YALE
INTERNATIONAL
CENTER
FOR FINANCE

Yale ICF Working Paper No. 04-30

May 2004

Convergence of Double Auctions to Pareto Optimal Allocations in the Edgeworth Box

Dhananjay (Dan) K. Gode
New York University
Stephen Spear
Carnegie Mellon University
Shyam Sunder
Yale School of Management

Convergence of Double Auctions to Pareto Optimal Allocations in the Edgeworth Box

Dan K. Gode, New York University

Stephen E. Spear, Carnegie Mellon University

Shyam Sunder, Yale University

Abstract

Double auctions with profit-motivated human traders as well as "zero-intelligence" programmed traders have previously been shown to converge to Pareto optimal allocations in partial equilibrium settings. We show that these results remain robust in two-good general equilibrium settings and elucidate how market structure, not optimization by traders, guides efficient resource allocation.

Keywords: *Pareto optimal allocations, Edgeworth Box, Double auction, Zero-intelligence traders*

JEL Codes: *C63, C68, D44, D51, D58, D61*

First Draft: June 1993

Revised Draft: May 2004

Please do not quote without the authors' permission. Email dgode@stern.nyu.edu, ss1f@andrew.cmu.edu, or shyam.sunder@yale.edu.

Convergence of Double Auctions to Pareto Optimal Allocations in the Edgeworth Box

The Edgeworth box is used to model exchanges among agents with given preferences and endowments. The contract curve and competitive equilibrium are derived by assuming that agents maximize their utility while staying within their opportunity sets. We show that agents need not maximize utility to achieve Pareto-efficient outcomes. Double auctions with "zero-intelligence" traders (ZI), who possess just enough rationality to stay within their opportunity sets (or are constrained by market rules to do so), achieve Pareto-efficient outcomes. Our simulations, which readers can download and run on their computers,[1] show how the elemental forces of want and scarcity cause Pareto-efficient outcomes even when agents do not maximize and when no evolutionary processes exist.

Beginning with Chamberlin (1948), economists have studied experimental economies with profit-motivated human subjects. Smith (1962) showed that double auctions, although different from Walrasian tatonnement, still yield prices and quantities close to competitive equilibrium. Gode and Sunder (1993, 1997) showed how even profit seeking is not necessary for approximate Pareto efficient allocations. Minimal rationality (avoid losses but do not worry about profits, termed "zero intelligence") suffices. Bosch and Sunder (2000) generalized the result to multiple interacting markets.

Prior studies, however, have examined partial equilibrium settings in which individual supply and demand functions are exogenous. This paper uses a two-good Edgeworth box in a general equilibrium setting to study double auctions with profit-motivated human traders and ZI traders.

[1] Readers can download simulations from http://www.som.yale.edu/faculty/sunder/exe/simul3.exe.

Economic Parameters

Group 1 agents' Cobb-Douglas utility function for two commodities is $U=c_1^{\alpha} c_2^{1-\alpha}$ and Group 2 agents' utility is $V = c_1^{\beta} c_2^{1-\beta}$. Group 1 endowment is (x_1, y_1).

The contract curve is given by:

$$(1-\alpha)(1-y)/\alpha y = (1-\beta)(1-x)/\beta x. \quad (1)$$

The competitive equilibrium holdings are given by:

$$(\alpha(x_1+y_1/p^e), (1-\alpha)(p^e x_1+y_1)). \quad (2)$$

And, the equilibrium price in units of y per unit of x is given by:

$$p^e = ((\alpha y_1+ \beta(1-y_1))/(1-\alpha)x_1+(1-\beta)(1-x_1)) \quad (3)$$

See Figure 1. The endowment point is A, the competitive equilibrium holding is C, and the competitive equilibrium price p^e is the slope of line AC.

I. Human Experiments

Two groups of 14 subjects each traded in one session. Each Group 1 member had an endowment of 44 green chips and 9 red chips and a 50 x 50 grid showing the utility of various combinations of chips (Table 1A) for utility function $U = c_1^{0.6} c_2^{0.4}$. Each Group 2 member had an endowment of 6 green and 41 red chips and another 50 x 50 grid (Table 1B) showing the utility of various combinations of chips for utility function $V = c_1^{0.8} c_2^{0.2}$. In both tables, the cell showing the initial endowment was circled. Subjects could walk about the room to find another trader they could trade with to increase their utility. In competitive equilibrium, each member of Group 1 holds 29 green and 39 red chips, which implies that each member of Group 2 holds 21 green and 11 red chips. The competitive equilibrium price is 0.49 green chips for each red chip, and it raises the utility of Group

1 members from 0.933 to 1.31 and raises the utility of Group 2 members from 0.35 to 0.73. The contract curve is given by the equation Red = 8 Green /(3 + 5 Green).

Figures 2 and 3 show the results of this single session experiment. Figure 2 shows the normalized Edgeworth box. The endowment point is A (0.88, 0.18) and the competitive equilibrium point is C (0.58, 0.79). Solid triangles show the actual final holdings of Group 1 members and hollow circles show the final holdings of Group 2 members. The average holdings of the two groups are given by the solid square (0.57, 0.69).

Group 1 members increased their utility from 0.933 to an average utility of 1.22, which is about 93 percent of the competitive equilibrium utility. Group 2 members increased their utility from 0.35 to an average utility 0.803, which is about 110 percent of the competitive equilibrium utility. All 28 participants traded and 26 of them improved their welfare. One Group 1 member remained on the indifference utility curve at endowment, and one Group 2 member traded to a lower utility level.

Figure 3 shows the transaction price sequence (number of green chips per red chip). Transaction prices are constrained because the number of green and red chips must be integers. The average transaction price of 0.64 was higher than the competitive equilibrium price of 0.49. Overall, in this single shot experiment, the subjects got close to the contract curve and competitive equilibrium, but fell short of achieving it. We expect human performance to improve with experience and unconstrained prices.

II. ZI Computational Experiments

A. ZI trader behavior

ZIs choose bids and offers randomly from a set, shown by the shaded area in Figure 4A, that is feasible and that does not make them worse off. Thus, the trader in Figure 4A either tries to buy X units to move in the southeast direction in the shaded area, or tries to sell X units to move in the northwest direction in the shaded area. Either move raises its utility.

The ZI traders are minimally rational in the sense that they simply try to climb their utility hill, choosing randomly from the available set of steps that takes them to a higher level. They act only locally. and do not even optimize within their local opportunity set. They have no memory, and they do not observe the actions of other market participants.

ZI traders are implemented as follows. At any time, each ZI trader submits a bid *B* that specifies the number of *Y* units the trader is willing to pay for one *X* unit. and an ask *A* that specifies the number of *Y* units the trader is willing to accept for one *X* unit. Thus, a trader's bids and asks and bids are slopes of lines measured in angles along which the trader is willing to move.

If each transaction involved infinitesimally small quantities, a trader in Group 1 would bid a uniformly distributed random angle between 0 and the slope (ω) of his indifference curve passing through his holdings and ask a uniformly distributed random angle between ω and $\pi/2$. However, to ensure that simulations finish in finite time, we require finite quantities per transaction. We specify the size of each transaction in terms of the Euclidean distance r between the pre- and post-transaction holdings; i.e., $r= (x^2 + y^2)^{1/2}$ where x and y are the quantities of the respective commodities exchanged in the transaction. For our simulations, we set r= 0.02. Specifying the size of the transaction in x or y exclusively would bias the results because of discreteness.

The finite transaction size r implies that Trader 1 can no longer bid between 0 and ω (slope of the indifference curve) and ask between ω and $\pi/2$ because such bids and asks may fall below the indifference curve. Instead, as shown in Figure 4B, the bids and asks of size r must lie on arcs Bid 1, Bid 2, Ask 1, and Ask 2, which are arcs of a circle of radius r bounded by utility increasing opportunity sets for each trader.

Therefore, a trader from Group 1 bids a random angle that is uniformly distributed over the angle subtended by the arc Bid 1, and asks a random angle that is uniformly distributed over the angle subtended by the arc Ask 1. Analogously, a trader from Group 2 bids a random angle that is uniformly distributed over the angle subtended by the arc Bid 2 and asks a random angle that is uniformly distributed over the angle subtended by the arc Ask 2. A transaction occurs when the highest bid exceeds the lowest ask and the transaction price is the midpoint of the two.

B. Economic Parameters

Figure 5 shows the four economies that we simulated with ZI traders. As is shown in Figure 1, the straight line *AC* joins the endowment point *A* to the competitive equilibrium point *C* on the contract curve. The slope of line *AC* is the competitive equilibrium price. A broken line *AD* is the bisection locus of the angle between the two indifference maps. Starting from the initial endowment point *A*, it intersects the contract curve at *D*. Table 2 lists the parameters of the four economies.

C. Convergence to Contract Curve

The short bars in Figure 6 show the path of holdings after each transaction starting from endowment point *A* to the end of the simulation. The distance between the consecutive bars is the size of the simulation step *r* (discrete transaction size), which was described earlier.

All computational economies terminate at the contract curve. The path of holdings may not end exactly at the contract curve due to the discreteness of the simulation. This deviation is never more than *r/2*, one half of the simulation step.

The reason why all paths end at the contract curve is easy to see from Figure 7, where *A* is the endowment and the two curves passing through this point are the indifference maps of the two groups of traders. The center of arc *BC* is *A* and the radius of arc *BC* is the transaction size *r*. All points on arc *BC* are feasible after the first transaction, but the double auction mechanism makes the points closer to the center of arc *BC* more likely than the points near the bounds set by the indifference curves. Suppose point *D* on arc *BC* is the holding after the first transaction. Arc *BC* which is centered at holding D and is bounded by the indifference maps passing through D now defines the feasible holdings after the second transaction, and so on. These iterations explain how the simulation reaches the contract curve (subject to the discrete step constraint discussed above), and terminate only when no further increases in utility through exchange are possible.

D. Competitive Equilibrium

Figure 6 shows that the computational economies reach the contract curve near, but not at, competitive equilibrium. Figure 8 shows the distribution of the ends of 100 simulations of the four economies from Figure 5. The short bars mark the path of one simulation for each economy. A cross indicates the mean or centroid of these 100 endpoints. Compared to Figure 6, the scale has been expanded to zoom in on the competitive equilibrium point C. As shown, the competitive equilibrium is not the centroid of endpoints of 100 simulations indicated by hollow circles. Instead, the endpoints are distributed around *D*, the point at which the contract curve intersects the locus of the curve bisecting the angle between the indifference maps originating at endowment *A*, as shown by the broken line *AD*. The final holdings are scattered around *D* in a band with an approximate width r, the discrete transaction size. A small downward bias is due to the random chance that the last transaction may not occur within the specified number of iterations.

E. Transaction Prices

Figure 9 shows the transaction price series for a single run of each of the four computational economies. In addition, the thick line shows the average and standard deviation of the i^{th} transaction over the 100 runs of each economy. The horizontal line is the competitive equilibrium price. The transaction prices tend to follow the bisection locus. This curve is convex for Economies 1 and 2, and their mean price increases from earlier to latter transactions. Since this curve is concave for Economies 3 and 4, the mean price decreases as trading proceeds. Finally, prices converge to a close neighborhood of the *CE* price, even though the final allocation deviates from the *CE* allocation. The small curvature of the contract curve causes the price at the intersection of contract curve and the bisection locus to be close to the *CE* price.

III. Concluding Remarks

Prior studies in partial equilibrium settings have shown that agents need not maximize their profits for markets to achieve Pareto efficient outcomes. It is not obvious whether such partial equilibrium results with exogenously fixed demand and supply extend to general equilibrium settings. In this paper, we examine a more realistic setting of a classical pure exchange economy modeled as an Edgeworth box. Theoretical results for such economies are derived using utility maximizing agents and Walrasian tatonnement. Our computational economies use a double auction mechanism with multiple rounds of trading among “zero-intelligence” (ZI) traders who bid and ask randomly within their opportunity sets. Our finding that these economies also achieve Pareto efficient allocations makes two contributions.

First, exchange economies may be Pareto efficient with agents whose decision rules fall far short of utility maximization. Rules of market institutions such as double auctions transform barely rational individual actions into rational aggregate outcomes. Second, our ZI simulations are a tractable way to model, analyze, and gain insights into how and why statistical interactions among simple individual decision rules yield efficient market outcomes.

Our results suggest another look at the critique of the maximization assumption in economics. Many question the descriptive validity of utility maximization-based economic theories because of the cognitive limitations of human beings. Our results show that even when agents do not maximize their utility, the allocations derived from utility maximizing models may remain valid in market institutions. We do, however, need to understand how and why this happens. North (1990) and Simon (1996) suggest identifying the performance characteristics of social and economic institutions that may be largely independent of the variations in participant behavior. Our method is a step in that direction.

References and Bibliography

Bosch, Antoni, and Shyam Sunder, 2000. Tracking the Invisible Hand: Convergence of Double Auctions to Competitive Equilibrium, Computational Economics 16:3 (December), pp. 257-284.

Chamberlin, Edward H., "An Experimental Imperfect Market," Journal of Political Economy, LVI (1948), 95-108.

Gode, Dhananjay K., and Shyam Sunder. (1993a). Allocative Efficiency of Markets with Zero Intelligence (ZI) Traders: Market as a Partial Substitute for Individual Rationality. Journal of Political Economy, 101: No.1. 119-137.

Gode, Dhananjay K. and Shyam Sunder, (1997). What Makes Markets Allocationally Efficient?" The Quarterly Journal of Economics 112 (2), pp. 603-630.

North, Douglass C., 1990. Institutions, Institutional Change and Economic Performance. Cambridge University Press, New York, NY.

Simon, Herbert A., 1996. The Sciences of the Artificial. Third Edition. MIT Press, Cambridge, MA.

Smith, Vernon. L., "An Experimental Study of Competitive Market Behavior," Journal of Political Economy, LXX (1962), 111-37.

Appendix: Viewing the Edgeworth Box Simulation Live on Your Computer

You can use your computer to look at the dynamics of convergence of a two-good Edgeworth Box economy to the contract curve with ZI traders as follows.

Software Name: Simul3.exe
Reference: D. K. Gode, Stephen E. Spear and Shyam Sunder, "Convergence of Double Auctions to Pareto Optimal Allocations in the Edgeworth Box," Revised May 2004. On the web: http://www.som.yale.edu/Faculty/sunder/Edgeworth/Edgeworth.pdf.

Hardware: Personal computer (DOS/Windows)

To Get the Software: Download a copy of file simul3.exe from http://www.som.yale.edu/faculty/sunder/zisoft.html and store it on your hard drive or a floppy. You may also run it directly from the abovementioned website.

To Run the Simulation:

When you run simul3.exe,
1. The computer prompts you for an initializing random seed. Enter any number between -32,768 and 32767, and press the ENTER key.
2. The computer prompts you for your choice of the speed of simulation. Enter any number between 1 (slow) to 100 (fast), and press the ENTER key.
3. The computer prompts you for how frequently you wish to have the indifference maps redrawn on the screen. Enter a number between 1 (to redraw the indifference maps after every iteration) and 501 (to never redraw the indifference maps) and press the ENTER key. Enjoy the show.

What You See on the Screen:

Edgeworth Box: The endowment point of trader type 1 is (0.8, 0.2). The initial Indifference map of trader type 1 (cyan color) and trader type 2 (brown color) are drawn through the endowment point.

A second set of indifference maps are drawn so they are tangential to each other. The point of tangency is the competitive equilibrium (CE) point and the slope of the straight line from endowment point and the CE point (not drawn on screen) is the CE price.
The contract curve is drawn in green.
The pink curve from endowment point to the contract curve is the bisection locus of the angle between the indifference maps.
Except when the indifference maps are symmetrical, the point of intersection of the bisection locus with the contract curve does not coincide with CE.

Simulation: The step size for simulated transactions is set at 0.02 (Euclidean distance $(x^2+y^2)^{0.5}$). As each transaction is completed, a black-green line shows the movement of endowment point in steps of size 0.02.
In the right bottom of the screen, the bids and asks are shown in dots and completed transactions are shown in a continuous gray line. Horizontal gray line marks the CE price as a benchmark. Cyan and brown lines show the current level of utility of trader types 1 and 2 respectively, and chart the change in utilities from the initial endowment level towards the end of simulation. The horizontal colored lines indicate the utility levels attained by the two types of traders at CE (cyan for Type 1 and brown for Type 2). The third colored line (purple) is the average utility for both types of traders.

Parameters for Simulations

Utility of Type 1 traders: $x^{0.4}y^{0.6}$
Utility of Type 2 traders: $x^{0.8}y^{0.2}$
Endowment point: (0.8, 0.2)
Simulation step size (transaction quantity in Euclidean distance $(x^2+y^2)^{0.5}$): 0.02
Number of Iterations 1000

Table 1A

	Value of Various Combinations of Green and Red Chips to Members of Group 1																																																		
Red	Green Chips																																																		
Chips	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	2.0	3.0	3.9	4.6	5.3	5.9	6.4	7.0	7.5	8.0	8.4	8.9	9.3	9.7	10.2	10.6	10.9	11.3	11.7	12.1	12.4	12.8	13.1	13.5	13.8	14.1	14.4	14.8	15.1	15.4	15.7	16.0	16.3	16.6	16.9	17.2	17.5	17.7	18.0	18.3	18.6	18.8	19.1	19.4	19.6	19.9	20.2	20.4	20.7	20.9
2	0.0	2.6	4.0	5.1	6.1	6.9	7.7	8.5	9.2	9.9	10.5	11.1	11.7	12.3	12.9	13.4	13.9	14.4	14.9	15.4	15.9	16.4	16.9	17.3	17.8	18.2	18.6	19.1	19.5	19.9	20.3	20.7	21.1	21.5	21.9	22.3	22.7	23.0	23.4	23.8	24.1	24.5	24.9	25.2	25.6	25.9	26.2	26.6	26.9	27.3	27.6
3	0.0	3.1	4.7	6.0	7.1	8.2	9.1	10.0	10.8	11.6	12.4	13.1	13.8	14.5	15.1	15.8	16.4	17.0	17.6	18.2	18.7	19.3	19.8	20.4	20.9	21.4	21.9	22.4	22.9	23.4	23.9	24.4	24.8	25.3	25.7	26.2	26.6	27.1	27.5	28.0	28.4	28.8	29.2	29.6	30.1	30.5	30.9	31.3	31.7	32.1	32.5
4	0.0	3.5	5.3	6.7	8.0	9.1	10.2	11.2	12.1	13.0	13.9	14.7	15.5	16.2	17.0	17.7	18.4	19.1	19.7	20.4	21.0	21.6	22.2	22.9	23.4	24.0	24.6	25.2	25.7	26.3	26.8	27.3	27.9	28.4	28.9	29.4	29.9	30.4	30.9	31.4	31.8	32.3	32.8	33.3	33.7	34.2	34.6	35.1	35.5	36.0	36.4
5	0.0	3.8	5.8	7.4	8.7	10.0	11.2	12.2	13.3	14.2	15.2	16.0	16.9	17.7	18.5	19.3	20.1	20.8	21.6	22.3	23.0	23.7	24.3	25.0	25.6	26.3	26.9	27.5	28.1	28.7	29.3	29.9	30.5	31.0	31.6	32.1	32.7	33.2	33.8	34.3	34.8	35.3	35.9	36.4	36.9	37.4	37.9	38.4	38.8	39.3	39.8
6	0.0	4.1	6.2	7.9	9.4	10.8	12.0	13.2	14.3	15.3	16.3	17.3	18.2	19.1	20.0	20.8	21.6	22.4	23.2	24.0	24.7	25.4	26.2	26.9	27.6	28.3	28.9	29.6	30.2	30.9	31.5	32.1	32.8	33.4	34.0	34.6	35.2	35.7	36.3	36.9	37.5	38.0	38.6	39.1	39.7	40.2	40.7	41.3	41.8	42.3	42.8
7	0.0	4.4	6.6	8.4	10.0	11.4	12.8	14.0	15.2	16.3	17.3	18.4	19.3	20.3	21.2	22.1	23.0	23.8	24.7	25.5	26.3	27.1	27.8	28.6	29.3	30.0	30.8	31.5	32.2	32.8	33.5	34.2	34.8	35.5	36.1	36.8	37.4	38.0	38.6	39.2	39.8	40.4	41.0	41.6	42.2	42.8	43.3	43.9	44.4	45.0	45.5
8	0.0	4.6	7.0	8.9	10.6	12.1	13.5	14.8	16.0	17.2	18.3	19.4	20.4	21.4	22.4	23.3	24.3	25.1	26.0	26.9	27.7	28.5	29.4	30.2	30.9	31.7	32.5	33.2	33.9	34.7	35.4	36.1	36.8	37.4	38.1	38.8	39.5	40.1	40.8	41.4	42.0	42.7	43.3	43.9	44.5	45.1	45.7	46.3	46.9	47.5	48.0
9	0.0	4.8	7.3	9.3	11.1	12.7	14.1	15.5	16.8	18.0	19.2	20.3	21.4	22.4	23.5	24.5	25.4	26.4	27.3	28.2	29.1	29.9	30.8	31.6	32.4	33.2	34.0	34.8	35.6	36.3	37.1	37.8	38.5	39.2	40.0	40.7	41.4	42.0	42.7	43.4	44.1	44.7	45.4	46.0	46.6	47.3	47.9	48.5	49.1	49.8	50.4
10	0.0	5.0	7.6	9.7	11.5	13.2	14.7	16.1	17.5	18.8	20.0	21.2	22.3	23.4	24.5	25.5	26.5	27.5	28.5	29.4	30.3	31.2	32.1	33.0	33.8	34.7	35.5	36.3	37.1	37.9	38.7	39.4	40.2	40.9	41.7	42.4	43.1	43.8	44.6	45.3	45.9	46.6	47.3	48.0	48.7	49.3	50.0	50.6	51.3	51.9	52.5
11	0.0	5.2	7.9	10.1	12.0	13.7	15.3	16.8	18.2	19.5	20.8	22.0	23.2	24.3	25.4	26.5	27.5	28.6	29.6	30.5	31.5	32.4	33.3	34.2	35.1	36.0	36.9	37.7	38.5	39.4	40.2	41.0	41.8	42.5	43.3	44.1	44.8	45.6	46.3	47.0	47.7	48.4	49.2	49.9	50.5	51.2	51.9	52.6	53.3	53.9	54.6
12	0.0	5.4	8.2	10.4	12.4	14.2	15.8	17.4	18.8	20.2	21.5	22.8	24.0	25.2	26.3	27.4	28.5	29.6	30.6	31.6	32.6	33.6	34.5	35.5	36.4	37.3	38.2	39.0	39.9	40.8	41.6	42.4	43.2	44.0	44.8	45.6	46.4	47.2	47.9	48.7	49.4	50.2	50.9	51.6	52.3	53.0	53.7	54.4	55.1	55.8	56.5
13	0.0	5.6	8.5	10.8	12.8	14.7	16.3	17.9	19.4	20.9	22.2	23.5	24.8	26.0	27.2	28.3	29.4	30.5	31.6	32.6	33.7	34.7	35.7	36.6	37.6	38.5	39.4	40.3	41.2	42.1	42.9	43.8	44.6	45.5	46.3	47.1	47.9	48.7	49.5	50.3	51.0	51.8	52.5	53.3	54.0	54.8	55.5	56.2	56.9	57.6	58.3
14	0.0	5.7	8.7	11.1	13.2	15.1	16.8	18.5	20.0	21.5	22.9	24.2	25.5	26.8	28.0	29.2	30.3	31.5	32.6	33.6	34.7	35.7	36.7	37.7	38.7	39.7	40.6	41.5	42.4	43.3	44.2	45.1	46.0	46.8	47.7	48.5	49.3	50.2	51.0	51.8	52.6	53.4	54.1	54.9	55.7	56.4	57.2	57.9	58.6	59.4	60.1
15	0.0	5.9	9.0	11.4	13.6	15.5	17.3	19.0	20.6	22.1	23.5	24.9	26.2	27.5	28.8	30.0	31.2	32.3	33.5	34.6	35.7	36.7	37.8	38.8	39.8	40.8	41.7	42.7	43.6	44.6	45.5	46.4	47.3	48.1	49.0	49.9	50.7	51.6	52.4	53.2	54.0	54.8	55.6	56.4	57.2	58.0	58.8	59.5	60.3	61.0	61.8
16	0.0	6.1	9.2	11.7	13.9	15.9	17.8	19.5	21.1	22.7	24.1	25.6	26.9	28.3	29.5	30.8	32.0	33.2	34.3	35.5	36.6	37.7	38.7	39.8	40.8	41.8	42.8	43.8	44.8	45.7	46.7	47.6	48.5	49.4	50.3	51.2	52.1	52.9	53.8	54.6	55.5	56.3	57.1	57.9	58.7	59.5	60.3	61.1	61.9	62.6	63.4
17	0.0	6.2	9.4	12.0	14.3	16.3	18.2	20.0	21.6	23.2	24.7	26.2	27.6	28.9	30.3	31.5	32.8	34.0	35.2	36.3	37.5	38.6	39.7	40.8	41.8	42.9	43.9	44.9	45.9	46.8	47.8	48.8	49.7	50.6	51.5	52.4	53.3	54.2	55.1	56.0	56.8	57.7	58.5	59.3	60.2	61.0	61.8	62.6	63.4	64.2	65.0
18	0.0	6.4	9.6	12.3	14.6	16.7	18.6	20.4	22.1	23.8	25.3	26.8	28.2	29.6	31.0	32.3	33.5	34.8	36.0	37.2	38.3	39.5	40.6	41.7	42.8	43.8	44.9	45.9	46.9	47.9	48.9	49.9	50.8	51.8	52.7	53.7	54.6	55.5	56.4	57.3	58.1	59.0	59.9	60.7	61.5	62.4	63.2	64.0	64.8	65.7	66.5
19	0.0	6.5	9.8	12.6	14.9	17.1	19.0	20.9	22.6	24.3	25.9	27.4	28.8	30.3	31.6	33.0	34.3	35.5	36.8	38.0	39.2	40.4	41.5	42.6	43.7	44.8	45.9	46.9	48.0	49.0	50.0	51.0	52.0	52.9	53.9	54.8	55.8	56.7	57.6	58.5	59.4	60.3	61.2	62.0	62.9	63.7	64.6	65.4	66.3	67.1	67.9
20	0.0	6.6	10.0	12.8	15.2	17.4	19.4	21.3	23.1	24.8	26.4	27.9	29.4	30.9	32.3	33.7	35.0	36.3	37.5	38.8	40.0	41.2	42.4	43.5	44.6	45.7	46.8	47.9	48.9	50.0	51.0	52.0	53.0	54.0	55.0	56.0	56.9	57.9	58.8	59.7	60.6	61.5	62.4	63.3	64.2	65.1	65.9	66.8	67.6	68.5	69.3
21	0.0	6.8	10.2	13.1	15.5	17.8	19.8	21.7	23.5	25.3	26.9	28.5	30.0	31.5	32.9	34.3	35.7	37.0	38.3	39.6	40.8	42.0	43.2	44.4	45.5	46.6	47.7	48.8	49.9	51.0	52.0	53.1	54.1	55.1	56.1	57.1	58.0	59.0	59.9	60.9	61.8	62.7	63.7	64.6	65.5	66.4	67.2	68.1	69.0	69.8	70.7
22	0.0	6.9	10.4	13.3	15.8	18.1	20.2	22.1	24.0	25.7	27.4	29.0	30.6	32.1	33.5	35.0	36.3	37.7	39.0	40.3	41.6	42.8	44.0	45.2	46.4	47.5	48.6	49.8	50.9	51.9	53.0	54.1	55.1	56.1	57.1	58.1	59.1	60.1	61.1	62.0	63.0	63.9	64.9	65.8	66.7	67.6	68.5	69.4	70.3	71.1	72.0
23	0.0	7.0	10.6	13.6	16.1	18.4	20.5	22.5	24.4	26.2	27.9	29.5	31.1	32.7	34.2	35.6	37.0	38.4	39.7	41.0	42.3	43.6	44.8	46.0	47.2	48.4	49.5	50.6	51.8	52.9	54.0	55.0	56.1	57.1	58.2	59.2	60.2	61.2	62.2	63.1	64.1	65.1	66.0	67.0	67.9	68.8	69.7	70.6	71.5	72.4	73.3
24	0.0	7.1	10.8	13.8	16.4	18.7	20.9	22.9	24.8	26.6	28.4	30.1	31.7	33.2	34.7	36.2	37.6	39.0	40.4	41.7	43.0	44.3	45.6	46.8	48.0	49.2	50.4	51.5	52.7	53.8	54.9	56.0	57.0	58.1	59.2	60.2	61.2	62.2	63.2	64.2	65.2	66.2	67.2	68.1	69.1	70.0	70.9	71.8	72.8	73.7	74.6
25	0.0	7.2	11.0	14.0	16.7	19.0	21.2	23.3	25.2	27.1	28.9	30.6	32.2	33.8	35.3	36.8	38.3	39.7	41.1	42.4	43.7	45.0	46.3	47.6	48.8	50.0	51.2	52.4	53.5	54.7	55.8	56.9	58.0	59.1	60.1	61.2	62.2	63.3	64.3	65.3	66.3	67.3	68.3	69.2	70.2	71.1	72.1	73.0	74.0	74.9	75.8
26	0.0	7.4	11.2	14.2	16.9	19.3	21.6	23.7	25.6	27.5	29.3	31.0	32.7	34.3	35.9	37.4	38.9	40.3	41.7	43.1	44.4	45.7	47.0	48.3	49.6	50.8	52.0	53.2	54.4	55.5	56.7	57.8	58.9	60.0	61.1	62.2	63.2	64.3	65.3	66.3	67.3	68.3	69.3	70.3	71.3	72.3	73.2	74.2	75.1	76.1	77.0
27	0.0	7.5	11.3	14.4	17.2	19.6	21.9	24.0	26.0	27.9	29.8	31.5	33.2	34.8	36.4	38.0	39.5	40.9	42.3	43.7	45.1	46.4	47.8	49.0	50.3	51.6	52.8	54.0	55.2	56.4	57.5	58.7	59.8	60.9	62.0	63.1	64.2	65.2	66.3	67.3	68.4	69.4	70.4	71.4	72.4	73.4	74.3	75.3	76.3	77.2	78.2
28	0.0	7.6	11.5	14.7	17.4	19.9	22.2	24.4	26.4	28.3	30.2	32.0	33.7	35.3	36.9	38.5	40.0	41.5	43.0	44.4	45.8	47.1	48.5	49.8	51.1	52.3	53.6	54.8	56.0	57.2	58.4	59.5	60.7	61.8	62.9	64.0	65.1	66.2	67.3	68.3	69.4	70.4	71.4	72.4	73.4	74.4	75.4	76.4	77.4	78.3	79.3
29	0.0	7.7	11.7	14.9	17.7	20.2	22.5	24.7	26.8	28.7	30.6	32.4	34.2	35.8	37.5	39.1	40.6	42.1	43.6	45.0	46.4	47.8	49.1	50.5	51.8	53.1	54.3	55.6	56.8	58.0	59.2	60.4	61.5	62.7	63.8	64.9	66.0	67.1	68.2	69.3	70.3	71.4	72.4	73.5	74.5	75.5	76.5	77.5	78.5	79.5	80.4
30	0.0	7.8	11.8	15.1	17.9	20.5	22.8	25.1	27.1	29.1	31.0	32.9	34.6	36.3	38.0	39.6	41.1	42.7	44.2	45.6	47.0	48.4	49.8	51.2	52.5	53.8	55.1	56.3	57.6	58.8	60.0	61.2	62.4	63.5	64.7	65.8	66.9	68.0	69.1	70.2	71.3	72.4	73.4	74.5	75.5	76.5	77.5	78.5	79.5	80.5	81.5
31	0.0	7.9	12.0	15.3	18.1	20.7	23.1	25.4	27.5	29.5	31.4	33.3	35.1	36.8	38.5	40.1	41.7	43.2	44.7	46.2	47.7	49.1	50.5	51.8	53.2	54.5	55.8	57.1	58.3	59.6	60.8	62.0	63.2	64.4	65.5	66.7	67.8	68.9	70.1	71.2	72.2	73.3	74.4	75.4	76.5	77.5	78.6	79.6	80.6	81.6	82.6
32	0.0	8.0	12.1	15.5	18.4	21.0	23.4	25.7	27.9	29.9	31.8	33.7	35.5	37.3	39.0	40.6	42.2	43.8	45.3	46.8	48.3	49.7	51.1	52.5	53.9	55.2	56.5	57.8	59.1	60.3	61.6	62.8	64.0	65.2	66.4	67.5	68.7	69.8	71.0	72.1	73.2	74.3	75.3	76.4	77.5	78.5	79.6	80.6	81.6	82.6	83.7
33	0.0	8.1	12.3	15.7	18.6	21.3	23.7	26.0	28.2	30.3	32.2	34.1	36.0	37.7	39.5	41.1	42.7	44.3	45.9	47.4	48.9	50.3	51.7	53.1	54.5	55.9	57.2	58.5	59.8	61.1	62.3	63.6	64.8	66.0	67.2	68.4	69.5	70.7	71.8	73.0	74.1	75.2	76.3	77.4	78.4	79.5	80.6	81.6	82.6	83.7	84.7
34	0.0	8.2	12.4	15.8	18.8	21.5	24.0	26.3	28.5	30.6	32.6	34.6	36.4	38.2	39.9	41.6	43.3	44.9	46.4	48.0	49.5	50.9	52.4	53.8	55.2	56.5	57.9	59.2	60.5	61.8	63.1	64.3	65.6	66.8	68.0	69.2	70.4	71.5	72.7	73.8	75.0	76.1	77.2	78.3	79.4	80.5	81.5	82.6	83.6	84.7	85.7
35	0.0	8.3	12.6	16.0	19.0	21.8	24.3	26.7	28.9	31.0	33.0	35.0	36.8	38.6	40.4	42.1	43.8	45.4	47.0	48.5	50.0	51.5	53.0	54.4	55.8	57.2	58.6	59.9	61.2	62.5	63.8	65.1	66.3	67.6	68.8	70.0	71.2	72.4	73.5	74.7	75.8	77.0	78.1	79.2	80.3	81.4	82.5	83.5	84.6	85.7	86.7
36	0.0	8.4	12.7	16.2	19.3	22.0	24.6	27.0	29.2	31.3	33.4	35.3	37.2	39.1	40.9	42.6	44.3	45.9	47.5	49.1	50.6	52.1	53.6	55.0	56.5	57.9	59.2	60.6	61.9	63.2	64.5	65.8	67.1	68.3	69.6	70.8	72.0	73.2	74.4	75.5	76.7	77.8	79.0	80.1	81.2	82.3	83.4	84.5	85.6	86.6	87.7
37	0.0	8.5	12.9	16.4	19.5	22.3	24.8	27.3	29.5	31.7	33.8	35.7	37.7	39.5	41.3	43.0	44.7	46.4	48.0	49.6	51.2	52.7	54.2	55.6	57.1	58.5	59.9	61.3	62.6	63.9	65.3	66.5	67.8	69.1	70.3	71.6	72.8	74.0	75.2	76.4	77.5	78.7	79.8	81.0	82.1	83.2	84.3	85.4	86.5	87.6	88.7
38	0.0	8.6	13.0	16.6	19.7	22.5	25.1	27.5	29.8	32.0	34.1	36.1	38.1	39.9	41.7	43.5	45.2	46.9	48.5	50.1	51.7	53.2	54.8	56.2	57.7	59.1	60.5	61.9	63.3	64.6	66.0	67.3	68.6	69.8	71.1	72.3	73.6	74.8	76.0	77.2	78.4	79.5	80.7	81.9	83.0	84.1	85.2	86.3	87.4	88.5	89.6
39	0.0	8.7	13.1	16.7	19.9	22.7	25.4	27.8	30.2	32.4	34.5	36.5	38.5	40.3	42.2	44.0	45.7	47.4	49.0	50.7	52.2	53.8	55.3	56.8	58.3	59.7	61.2	62.6	63.9	65.3	66.6	68.0	69.3	70.6	71.8	73.1	74.3	75.6	76.8	78.0	79.2	80.4	81.5	82.7	83.9	85.0	86.1	87.2	88.3	89.4	90.5
40	0.0	8.7	13.3	16.9	20.1	23.0	25.6	28.1	30.5	32.7	34.8	36.9	38.8	40.8	42.6	44.4	46.2	47.9	49.5	51.2	52.8	54.3	55.9	57.4	58.9	60.3	61.8	63.2	64.6	66.0	67.3	68.7	70.0	71.3	72.6	73.8	75.1	76.3	77.6	78.8	80.0	81.2	82.4	83.5	84.7	85.9	87.0	88.1	89.2	90.4	91.5
41	0.0	8.8	13.4	17.1	20.3	23.2	25.9	28.4	30.8	33.0	35.2	37.2	39.2	41.2	43.0	44.9	46.6	48.4	50.0	51.7	53.3	54.9	56.4	58.0	59.5	60.9	62.4	63.8	65.2	66.6	68.0	69.3	70.7	72.0	73.3	74.6	75.8	77.1	78.3	79.6	80.8	82.0	83.2	84.4	85.5	86.7	87.9	89.0	90.1	91.3	92.4
42	0.0	8.9	13.5	17.2	20.5	23.4	26.1	28.7	31.1	33.3	35.5	37.6	39.6	41.6	43.5	45.3	47.1	48.8	50.5	52.2	53.8	55.4	57.0	58.5	60.0	61.5	63.0	64.4	65.9	67.3	68.6	70.0	71.4	72.7	74.0	75.3	76.6	77.8	79.1	80.3	81.6	82.8	84.0	85.2	86.4	87.6	88.7	89.9	91.0	92.1	93.3
43	0.0	9.0	13.6	17.4	20.7	23.6	26.4	28.9	31.4	33.6	35.8	38.0	40.0	42.0	43.9	45.7	47.5	49.3	51.0	52.7	54.3	55.9	57.5	59.1	60.6	62.1	63.6	65.0	66.5	67.9	69.3	70.7	72.0	73.4	74.7	76.0	77.3	78.6	79.9	81.1	82.3	83.6	84.8	86.0	87.2	88.4	89.6	90.7	91.9	93.0	94.1
44	0.0	9.1	13.8	17.6	20.9	23.9	26.6	29.2	31.6	34.0	36.2	38.3	40.4	42.3	44.3	46.1	48.0	49.7	51.5	53.2	54.8	56.5	58.1	59.6	61.2	62.7	64.2	65.6	67.1	68.5	69.9	71.3	72.7	74.0	75.4	76.7	78.0	79.3	80.6	81.9	83.1	84.3	85.6	86.8	88.0	89.2	90.4	91.6	92.7	93.9	95.0
45	0.0	9.2	13.9	17.7	21.1	24.1	26.9	29.5	31.9	34.3	36.5	38.7	40.7	42.7	44.7	46.6	48.4	50.2	51.9	53.6	55.3	57.0	58.6	60.2	61.7	63.3	64.8	66.2	67.7	69.1	70.6	72.0	73.4	74.7	76.1	77.4	78.7	80.0	81.3	82.6	83.9	85.1	86.4	87.6	88.8	90.0	91.2	92.4	93.6	94.7	95.9
46	0.0	9.2	14.0	17.9	21.3	24.3	27.1	29.7	32.2	34.6	36.8	39.0	41.1	43.1	45.1	47.0	48.8	50.6	52.4	54.1	55.8	57.5	59.1	60.7	62.3	63.8	65.3	66.8	68.3	69.8	71.2	72.6	74.0	75.4	76.7	78.1	79.4	80.7	82.0	83.3	84.6	85.9	87.1	88.4	89.6	90.8	92.0	93.2	94.4	95.6	96.7
47	0.0	9.3	14.1	18.0	21.4	24.5	27.3	30.0	32.5	34.9	37.1	39.3	41.4	43.5	45.5	47.4	49.2	51.1	52.8	54.6	56.3	58.0	59.6	61.2	62.8	64.4	65.9	67.4	68.9	70.4	71.8	73.2	74.6	76.0	77.4	78.8	80.1	81.4	82.7	84.0	85.3	86.6	87.9	89.1	90.4	91.6	92.8	94.0	95.2	96.4	97.6
48	0.0	9.4	14.3	18.2	21.6	24.7	27.6	30.2	32.8	35.2	37.5	39.7	41.8	43.8	45.8	47.8	49.7	51.5	53.3	55.1	56.8	58.5	60.1	61.7	63.3	64.9	66.5	68.0	69.5	71.0	72.4	73.8	75.3	76.7	78.1	79.4	80.8	82.1	83.4	84.8	86.1	87.3	88.6	89.9	91.1	92.4	93.6	94.8	96.0	97.2	98.4
49	0.0	9.5	14.4	18.3	21.8	24.9	27.8	30.5	33.0	35.5	37.8	40.0	42.1	44.2	46.2	48.2	50.1	51.9	53.7	55.5	57.2	58.9	60.6	62.3	63.9	65.4	67.0	68.5	70.0	71.5	73.0	74.5	75.9	77.3	78.7	80.1	81.4	82.8	84.1	85.5	86.8	88.1	89.3	90.6	91.9	93.1	94.4	95.6	96.8	98.0	99.2
50	0.0	9.6	14.5	18.5	22.0	25.1	28.0	30.7	33.3	35.7	38.1	40.3	42.5	44.6	46.6	48.6	50.5	52.3	54.2	56.0	57.7	59.4	61.1	62.8	64.4	66.0	67.5	69.1	70.6	72.1	73.6	75.1	76.5	77.9	79.3	80.7	82.1	83.5	84.8	86.2	87.5	88.8	90.1	91.3	92.6	93.9	95.1	96.4	97.6	98.8	100.0

Table 1B

													Value of Various Combinations of Green and Red Chips to Members of Group 2																																						
Red	Green Chips																																																		
Chips	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	2.0	3.5	4.8	6.1	7.2	8.4	9.5	10.6	11.6	12.6	13.6	14.6	15.6	16.5	17.5	18.4	19.3	20.2	21.1	22.0	22.8	23.7	24.6	25.4	26.3	27.1	27.9	28.8	29.6	30.4	31.2	32.0	32.8	33.6	34.4	35.2	35.9	36.7	37.5	38.3	39.0	39.8	40.5	41.3	42.0	42.8	43.5	44.3	45.0	45.7
2	0.0	2.3	4.0	5.5	7.0	8.3	9.6	10.9	12.1	13.3	14.5	15.6	16.8	17.9	19.0	20.0	21.1	22.2	23.2	24.2	25.2	26.2	27.2	28.2	29.2	30.2	31.1	32.1	33.0	34.0	34.9	35.8	36.8	37.7	38.6	39.5	40.4	41.3	42.2	43.1	43.9	44.8	45.7	46.6	47.4	48.3	49.1	50.0	50.8	51.7	52.5
3	0.0	2.5	4.3	6.0	7.6	9.0	10.4	11.8	13.2	14.4	15.7	17.0	18.2	19.4	20.6	21.7	22.9	24.0	25.2	26.3	27.4	28.5	29.5	30.6	31.7	32.7	33.8	34.8	35.8	36.8	37.9	38.9	39.9	40.9	41.8	42.8	43.8	44.8	45.7	46.7	47.7	48.6	49.6	50.5	51.4	52.4	53.3	54.2	55.1	56.1	57.0
4	0.0	2.6	4.6	6.4	8.0	9.6	11.1	12.5	13.9	15.3	16.7	18.0	19.3	20.5	21.8	23.0	24.3	25.5	26.6	27.8	29.0	30.1	31.3	32.4	33.5	34.7	35.8	36.9	37.9	39.0	40.1	41.2	42.2	43.3	44.3	45.4	46.4	47.4	48.4	49.5	50.5	51.5	52.5	53.5	54.5	55.5	56.4	57.4	58.4	59.4	60.3
5	0.0	2.8	4.8	6.6	8.4	10.0	11.6	13.1	14.6	16.0	17.4	18.8	20.1	21.5	22.8	24.1	25.4	26.6	27.9	29.1	30.3	31.5	32.7	33.9	35.1	36.2	37.4	38.5	39.7	40.8	41.9	43.0	44.2	45.3	46.3	47.4	48.5	49.6	50.7	51.7	52.8	53.8	54.9	55.9	57.0	58.0	59.0	60.0	61.1	62.1	63.1
6	0.0	2.9	5.0	6.9	8.7	10.4	12.0	13.6	15.1	16.6	18.1	19.5	20.9	22.3	23.6	25.0	26.3	27.6	28.9	30.2	31.4	32.7	33.9	35.2	36.4	37.6	38.8	40.0	41.2	42.3	43.5	44.6	45.8	46.9	48.1	49.2	50.3	51.4	52.5	53.6	54.7	55.8	56.9	58.0	59.1	60.1	61.2	62.3	63.3	64.4	65.4
7	0.0	3.0	5.1	7.1	8.9	10.7	12.4	14.0	15.6	17.1	18.6	20.1	21.5	23.0	24.4	25.8	27.1	28.5	29.8	31.1	32.4	33.7	35.0	36.3	37.5	38.8	40.0	41.2	42.4	43.6	44.8	46.0	47.2	48.4	49.6	50.7	51.9	53.0	54.2	55.3	56.5	57.6	58.7	59.8	60.9	62.0	63.1	64.2	65.3	66.4	67.5
8	0.0	3.0	5.3	7.3	9.2	11.0	12.7	14.4	16.0	17.6	19.1	20.6	22.1	23.6	25.0	26.5	27.9	29.2	30.6	32.0	33.3	34.6	35.9	37.2	38.5	39.8	41.1	42.3	43.6	44.8	46.1	47.3	48.5	49.7	50.9	52.1	53.3	54.5	55.7	56.8	58.0	59.1	60.3	61.4	62.6	63.7	64.8	66.0	67.1	68.2	69.3
9	0.0	3.1	5.4	7.5	9.4	11.2	13.0	14.7	16.4	18.0	19.6	21.1	22.7	24.2	25.6	27.1	28.5	29.9	31.3	32.7	34.1	35.5	36.8	38.1	39.5	40.8	42.1	43.3	44.6	45.9	47.2	48.4	49.7	50.9	52.1	53.3	54.6	55.8	57.0	58.2	59.4	60.5	61.7	62.9	64.1	65.2	66.4	67.5	68.7	69.8	71.0
10	0.0	3.2	5.5	7.6	9.6	11.5	13.3	15.0	16.7	18.4	20.0	21.6	23.1	24.7	26.2	27.7	29.1	30.6	32.0	33.4	34.8	36.2	37.6	38.9	40.3	41.6	43.0	44.3	45.6	46.9	48.2	49.4	50.7	52.0	53.2	54.5	55.7	57.0	58.2	59.4	60.6	61.8	63.0	64.2	65.4	66.6	67.8	69.0	70.1	71.3	72.5
11	0.0	3.2	5.6	7.8	9.8	11.7	13.5	15.3	17.1	18.7	20.4	22.0	23.6	25.1	26.7	28.2	29.7	31.2	32.6	34.1	35.5	36.9	38.3	39.7	41.1	42.4	43.8	45.1	46.5	47.8	49.1	50.4	51.7	53.0	54.3	55.5	56.8	58.1	59.3	60.6	61.8	63.0	64.3	65.5	66.7	67.9	69.1	70.3	71.5	72.7	73.9
12	0.0	3.3	5.7	7.9	10.0	11.9	13.8	15.6	17.4	19.1	20.7	22.4	24.0	25.6	27.1	28.7	30.2	31.7	33.2	34.7	36.1	37.6	39.0	40.4	41.8	43.2	44.5	45.9	47.3	48.6	50.0	51.3	52.6	53.9	55.2	56.5	57.8	59.1	60.4	61.6	62.9	64.1	65.4	66.6	67.9	69.1	70.3	71.5	72.8	74.0	75.2
13	0.0	3.3	5.8	8.0	10.1	12.1	14.0	15.8	17.6	19.4	21.1	22.7	24.4	26.0	27.6	29.2	30.7	32.2	33.7	35.2	36.7	38.2	39.6	41.0	42.5	43.9	45.3	46.7	48.0	49.4	50.8	52.1	53.4	54.8	56.1	57.4	58.7	60.0	61.3	62.6	63.9	65.2	66.4	67.7	69.0	70.2	71.5	72.7	73.9	75.2	76.4
14	0.0	3.4	5.9	8.2	10.3	12.3	14.2	16.1	17.9	19.7	21.4	23.1	24.8	26.4	28.0	29.6	31.2	32.7	34.2	35.7	37.2	38.7	40.2	41.7	43.1	44.5	45.9	47.4	48.8	50.1	51.5	52.9	54.2	55.6	56.9	58.3	59.6	60.9	62.2	63.5	64.8	66.1	67.4	68.7	70.0	71.3	72.5	73.8	75.0	76.3	77.5
15	0.0	3.4	6.0	8.3	10.4	12.5	14.4	16.3	18.1	19.9	21.7	23.4	25.1	26.8	28.4	30.0	31.6	33.2	34.7	36.2	37.8	39.3	40.8	42.2	43.7	45.1	46.6	48.0	49.4	50.8	52.2	53.6	55.0	56.4	57.7	59.1	60.4	61.8	63.1	64.4	65.8	67.1	68.4	69.7	71.0	72.2	73.5	74.8	76.1	77.3	78.6
16	0.0	3.5	6.1	8.4	10.6	12.6	14.6	16.5	18.4	20.2	22.0	23.7	25.4	27.1	28.8	30.4	32.0	33.6	35.2	36.7	38.3	39.8	41.3	42.8	44.3	45.7	47.2	48.6	50.1	51.5	52.9	54.3	55.7	57.1	58.5	59.9	61.2	62.6	63.9	65.3	66.6	67.9	69.3	70.6	71.9	73.2	74.5	75.8	77.1	78.3	79.6
17	0.0	3.5	6.1	8.5	10.7	12.8	14.8	16.7	18.6	20.4	22.2	24.0	25.7	27.4	29.1	30.8	32.4	34.0	35.6	37.2	38.7	40.3	41.8	43.3	44.8	46.3	47.8	49.2	50.7	52.1	53.6	55.0	56.4	57.8	59.2	60.6	62.0	63.3	64.7	66.1	67.4	68.8	70.1	71.4	72.8	74.1	75.4	76.7	78.0	79.3	80.6
18	0.0	3.6	6.2	8.6	10.8	12.9	14.9	16.9	18.8	20.7	22.5	24.3	26.0	27.7	29.4	31.1	32.8	34.4	36.0	37.6	39.2	40.7	42.3	43.8	45.3	46.8	48.3	49.8	51.3	52.7	54.2	55.6	57.0	58.5	59.9	61.3	62.7	64.1	65.5	66.8	68.2	69.6	70.9	72.3	73.6	74.9	76.3	77.6	78.9	80.2	81.5
19	0.0	3.6	6.3	8.7	10.9	13.1	15.1	17.1	19.0	20.9	22.7	24.5	26.3	28.1	29.8	31.5	33.1	34.8	36.4	38.0	39.6	41.2	42.7	44.3	45.8	47.3	48.8	50.3	51.8	53.3	54.8	56.2	57.7	59.1	60.5	61.9	63.4	64.8	66.2	67.6	68.9	70.3	71.7	73.0	74.4	75.7	77.1	78.4	79.8	81.1	82.4
20	0.0	3.6	6.3	8.8	11.0	13.2	15.3	17.3	19.2	21.1	23.0	24.8	26.6	28.3	30.1	31.8	33.5	35.1	36.8	38.4	40.0	41.6	43.2	44.7	46.3	47.8	49.3	50.9	52.4	53.8	55.3	56.8	58.3	59.7	61.2	62.6	64.0	65.4	66.8	68.2	69.6	71.0	72.4	73.8	75.2	76.5	77.9	79.2	80.6	81.9	83.3
21	0.0	3.7	6.4	8.9	11.1	13.3	15.4	17.4	19.4	21.3	23.2	25.0	26.8	28.6	30.4	32.1	33.8	35.5	37.1	38.8	40.4	42.0	43.6	45.2	46.7	48.3	49.8	51.4	52.9	54.4	55.9	57.4	58.8	60.3	61.8	63.2	64.6	66.1	67.5	68.9	70.3	71.7	73.1	74.5	75.9	77.3	78.6	80.0	81.4	82.7	84.1
22	0.0	3.7	6.5	8.9	11.3	13.4	15.6	17.6	19.6	21.5	23.4	25.3	27.1	28.9	30.6	32.4	34.1	35.8	37.5	39.1	40.8	42.4	44.0	45.6	47.2	48.7	50.3	51.8	53.4	54.9	56.4	57.9	59.4	60.9	62.3	63.8	65.2	66.7	68.1	69.6	71.0	72.4	73.8	75.2	76.6	78.0	79.4	80.8	82.1	83.5	84.9
23	0.0	3.7	6.5	9.0	11.4	13.6	15.7	17.8	19.8	21.7	23.6	25.5	27.3	29.1	30.9	32.7	34.4	36.1	37.8	39.5	41.1	42.8	44.4	46.0	47.6	49.2	50.7	52.3	53.8	55.4	56.9	58.4	59.9	61.4	62.9	64.4	65.8	67.3	68.7	70.2	71.6	73.0	74.5	75.9	77.3	78.7	80.1	81.5	82.9	84.2	85.6
24	0.0	3.8	6.6	9.1	11.4	13.7	15.8	17.9	19.9	21.9	23.8	25.7	27.6	29.4	31.2	33.0	34.7	36.4	38.1	39.8	41.5	43.1	44.8	46.4	48.0	49.6	51.2	52.7	54.3	55.8	57.4	58.9	60.4	61.9	63.4	64.9	66.4	67.9	69.3	70.8	72.2	73.7	75.1	76.5	78.0	79.4	80.8	82.2	83.6	85.0	86.3
25	0.0	3.8	6.6	9.2	11.5	13.8	16.0	18.1	20.1	22.1	24.0	25.9	27.8	29.6	31.4	33.2	35.0	36.7	38.4	40.1	41.8	43.5	45.1	46.8	48.4	50.0	51.6	53.2	54.7	56.3	57.9	59.4	60.9	62.4	63.9	65.4	66.9	68.4	69.9	71.4	72.8	74.3	75.7	77.2	78.6	80.0	81.4	82.9	84.3	85.7	87.1
26	0.0	3.8	6.7	9.2	11.6	13.9	16.1	18.2	20.3	22.3	24.2	26.1	28.0	29.9	31.7	33.5	35.3	37.0	38.7	40.5	42.2	43.8	45.5	47.1	48.8	50.4	52.0	53.6	55.2	56.7	58.3	59.9	61.4	62.9	64.4	66.0	67.5	69.0	70.4	71.9	73.4	74.9	76.3	77.8	79.2	80.6	82.1	83.5	84.9	86.3	87.7
27	0.0	3.9	6.7	9.3	11.7	14.0	16.2	18.3	20.4	22.4	24.4	26.3	28.2	30.1	31.9	33.7	35.5	37.3	39.0	40.8	42.5	44.2	45.8	47.5	49.1	50.8	52.4	54.0	55.6	57.2	58.7	60.3	61.9	63.4	64.9	66.5	68.0	69.5	71.0	72.5	74.0	75.4	76.9	78.4	79.8	81.3	82.7	84.1	85.6	87.0	88.4
28	0.0	3.9	6.8	9.4	11.8	14.1	16.3	18.5	20.6	22.6	24.6	26.5	28.4	30.3	32.2	34.0	35.8	37.6	39.3	41.1	42.8	44.5	46.2	47.8	49.5	51.1	52.8	54.4	56.0	57.6	59.2	60.8	62.3	63.9	65.4	66.9	68.5	70.0	71.5	73.0	74.5	76.0	77.5	78.9	80.4	81.9	83.3	84.8	86.2	87.6	89.1
29	0.0	3.9	6.8	9.4	11.9	14.2	16.4	18.6	20.7	22.7	24.7	26.7	28.6	30.5	32.4	34.2	36.0	37.8	39.6	41.4	43.1	44.8	46.5	48.2	49.9	51.5	53.1	54.8	56.4	58.0	59.6	61.2	62.8	64.3	65.9	67.4	69.0	70.5	72.0	73.5	75.0	76.5	78.0	79.5	81.0	82.4	83.9	85.3	86.8	88.2	89.7
30	0.0	3.9	6.9	9.5	12.0	14.3	16.6	18.7	20.8	22.9	24.9	26.9	28.8	30.7	32.6	34.5	36.3	38.1	39.9	41.6	43.4	45.1	46.8	48.5	50.2	51.9	53.5	55.1	56.8	58.4	60.0	61.6	63.2	64.8	66.3	67.9	69.4	71.0	72.5	74.0	75.5	77.0	78.5	80.0	81.5	83.0	84.5	85.9	87.4	88.8	90.3
31	0.0	4.0	6.9	9.6	12.0	14.4	16.7	18.9	21.0	23.1	25.1	27.1	29.0	30.9	32.8	34.7	36.5	38.3	40.1	41.9	43.7	45.4	47.1	48.8	50.5	52.2	53.9	55.5	57.2	58.8	60.4	62.0	63.6	65.2	66.8	68.3	69.9	71.4	73.0	74.5	76.0	77.5	79.0	80.6	82.0	83.5	85.0	86.5	88.0	89.4	90.9
32	0.0	4.0	7.0	9.6	12.1	14.5	16.8	19.0	21.1	23.2	25.2	27.2	29.2	31.1	33.0	34.9	36.8	38.6	40.4	42.2	43.9	45.7	47.4	49.1	50.8	52.5	54.2	55.9	57.5	59.2	60.8	62.4	64.0	65.6	67.2	68.8	70.3	71.9	73.4	75.0	76.5	78.0	79.6	81.1	82.6	84.1	85.6	87.0	88.5	90.0	91.5
33	0.0	4.0	7.0	9.7	12.2	14.6	16.9	19.1	21.2	23.3	25.4	27.4	29.4	31.3	33.2	35.1	37.0	38.8	40.6	42.4	44.2	46.0	47.7	49.4	51.2	52.9	54.5	56.2	57.9	59.5	61.2	62.8	64.4	66.0	67.6	69.2	70.8	72.3	73.9	75.4	77.0	78.5	80.0	81.6	83.1	84.6	86.1	87.6	89.1	90.6	92.0
34	0.0	4.0	7.0	9.8	12.3	14.7	17.0	19.2	21.4	23.5	25.5	27.6	29.6	31.5	33.4	35.3	37.2	39.1	40.9	42.7	44.5	46.2	48.0	49.7	51.5	53.2	54.9	56.5	58.2	59.9	61.5	63.2	64.8	66.4	68.0	69.6	71.2	72.8	74.3	75.9	77.4	79.0	80.5	82.1	83.6	85.1	86.6	88.1	89.6	91.1	92.6
35	0.0	4.1	7.1	9.8	12.3	14.8	17.1	19.3	21.5	23.6	25.7	27.7	29.7	31.7	33.6	35.5	37.4	39.3	41.1	42.9	44.7	46.5	48.3	50.0	51.8	53.5	55.2	56.9	58.6	60.2	61.9	63.5	65.2	66.8	68.4	70.0	71.6	73.2	74.8	76.3	77.9	79.4	81.0	82.5	84.1	85.6	87.1	88.6	90.1	91.6	93.1
36	0.0	4.1	7.1	9.9	12.4	14.8	17.2	19.4	21.6	23.8	25.8	27.9	29.9	31.9	33.8	35.7	37.6	39.5	41.4	43.2	45.0	46.8	48.6	50.3	52.1	53.8	55.5	57.2	58.9	60.6	62.2	63.9	65.5	67.2	68.8	70.4	72.0	73.6	75.2	76.8	78.3	79.9	81.4	83.0	84.5	86.1	87.6	89.1	90.6	92.1	93.6
37	0.0	4.1	7.2	9.9	12.5	14.9	17.3	19.5	21.7	23.9	26.0	28.0	30.1	32.0	34.0	35.9	37.8	39.7	41.6	43.4	45.2	47.0	48.8	50.6	52.3	54.1	55.8	57.5	59.2	60.9	62.6	64.2	65.9	67.5	69.2	70.8	72.4	74.0	75.6	77.2	78.8	80.3	81.9	83.5	85.0	86.5	88.1	89.6	91.1	92.6	94.2
38	0.0	4.1	7.2	10.0	12.5	15.0	17.4	19.6	21.9	24.0	26.1	28.2	30.2	32.2	34.2	36.1	38.0	39.9	41.8	43.7	45.5	47.3	49.1	50.9	52.6	54.4	56.1	57.8	59.5	61.2	62.9	64.6	66.2	67.9	69.5	71.2	72.8	74.4	76.0	77.6	79.2	80.8	82.3	83.9	85.5	87.0	88.6	90.1	91.6	93.1	94.7
39	0.0	4.2	7.2	10.0	12.6	15.1	17.4	19.7	22.0	24.1	26.3	28.3	30.4	32.4	34.4	36.3	38.2	40.1	42.0	43.9	45.7	47.5	49.3	51.1	52.9	54.7	56.4	58.1	59.8	61.5	63.2	64.9	66.6	68.2	69.9	71.5	73.2	74.8	76.4	78.0	79.6	81.2	82.8	84.3	85.9	87.5	89.0	90.6	92.1	93.6	95.2
40	0.0	4.2	7.3	10.1	12.7	15.2	17.5	19.8	22.1	24.3	26.4	28.5	30.5	32.6	34.5	36.5	38.4	40.3	42.2	44.1	45.9	47.8	49.6	51.4	53.2	54.9	56.7	58.4	60.1	61.9	63.6	65.2	66.9	68.6	70.2	71.9	73.5	75.2	76.8	78.4	80.0	81.6	83.2	84.8	86.3	87.9	89.5	91.0	92.6	94.1	95.6
41	0.0	4.2	7.3	10.1	12.7	15.2	17.6	19.9	22.2	24.4	26.5	28.6	30.7	32.7	34.7	36.7	38.6	40.5	42.4	44.3	46.2	48.0	49.8	51.6	53.4	55.2	57.0	58.7	60.4	62.2	63.9	65.6	67.3	68.9	70.6	72.3	73.9	75.5	77.2	78.8	80.4	82.0	83.6	85.2	86.8	88.3	89.9	91.5	93.0	94.6	96.1
42	0.0	4.2	7.4	10.2	12.8	15.3	17.7	20.0	22.3	24.5	26.6	28.8	30.8	32.9	34.9	36.9	38.8	40.7	42.6	44.5	46.4	48.2	50.1	51.9	53.7	55.5	57.2	59.0	60.7	62.5	64.2	65.9	67.6	69.3	70.9	72.6	74.3	75.9	77.5	79.2	80.8	82.4	84.0	85.6	87.2	88.8	90.3	91.9	93.5	95.0	96.6
43	0.0	4.2	7.4	10.2	12.9	15.4	17.8	20.1	22.4	24.6	26.8	28.9	31.0	33.0	35.0	37.0	39.0	40.9	42.8	44.7	46.6	48.5	50.3	52.1	53.9	55.7	57.5	59.3	61.0	62.8	64.5	66.2	67.9	69.6	71.3	72.9	74.6	76.3	77.9	79.5	81.2	82.8	84.4	86.0	87.6	89.2	90.8	92.3	93.9	95.5	97.0
44	0.0	4.3	7.4	10.3	12.9	15.4	17.9	20.2	22.5	24.7	26.9	29.0	31.1	33.2	35.2	37.2	39.2	41.1	43.0	44.9	46.8	48.7	50.5	52.4	54.2	56.0	57.8	59.5	61.3	63.0	64.8	66.5	68.2	69.9	71.6	73.3	74.9	76.6	78.3	79.9	81.5	83.2	84.8	86.4	88.0	89.6	91.2	92.8	94.3	95.9	97.5
45	0.0	4.3	7.5	10.3	13.0	15.5	18.0	20.3	22.6	24.8	27.0	29.2	31.3	33.3	35.4	37.4	39.4	41.3	43.2	45.2	47.0	48.9	50.8	52.6	54.4	56.2	58.0	59.8	61.6	63.3	65.1	66.8	68.5	70.2	71.9	73.6	75.3	77.0	78.6	80.3	81.9	83.5	85.2	86.8	88.4	90.0	91.6	93.2	94.8	96.3	97.9
46	0.0	4.3	7.5	10.4	13.0	15.6	18.0	20.4	22.7	24.9	27.1	29.3	31.4	33.5	35.5	37.5	39.5	41.5	43.4	45.4	47.3	49.1	51.0	52.8	54.7	56.5	58.3	60.1	61.8	63.6	65.4	67.1	68.8	70.5	72.2	73.9	75.6	77.3	79.0	80.6	82.3	83.9	85.5	87.2	88.8	90.4	92.0	93.6	95.2	96.8	98.3
47	0.0	4.3	7.5	10.4	13.1	15.7	18.1	20.5	22.8	25.1	27.3	29.4	31.5	33.6	35.7	37.7	39.7	41.7	43.6	45.5	47.5	49.3	51.2	53.1	54.9	56.7	58.5	60.3	62.1	63.9	65.6	67.4	69.1	70.8	72.5	74.3	75.9	77.6	79.3	81.0	82.6	84.3	85.9	87.5	89.2	90.8	92.4	94.0	95.6	97.2	98.8
48	0.0	4.3	7.6	10.4	13.2	15.7	18.2	20.6	22.9	25.2	27.4	29.5	31.7	33.8	35.8	37.9	39.9	41.8	43.8	45.7	47.7	49.6	51.4	53.3	55.1	57.0	58.8	60.6	62.4	64.1	65.9	67.7	69.4	71.1	72.9	74.6	76.3	78.0	79.6	81.3	83.0	84.6	86.3	87.9	89.5	91.2	92.8	94.4	96.0	97.6	99.2
49	0.0	4.4	7.6	10.5	13.2	15.8	18.3	20.7	23.0	25.3	27.5	29.7	31.8	33.9	36.0	38.0	40.0	42.0	44.0	45.9	47.9	49.8	51.6	53.5	55.4	57.2	59.0	60.8	62.6	64.4	66.2	67.9	69.7	71.4	73.2	74.9	76.6	78.3	80.0	81.6	83.3	85.0	86.6	88.3	89.9	91.5	93.2	94.8	96.4	98.0	99.6
50	0.0	4.4	7.6	10.5	13.3	15.8	18.3	20.7	23.1	25.4	27.6	29.8	31.9	34.0	36.1	38.2	40.2	42.2	44.2	46.1	48.0	50.0	51.9	53.7	55.6	57.4	59.3	61.1	62.9	64.7	66.5	68.2	70.0	71.7	73.5	75.2	76.9	78.6	80.3	82.0	83.7	85.3	87.0	88.6	90.3	91.9	93.5	95.2	96.8	98.4	100.0

Table 2

Parameters of Four Computational Economies
(Cobb-Douglas Utility Function)

Parameter	Economy 1	Economy 2	Economy 3	Economy 4
α	0.4	.7	.3	.4
β	0.8	.4	.5	.2
Initial Endowment	(0.8, 0.1)	(0.8, 0.1)	(0.8, 0.1)	(0.8, 0.1)
Number of Iterations	2,000	2,000	2,000	2,000
Discrete Transaction Size	0.02	0.02	0.02	0.02

Figure 1: Competitive Equilibrium in Edgeworth Box with Cobb Douglas Utility

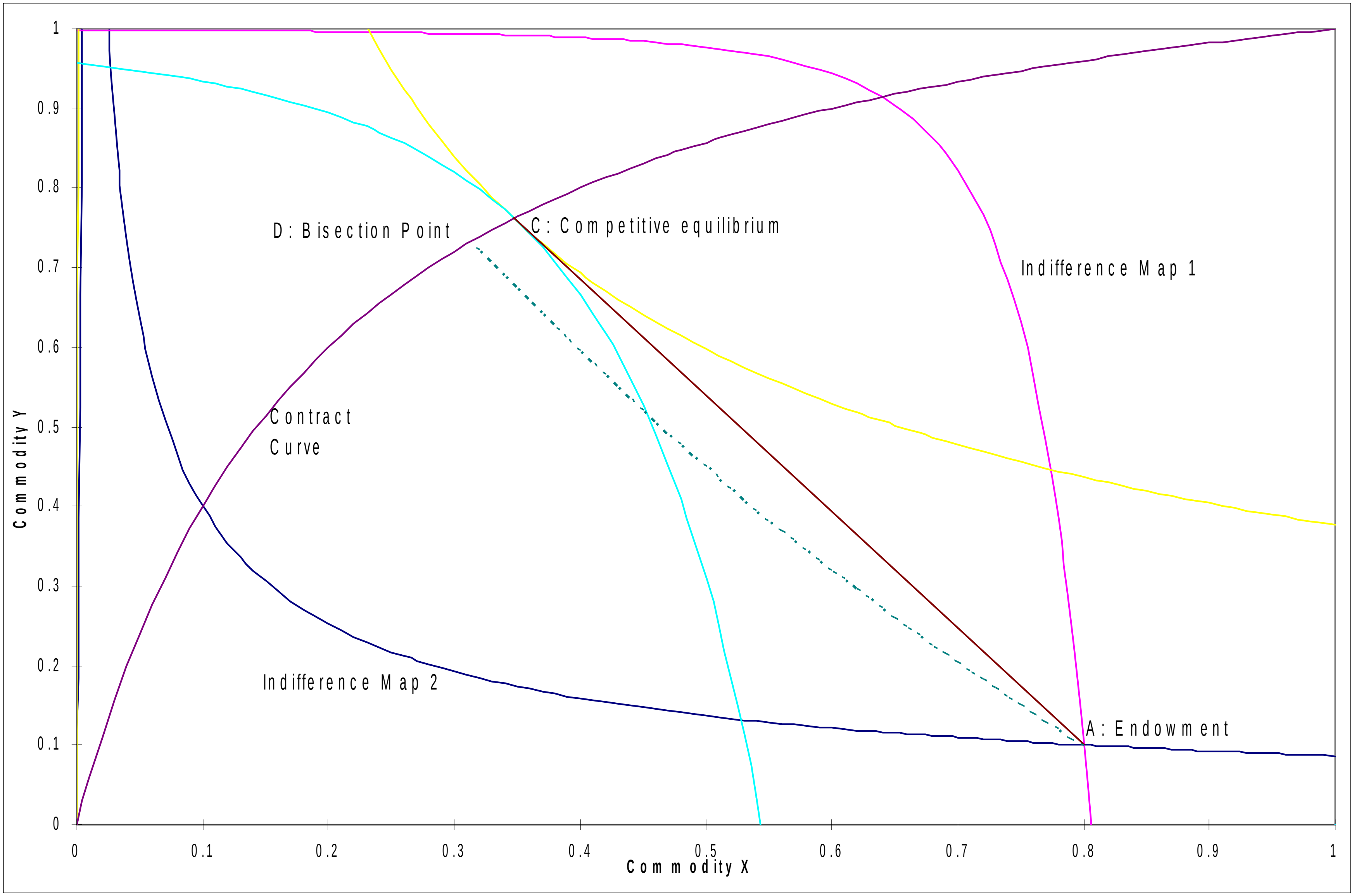

Figure 2: Final Trading Positions in Edgeworth Box for Two Groups of 14 Human Traders

(Initial Endowment: 0.88,0.18; Competitive Equilibrium: 0.58,0.79; Average Holdings at the end: 0.57, 0.69)
(Group 1: Initial Utility = 0.933, CE Utility = 1.31, Average Utility at the end = 1.22)
(Group 2: Initial Utility = 0.0.35, CE Utility = 0.73, Average Utility at the end = 0.803)
Legend: Group 1 Traders in solid triangles, Group 2 Traders in hollow circles)

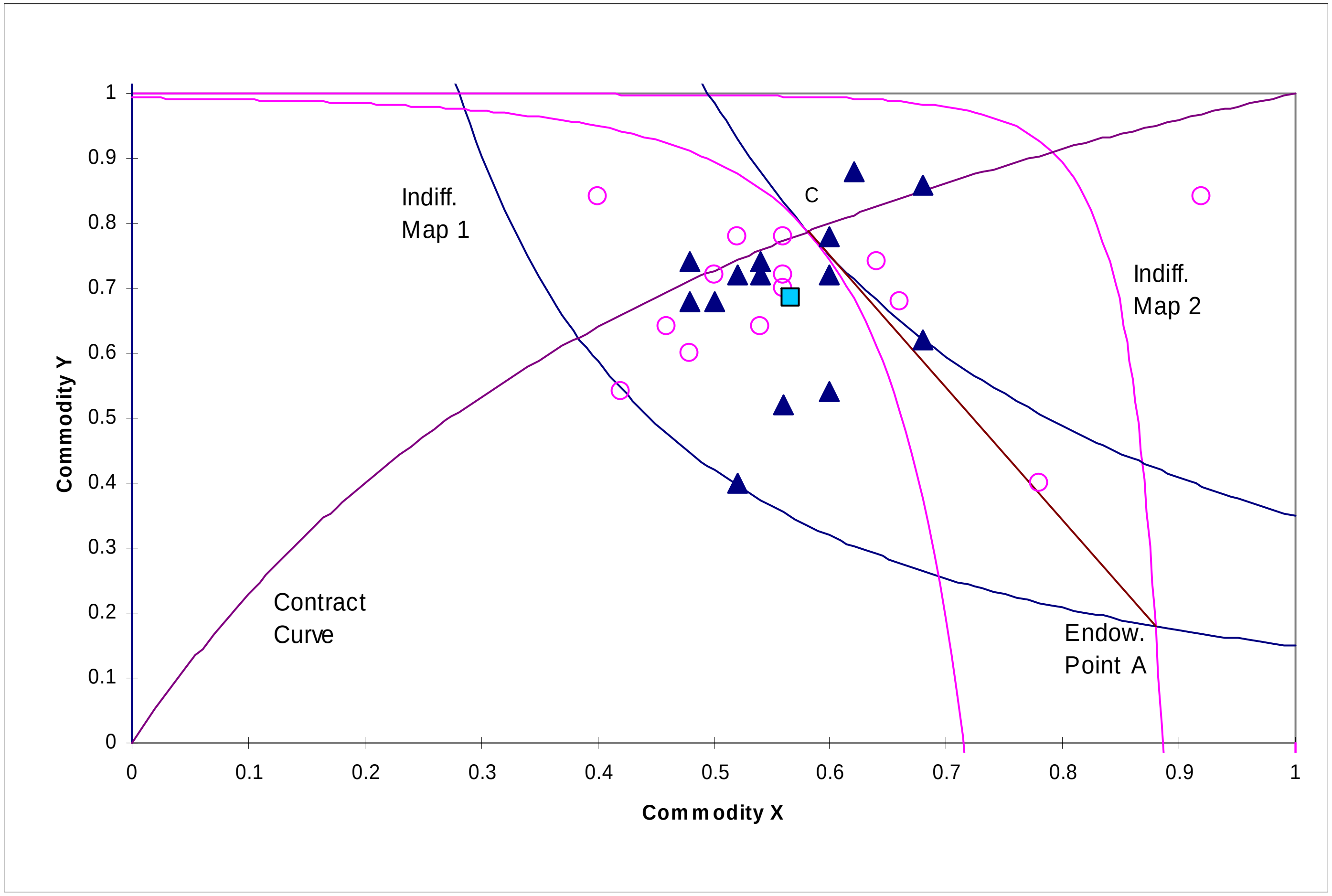

Figure 3: Time Series of Prices (Green Chips per Red Chip) Reported in Edgeworth Box Human Trader Experiment (Competitive Equilibrium Price = 0.49, Average Price = 0.64, 55 transactions among 14 pairs of traders)

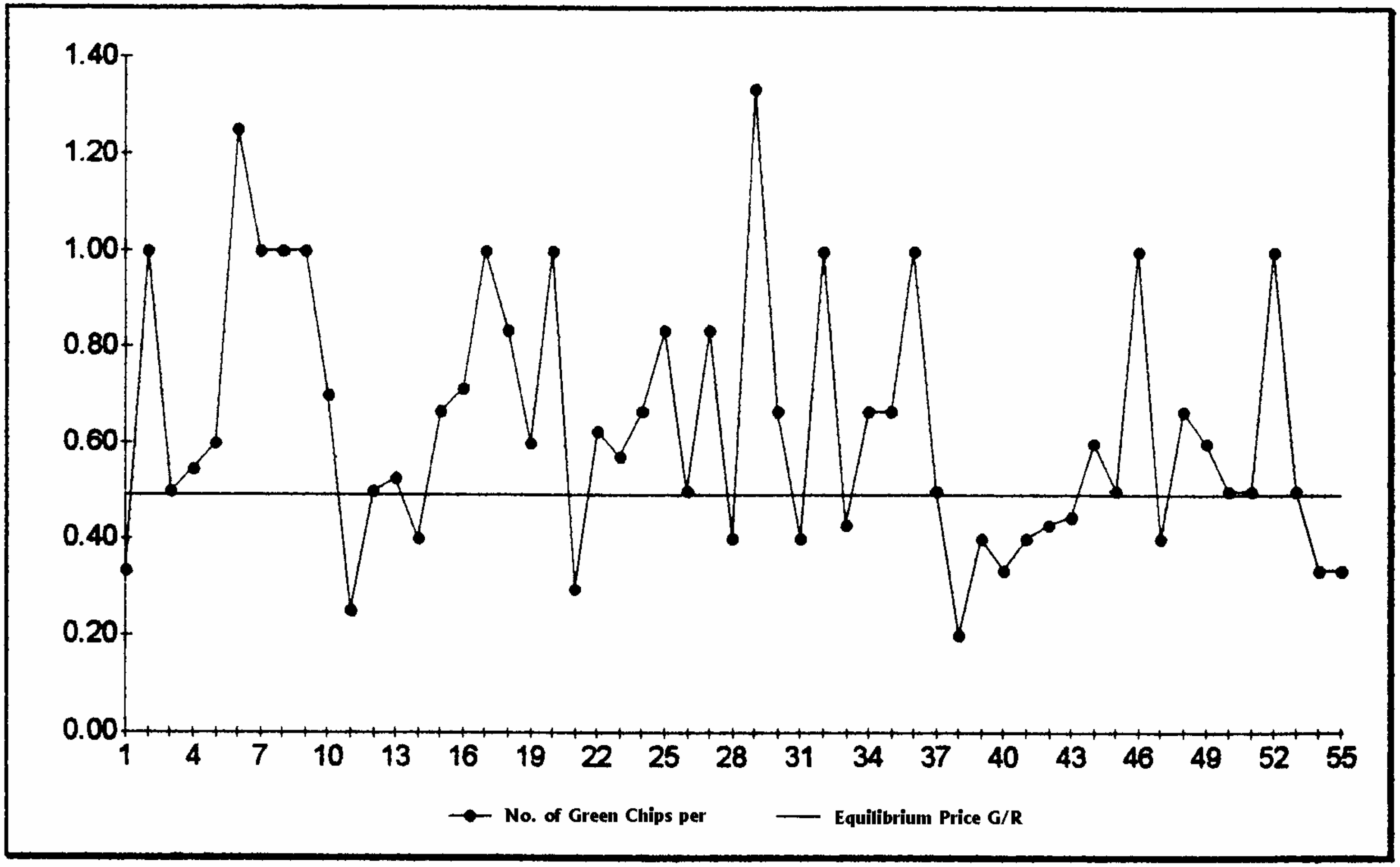

Figure 4: Zero-Intelligence Bids and Asks

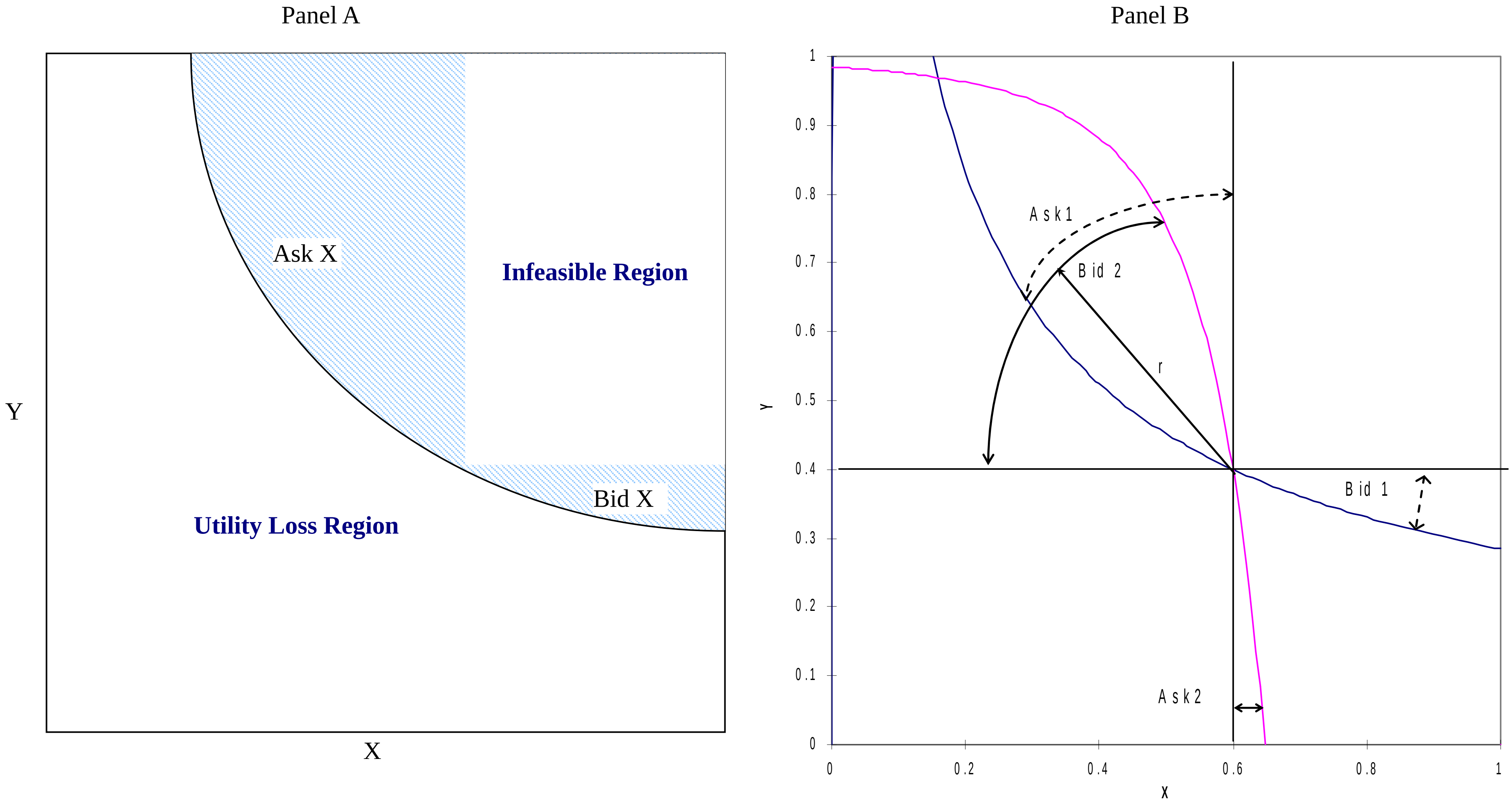

Figure 5: Four Computational Economies for Double Auctions with Zero-Intelligence Traders

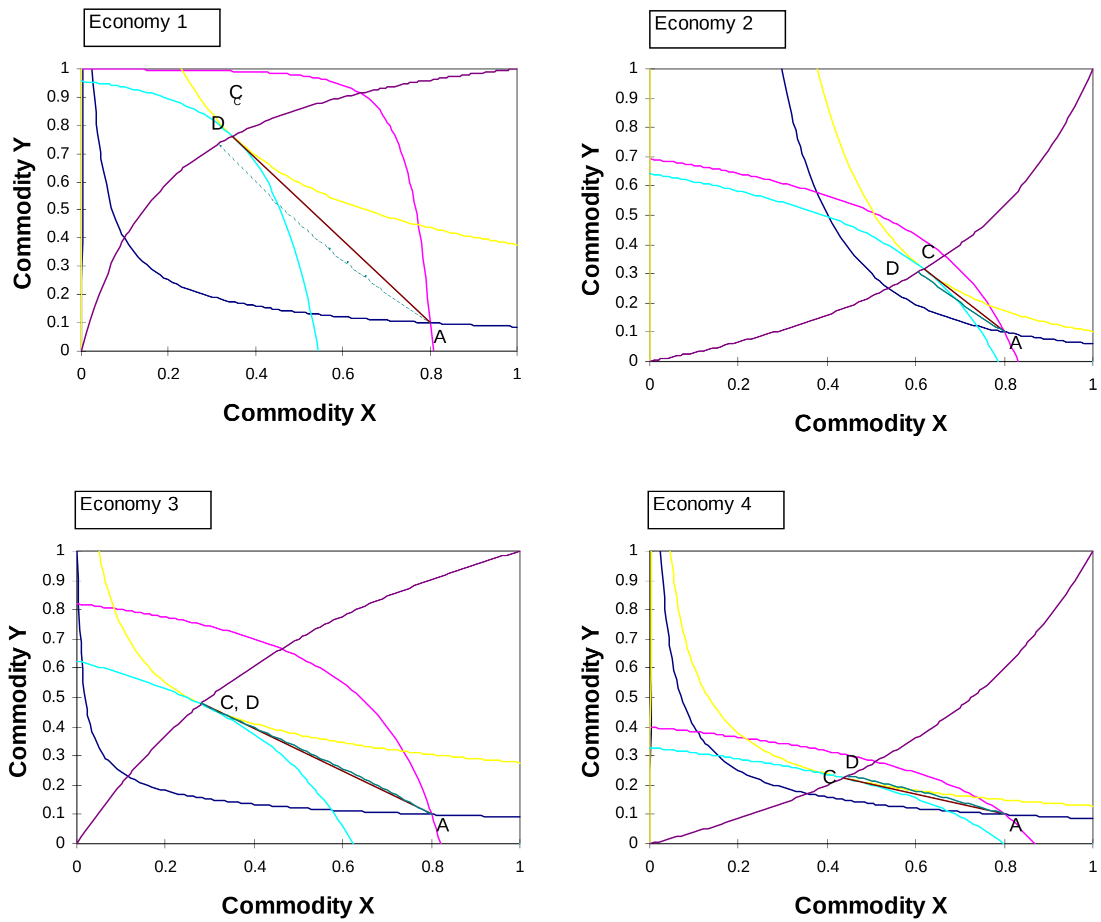

Figure 6: Endowment Paths from Initial to Contract Curve in Four Computational Economies

(Double Auctions with Zero-Intelligence Traders)

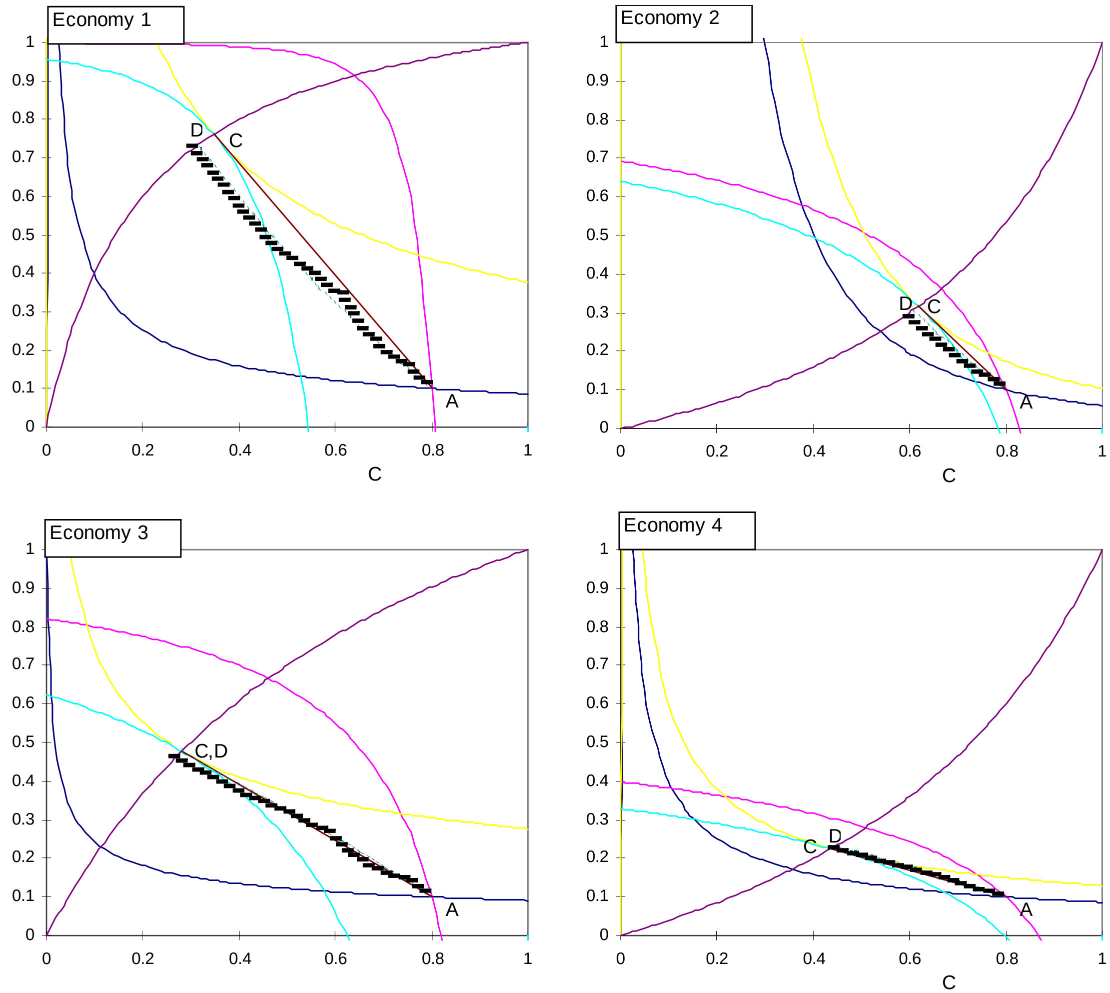

Figure 7: Movement of the Economy to the Contract Curve in a Double Auction

(Not Drawn to Scale)

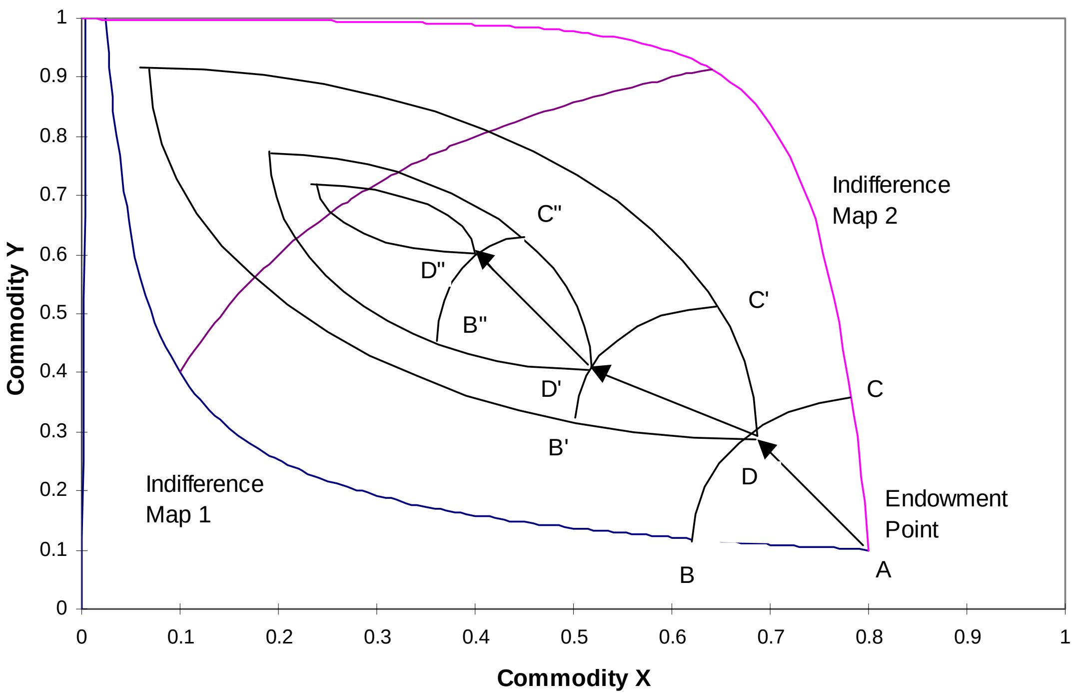

Figure 8: Distribution of End Points of the Endowment Paths around Contract Curves in Four Computational Economies (Double Auctions with ZI Traders)

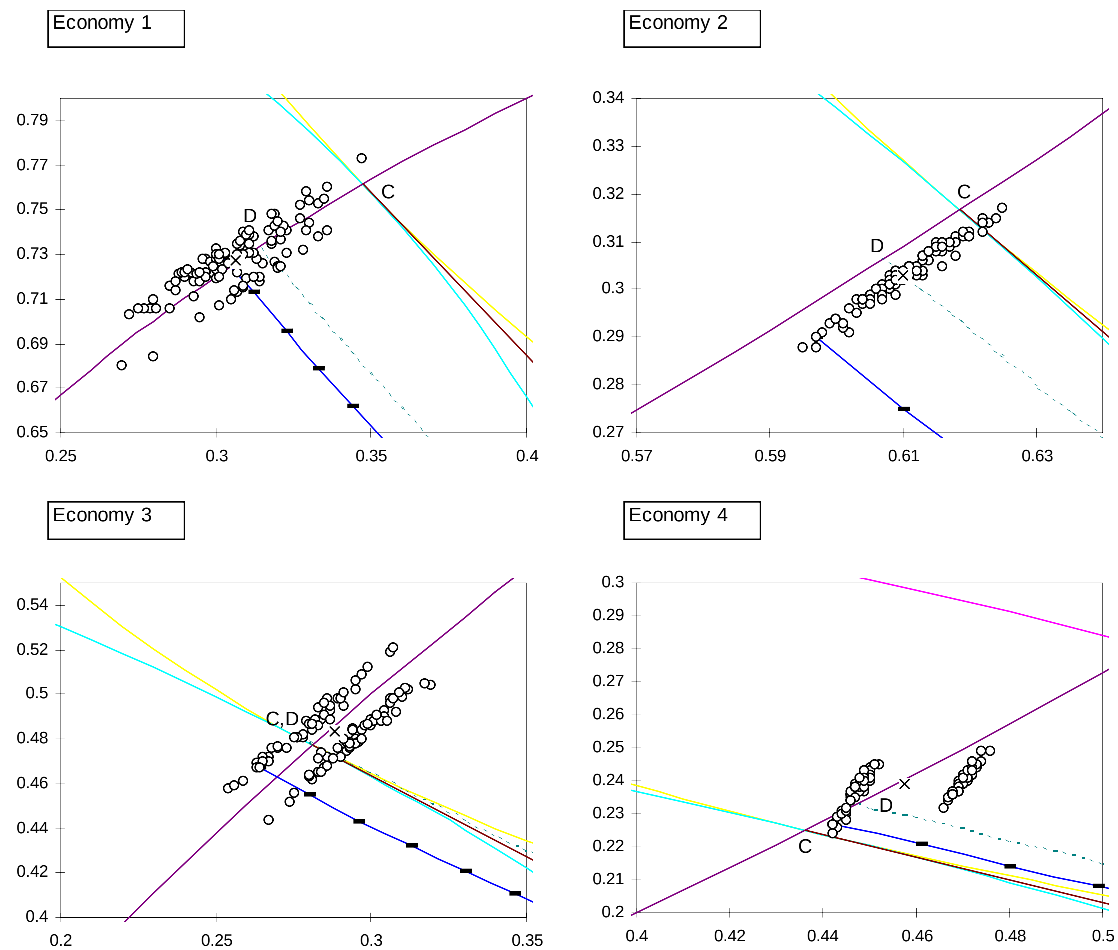

Figure 9: Transaction Price Paths including Mean and Standard Deviation of Prices in Four Computational Economies (Double Auctions with ZI Traders)