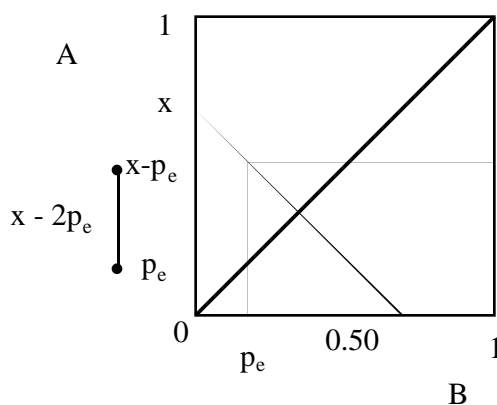
What makes the analysis more challenging is that the optimal response of the entrant is not to come in at price $1/2$. In the case of perfect positive correlation, if the incumbent charges x for the bundle, the entrant's profits are $\Pi_e = p_e(x - 2p_e), p_e \leq x/2$, while the incumbent makes $\Pi_{inc|entry} = x(1 - x + p_e)$. Thus the optimal entry price is $p_e = x/4$ and anticipating this the incumbent should charge a bundle price x to maximize $x(1 - 3x/4)$. The result is $x^* = 2/3$ and $p_e = 1/6$. The fact that the entrant charges such a low price is very costly to the incumbent. On the other hand, the entrant makes very little money: $\Pi_e = 1/6(1/3) = 1/18$ and hence is easily deterred.

This implies that the incumbent can do no worse than to earn $(2/3)(1/2) = 1/3$ even if entry occurs. The incumbent would earn $1/3$ without entry by charging an amount

$$x(1 - x/2) = 1/3 \quad \text{which implies } x = 1 - \sqrt{3}/3 \approx 0.423.$$

By charging that amount, the entrant would earn only $x^2/8 = 0.0224$. Thus if the potential challenger's entry costs are below 0.0224 the incumbent should price so as to keep out the entrant. However, if entry costs are so low as to make deterrence impractical, then the incumbent actually *raises* its price above what it would charge absent entry. It does this so as to mitigate the entrant's incentive to price low and destroy all of the market's profits.

Correlation = +1



No entry

$$\Pi_{\text{inc}} = x(1-x/2)$$

$$x^* = 1; \Pi = 1/2$$

Entry

$$\Pi_e = p_e (x - 2p_e)$$

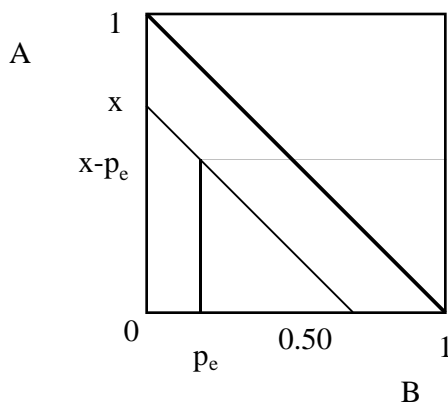
$$\Pi_{\text{inc}|e} = x(1-x+p_e)$$

$$P_e = x/4; \Pi_e = x^*x/8$$

$$\Pi_{\text{inc}}|e = x(1-3/4x)$$

$$x^* = 2/3; \Pi_{\text{inc}}|e = 1/3$$

Correlation = -1



No entry

$$\Pi_{\text{inc}} = x; x < 1$$

$$x^* = 1; \Pi = 1$$

Entry

$$\Pi_e = p_e (x - p_e)$$

$$\Pi_{\text{inc}}|e = x(1-x+p_e)$$

$$P_e = x/2; \Pi_e = x^*x/4$$

$$\Pi_{\text{inc}}|e = x(1-x/2)$$

$$x^* = 1; \Pi_{\text{inc}}|e = 1/2$$

Bundling complements

Often times the products in a bundle complement each other. This is separate from and potentially in addition to the existence of positive correlation in the value of the bundled products. Thus a high valuation for Excel may indicate a high income or a business use and this correlation would suggest a high value for PowerPoint. With

complementarity, the value of PowerPoint plus Excel is higher than Word alone plus Excel alone. In the case of the components of Microsoft Office, we suspect that both arguments are true.

In this section, we show that complementarities greatly amplifies the advantages of bundling. We say A and B are complements if $V_{a+b} = (1 + \delta)(V_a + V_b)$, $\delta > 0$. If $\delta < 0$ then A and B are substitutes.

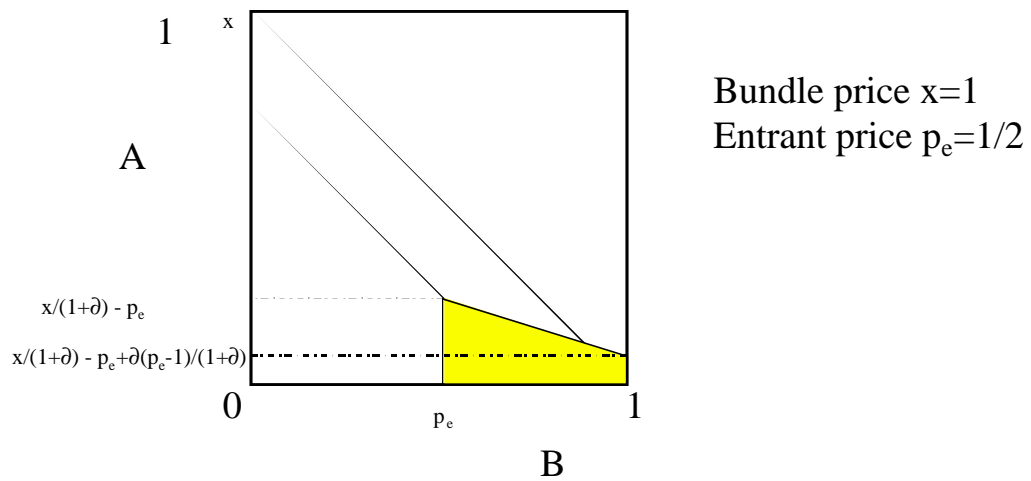
Pure Bundling Effect with Complementarity: At equivalent prices, the act of putting two complementary products into a bundle reduces the entrant's profits from $\frac{1}{4}$ to $\frac{1}{8} \frac{1}{1+\delta} [1 - 3\delta/2]$, where δ is the measure of complementarity.

Thus if $\delta = 0$, the reduction in profits is 50%, from 0.25 to 0.125. This is our original pure bundling result. If $\delta = 0.10$ then profits are reduced to 40% of their previous level, from 0.25 to 0.10. If $\delta = 0.25$ then profits are reduced to 25% of their previous level, from 0.25 to 0.0625.

Conversely, if the two products are substitutes, bundling is less effective. If $\delta = -0.10$ then profits are reduced by 36%, from 0.25 to 0.16. If $\delta = -0.25$ then profits are only reduced by 12%, from 0.25 to 0.22.

To demonstrate the pure bundling effect with complementarity, assume the entrant offers good B . The set of consumers who are indifferent between purchasing the bundle and the entrant's B offering is determined by the line $(\alpha_a + \alpha_b)(1 + \delta) - x = \alpha_b - p_e$. This is illustrated in Figure 6

Figure 6



The entrant's profits are

$$\begin{aligned}\Pi_e &= p_e * (1 - p_e) * \left[\frac{1}{2} \left(\frac{x}{1 + \delta} - p_e + \frac{x}{1 + \delta} - p_e + \frac{\delta(p_e - 1)}{1 + \delta} \right) \right] \\ \Pi_e &= \frac{p_e * (1 - p_e)}{2(1 + \delta)} * [2x - 2p_e(1 + \delta) + \delta(p_e - 1)] \\ \Pi_e|_{(x=1; p_e=\frac{1}{2})} &= \frac{1}{8(1 + \delta)} * \left[1 - \frac{3\delta}{2} \right].\end{aligned}$$

We can also calculate the optimal response to the incumbent's price. The first-order condition for the entrant's profits is

$$(1 - 2p_e) \left[x - \frac{\delta}{2} - p_e \left(1 + \frac{\delta}{2} \right) \right] - p_e(1 - p_e) \left[1 + \frac{\delta}{2} \right] = 0.$$

Using the quadratic formula leads to:

$$p_e^* = \frac{1}{3(1 + \frac{\delta}{2})} \left[(1 + x) - \sqrt{1 - x + x^2 + \frac{3\delta}{2} \left(1 - x + \frac{\delta}{2} \right)} \right].$$

For $\delta = 0$, this reduces to our previous expression for the optimal entrant's response.

The incumbent's profits given entry are

$$\Pi_I|_{entry} = x * \left[1 - \frac{x}{1 + \delta} + p_e - p_e^2/2 + \frac{1}{2}(1 - p_e)^2 \frac{\delta}{1 + \delta} \right].$$

At $\delta = 0$ this is $\Pi_I|_{entry} = x * (1 - x + p_e - p_e^2/2)$ as required.

Cournot: There is another reason to bundle that dates back to Cournot. While Cournot is best known for his analysis of quantity competition between two firms selling perfect substitutes, he also introduced the case of “complementary monopoly” [Cournot, 1838, chapter IX]. In this case, A and B are consumed together, but made by separate firms.

The present paper has focused on the case in which one firm can produce both A and B . There are also incentives to sell A and B as a bundle when the two products are sold by different firms, each with market power. This situation arises when the two firms are complementors, as opposed to competitors [see Brandenburger and Nalebuff (1996)]. Think of Microsoft and Intel selling hardware and software, or a ski resort and an airline selling a holiday package.²¹

²¹ The issue of selling complements and choosing the compatibility of potential complements is studied in Matutes and Regibeau (1992), Brandenburger and Nalebuff (1996), Economides (1998), and Sonnenschein (1968).

The intuition for bundling is demonstrated in a class exercise written by Brandenburger following Sonnenschein (1968). His example looks at the limiting case in which all consumers either buy both A and B or buy neither, but no one buys one or the other. Thus, consumers care only about the combined price, $p_a + p_b$. In the case of linear demand,

$$Q = 100 - (p_a + p_b).$$

There are two firms. One sets the price of A and the other sets the price of B . The equilibrium has each firm setting a price of $33\frac{1}{3}$, and total sales would also be $33\frac{1}{3}$. This result should be familiar, as the mathematics are exactly the same as with the standard Cournot-Nash equilibrium, only here we have switched prices for quantities and complements for substitutes. Cournot observed that if the two firms got together and coordinated their pricing decision, they would choose $(p_a + p_b) = 0.5$, and joint profits would rise from 0.22 to 0.25.

While it is not surprising that coordinated pricing leads to higher profits, what might be surprising is that coordinated pricing leads to a reduction in prices.²² Both consumers and firms are better off. The reason is that in the case of complementors, when one firm lowers its price, the other firm's sales increase, an externality that is not taken into account with uncoordinated pricing. Thus, another situation that calls for bundling is when two firms each have market power, but each is missing one of the complementary products.

It is interesting to note that Posner (1972) looked at the case for bundling pure complements — A and B only have any value when consumed together. He concluded that as consumers care only about the price of the bundle, there would be no point in trying to leverage a monopoly in A to B so as to raise the price of B . Raising the price of A would do just as well. The surprise is that this argument no longer holds when B is sold by an oligopoly. Because its price lies above the competitive level, one might want to use leverage to *lower* the price of B .

Competition Against An Existing Rival

Up until this point, we have focused on the potential profits of an entrant and the consequences that deterring or accommodating entry would have upon an incumbent. We now consider competition between two companies, each established in the market. Company 1 sells both products A and B , and company 2 sells only product B . The question is whether company 1 (with its monopoly in good A) should extract all of its

²² The fact that coordination leads to higher profits suggests that a company that sells two complementary products will have a higher incentive to innovate than when the products are sold separately, see Heeb (1998).

monopoly rents in good A directly, or, would it do better by leveraging that monopoly and selling an A - B bundle?

The results in the previous sections have already answered that question in the context of a Stackelberg model. However, in a market with two established firms, the leader-follower dynamic may be less appealing. Here, we demonstrate that the benefits to bundling continue to hold when we consider the Nash equilibrium in the pricing game. Firm 1 becomes more aggressive in its bundle price, since raising price no longer leads firm 2 to follow suit. Yet, firm 1's profits are still nearly 50% higher than what it can extract by simply exploiting its monopoly in good A . A large part of the gain comes from a byproduct of the pure bundling effect—a competitor does much less damage against a bundle than against a single product.

Nash Solution

The case in which the two goods are sold independently is straightforward. The two B goods are perfect substitutes, and so the Bertrand-Nash equilibrium price is 0. As for good A , the monopoly price is $1/2$. Thus, profits of firm 1 are $1/4$, and profits of firm 2 are 0.

Next, we consider the Nash equilibrium when firm 1 sells only an A - B bundle and firm 2 sells B . Firm 2's optimal response to a bundle price of x is the same as in the Stackelberg case. Its profits are maximized when it charges p_b :

$$p_b^* = \frac{1+x}{3} - \frac{1}{3}\sqrt{1-x+x^2}.$$

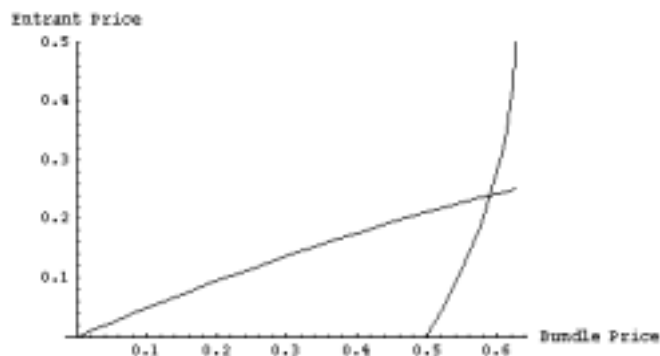
Firm 1's profits given a price of p_b for good B are

$$\Pi_1|entry = x * (1 - x + p_b - p_b^2/2).$$

Profits are maximized at

$$p_e^2 - p_e + 2x - 1 = 0 \quad \text{or} \quad x^* = (1 + p_e - p_e^2)/2.$$

As illustrated in the graph below, the Nash equilibrium is $x^* = 0.59$, $p_b = 0.24$.



Equilibrium profits are 0.366 for firm 1 and 0.064 for firm 2. When compared to independent pricing, the gain in profits is 46.8% for firm 1 and an infinite percent for firm 2 (which previously made 0). In absolute terms, it is the incumbent who has the greater gain, almost two to one.²³

We note that the bundling equilibrium is also more efficient than independent pricing. Profits rise by more than the fall in consumer surplus.

Proposition 3: Total surplus is 0.915 in the bundling Nash equilibrium, up from 0.875 when A and B are sold independently.

Proof: With independent pricing, all consumers purchase good B . The surplus from this transaction is 0.5, as this is the mean valuation of good B . As for the surplus from the sale of A , only half the market, namely those consumers with valuations above 0.5, purchase A . Their mean value for A is 0.75. Thus, 0.5 purchases with a mean value of 0.75 adds 0.375 to the 0.5 surplus from the B sales for a total of 0.875.

With bundling, the mean valuation for B among the consumers who buy only B is $(1 + 0.24)/2 = 0.62$. Sales are 0.266, which generates a surplus of $0.62 * 0.266 = 0.165$. The average value of the bundle among those who purchase the bundle from firm 1 is 1.21. Bundle sales are 0.62, which leads to a total surplus of 0.75 on bundled sales. Combining this with the 0.165 surplus from B sales results in aggregate surplus of 0.915.

4: LITERATURE REVIEW, PART 2

Having presented the model and its extensions, we can now expand on the connections to the related work of Whinston (1990), Chen (1997), and Choi (1998). Similar

²³ In this context, bundling might be viewed as a facilitating device in that it raises the post-entry profits of both the incumbent and the entrant [see Carbajo, De Meza, and Seidman (1990) and Chen (1997)]. While the entrant does better than 0, that is an unrealistically tough benchmark to beat.

to the Nash model just presented, Whinston considers a monopoly in product A and a duopoly in product B . In his model, the two B goods are differentiated. More importantly, in the first part of his paper, there is no heterogeneity in the valuations of A . Thus, if the entrant were to get any B customers, the incumbent would lose *all* of its A customers. (This is because all customers have the same extra value from getting a bundle with A .) Thus, in a second-stage pricing game, the incumbent has a great incentive to price the bundle low so as to preserve the value it creates in the A market. The end result is that firm 2 finds it hard to beat the A - B bundle and chooses not to enter the market.

Note that in Whinston's model, the pure bundling effect not only disappears, it is reversed. Assume that the common value of A is $1/2$ and that the value of B is uniformly distributed on $[0,1]$. Against a bundle price of 1, an entrant who comes in at $1/2$ (minus epsilon) sells to half the market, up from $1/4$. And, instead of taking away only $1/4$ of the incumbent's sales, it takes away 100% of the incumbent's sales.

The point is that bundling doesn't help the incumbent defend itself once the entrant is in the market — quite the contrary. However, this weakness is turned into strength as an entry deterrent.²⁴ The only way that the incumbent can earn any money is to ensure that the entrant makes zero sales. Even an entrant with a cost advantage (or superior product) in B is deterred, as the incumbent uses its value in good A to cross-subsidize the B good in the bundle and thereby deny the entrant any sales.²⁵ This approach requires the incumbent to make a credible commitment to selling its products only as part of a bundle; if entry occurs, the incumbent would prefer not to bundle.

Credibility is not an issue in our approach (even in the Nash game) because the incumbent's post-entry profits are higher with a bundle than without. This is because we allow for heterogeneity in consumer values of A .²⁶ As a result, the entrant can

²⁴ There is the old saying: My enemy's enemy is my friend. In a two-stage game, weakness in the second stage often translates into strength in the first. When the post-entry market is more competitive, then there is less incentive to enter the market [see, for example, Bernheim (1984)].

²⁵ For example, imagine the incumbent can produce A at a cost of 0.20 while all consumers value it at $1/2$. The rival can only produce good B , but can do so at a cost advantage — the rival's unit cost is 0.10 while the incumbent's unit cost is 0.30. With independent sales, the price of B would be 0.30 and the rival would expect to earn a margin of 0.20 on sales of 0.70. If the incumbent only sold A as part of a bundle, it would be forced to sell the bundle at a price of 0.60 in order to make any sales. It would use 0.20 of the surplus it generates in sales of A to subsidize its cost disadvantage in B . The result is that the entrant would not be able to attract any customers and, anticipating this, would not enter the market.

²⁶ Whinston also considers the case in which there is heterogeneity in the valuation of A . He finds that a high dispersion in the value of good A and a low differentiation in B goods are necessary for bundling to raise the entrant's profits.

take a small number of customers without threatening all of the incumbent's sales. Secondly, the entrant doesn't have a cost advantage in B and so the entrant isn't forced to cross-subsidize the bundle price. Of course, in Whinston's model if the incumbent doesn't have a cost disadvantage in B then it can fight the entrant better when the goods are sold independently and so there is no gain from bundling. We saw this in the previous section. When all costs are zero, the entrant's profits rise from 0 with independent pricing to 0.064 when the incumbent bundles.

In this sense, bundling is less effective at deterring entry. But bundling is only a less effective entry deterrent when the solution to the post-entry pricing game is Bertrand-Nash equilibrium. If one believes that the incumbent will have some delay in responding (or that the entrant believes that the incumbent will be slow to respond), then a Stackelberg solution is more appropriate. And, in a Stackelberg game, bundling is a very effective tool for lowering a rival's potential post-entry profit even when costs are identical.

Choi (1998) builds on Whinston by considering how bundling changes the dynamic incentives to engage in cost-cutting. If entry is not deterred, the incumbent then has a bigger incentive to eliminate its cost disadvantage. For simplicity, consider a case where all consumers value A and B at $1/2$. Let the incumbent firm have unit costs of 0.10 and 0.50 for products A and B , respectively, while the rival firm has unit costs of 0.50 and 0.20. If they compete product-by-product, then the price of each product will be 0.50. The incumbent will make 0.40 while the rival will make 0.30. If the two firms compete bundle-to-bundle, their bundle costs are the 0.60 versus 0.70 and hence Bertrand competition will eliminate all the rival's profits, while knocking the incumbent down to 0.10. This is both good and bad. It is good in that it makes entry much less attractive. It is bad in that if entry occurs, post-entry profits are much reduced.

But, if entry cannot be avoided, bundling creates other advantages to the incumbent as regard to its incentives to engage in cost-reducing R&D. Normally the incumbent with unit B costs of 0.50 would have no incentive to lower its costs to 0.40 when a rival has costs of 0.20. Once the firm is committed to selling its products through a bundle then a cost reduction on either product is of equal value. In contrast, until the entrant can achieve total cost parity with the incumbent, it has no incentive to lower costs. Thus in a bundling environment, a firm with the lowest overall cost structure will also have the greatest incentive to pursue R&D and thus maintain its dominant position.

Chen's (1997) paper highlights the potential for bundling to reduce competition through differentiation. In Whinston's approach, bundling A and B commits firm 1 to

being a tougher competitor in the B market (where its higher costs would otherwise make it a poor competitor). This keeps firm 2 out of the game and allows firm 1 to charge high prices. In contrast, Chen's firm 1 chooses an A - B bundle to prevent itself from going head-to-head with firm 2, which is in the market with an A product (which it had chosen so as not to go head-to-head with firm 1's A - B bundle).

The current paper is different from both approaches. In the Stackelberg game, differentiation works against the entrant. While the entrant's B product is not differentiated, its ability to undercut the incumbent gives it a large advantage. Bundling makes the two offerings more differentiated and this makes B 's undercutting much less effective.

The entrant finds it hard to compete not because the monopolist is committed to engaging in a price war. Quite the contrary. The incumbent has to pick its prices in advance and, therefore, can't increase its profits once entry is deterred. The one-product entrant finds it hard to compete because it has only a limited customer base, namely those customers who are partial to B and who also don't place a high value on A . This effect is amplified by the incumbent's ability to charge a low price for the bundle while sacrificing very little profit.

5: CONCLUSIONS

Although creating a bundle doesn't stop competition, it forces competitors to play the game bundle against bundle. A firm that has only some components of a bundle will find it hard to enter against an incumbent who sells a package solution at a discount. This will be especially true when the consumers have positively correlated values for the components of the package or when the components are complements. Bundling also softens the harm done by a one-product (or limited line) competitor. The rival takes fewer customers away and prices don't fall as far.

A monopolist, even without fear of entry, has incentives to bundle, either as a way to achieve better price discrimination (when values have a negative correlation) or to help save costs (when valuations are positively correlated). But most important to a firm with market power is preserving that power, by deterring a potential entrant or reducing the impact of an existing (one-product) rival. It is in this role that bundling truly shines. Entry is deterred more easily, in which case profits are more than doubled. And when entry deterrence fails, post-profits are still more than 50 percent higher when products are sold as a bundle.

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7. APPENDIX: BUNDLING THREE OR MORE ITEMS

In this appendix, we consider the potential profits of an entrant when the incumbent creates a bundle with three or more items. We continue with the assumption that $f(\alpha)$ is uniform over the n -dimensional unit cube. Thus, consumers' values for the n goods are independent and uniformly distributed over $[0,1]$ for each of the n goods.

The pure bundling effect now compares the following two incumbent strategies:

- (i) Sell each of n items at a price of $1/2$.
- (ii) Sell an n -good bundle for a price of $n/2$.

Absent entry, both strategies lead to identical profits for the incumbent. In the first case, firm 1 collects $1/2$ on sales of $1/2$ for each of n goods. In the second case, firm 1 sells a bundle to half the population and collects a price of $n/2$.

The surprising result is that the potential profits of an entrant at $p_e = 1/2$ are identical for $n = 2$ and $n = 3, 4, \dots$ (and the cost of entry to the incumbent). When items are sold individually, the entrant can make $1/4$ simply by undercutting the incumbent in one market.

Proposition 3: For all $n \geq 2$, with $x = n/2$ and $p_e = 1/2$, the challenger has sales of $1/4$ units and revenue of $1/8$.

Proof: Without loss of generality, assume that the entrant offers product 1. A consumer buys from the entrant if and only if

$$\alpha_1 - \frac{1}{2} \geq \alpha \cdot 1 - \frac{n}{2}.$$

Rearranging this inequality leads to

$$\sum_2^n \alpha_i \leq \frac{n-1}{2}.$$

It follows from this directly that the challenger's sales always equal $1/4$. Among the population that values good 1 at more than $1/2$, the challenger sells to half the population and the incumbent sells to the other half.

In the case of a single item being sold, the challenger can obtain sales of $1/2$ and revenue of $1/4$. Creating the two-item bundle knocks the value down by 50 percent. But adding more elements to the bundle has no incremental effect.

Of course, the pricing of the multi-unit bundle is too high, as is the entrant's price. For price-discrimination reasons, the incumbent would prefer to charge a price below

$n/2$ for the bundle (but not that much below!). And given a price of $n/2$ or below, the entrant would want to charge less than $1/2$.

As we will see below, there are gains from bundling more than two items, but the gains ultimately come from being able to do a better job price discriminating, as opposed to simply creating the bundle.

Optimal multi-good bundled pricing

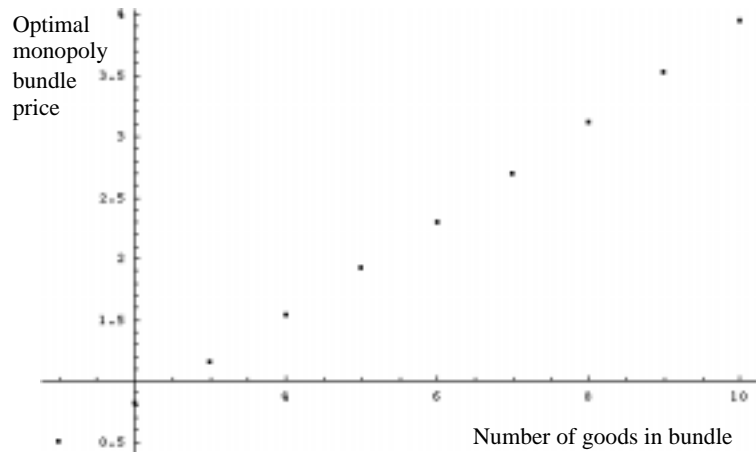
With n goods in the bundle and a price of x the demand is

$$D(x, n) = 1 - \frac{1}{n!} \sum_{k=0}^n (-1)^k \binom{n}{k} \max[0, x - k]^n.$$

Thus for $n = 2$, demand is $1 - (1/2)x^2 + \max(0, x - 1)^2$. As we have been focussing only on $x \leq 1$, this simplifies to $1 - x^2/2$.

For $n = 3$ and $x \leq 3/2$, demand is $1 - (1/6)x^3 + (1/2)(x - 1)^3$.

We can find the optimal uncontested monopoly price using Mathematica. Profits are maximized at $x = 1.162$, a unit price of almost 0.39. By the time there are 100 goods in the bundle, the optimal bundle price is approximately 44.5, suggesting a unit price of 0.44.



$n = 10$, $x = 3.95$, profits are 3.45.

$n = 25$, $x = 10.4$, profits are 9.6.

$n = 50$, $x = 21.5$, profits are 20.6.

$n = 75$, $x = 33$, profits are 31.5.

$n = 100$, $x = 44.5$, profits are 42.

The challenger's profits are

$$p_e * (1 - p_e) * \frac{1}{(n-1)!} \sum_{k=0}^{n-1} (-1)^k \binom{n-1}{k} \max[0, x - p_e - k]^{n-1}.$$

Essentially this is the entrant's price multiplied by the fraction of the population who value the entrant's good at above p_e (the $1 - p_e$ term) multiplied by the fraction who value the other $n - 1$ goods at less than $x - p_e$.

Against an 3-good bundle incumbent price of 1.16, the best response of the entrant is to charge 0.28 and its profits are reduced to 0.078. (In comparison, with two goods, the entrant charged 0.3 and its profits were 0.105.) With four goods in the bundle, the entrant charges 0.3, but earns only 0.065. Against a 10-item bundle priced at 3.95, the entrant would charge 0.31 but earns only 0.035. The optimal entry price against a 50-item bundle priced at 21.5 is 0.375, and entry profits are reduced to 0.011.

N	Ind. Π no entry	Ind. Π entry	Bundle Π no entry	Bundle Π entry	Entrant's Π
2	0.50	0.25	0.54	0.36	0.10
3	0.75	0.50	0.86	0.67	0.078
4	1.00	0.75	1.2	1.00	0.065
10	2.25	2.50	3.45	3.24	0.035
50	12.50	12.25	20.6	20.4	0.011

This is the hard way to attack the problem. For very large bundles, Bakos and Brynjolfsson (1999a) offer an elegant approach. They take advantage of the fact that the distribution of consumer valuations will be normal will mean $n/2$ and standard deviation $\sqrt{n/12}$. The law of large numbers then works to the monopolist's great advantage. Since the mass of the distribution is converging to the mean, any small discount from 0.50 per item will (in the limit) attract all the consumers. Thus, in the limit, the monopolist will set a bundled price so as to attract 100% of customers. Moreover this can be done while collecting 100% of consumer surplus. We can see this from the trend above. In a two-good bundle the per-item price is 0.41 and 68% of consumers buy the bundle. By the time the bundle has 100 goods, the per-item price is up to 0.44 and now over 94% of consumers will buy the bundle.

This also means that once the bundle gets large, the incumbent is pretty much unaffected by entry. At most, the entrant can attract the set of consumers with surplus of less than 1 from the bundle. Returning to the 100 good bundle, the incumbent prices

the bundle at 89, which is almost 4 standard deviations below the mean valuation. The set of potential customers for the entrant, even with a zero price, would be those with valuations below 90, which is 3.5 standard deviations below the mean.

In the limit, bundling denies the entrant any market. Bakos and Brynjolfsson (1999b) reach a slightly different conclusion as they allow the entrant to offer a differentiated product and thus make sales to consumers who are willing to buy the differentiated product in addition to the bundle.

It is worth emphasizing that the law of large numbers works slowly here. Although the per-unit price eventually converges to 0.50, it actually falls from 0.41 in the two-good bundle case to 0.39 in the three-good bundle case. Even with ten goods in the bundle, the unit price is 0.395, still below 0.41. Creating the two-good bundle causes the entrant's profits to fall by 60%, from 0.25 to 0.10. It takes adding another 8 goods to the bundle to bring profits down another 65%.