YALE INTERNATIONAL CENTER FOR FINANCE

Yale ICF Working Paper No. 05-11
May 2005

HUMAN CAPITAL, ASSET ALLOCATION, AND LIFE INSURANCE

Roger G. Ibbotson, Yale School of Management, Yale University
Peng Chen, Ibbotson Associates
Moshe Milevsky, Schulich School of Business, York University
Xingnong Zhu, Ibbotson Associates

Human Capital, Asset Allocation, and Life Insurance

Peng Chen, Roger G. Ibbotson, Moshe A. Milevsky, Xingnong K. Zhu[1]

Version: January 25, 2005

Abstract

Financial planners and advisors have recently started to recognize that human capital must be taken into account when building optimal portfolios for individual investors. But human capital is not just another pre-endowed asset class that must be included as part of the portfolio frontier. An investor's human capital contains a unique mortality risk, which is the loss of all future income and wages in the unfortunate event of premature death. However, life insurance in its various guises and incarnations can hedge against this mortality risk. Thus, human capital affects both the optimal asset allocation and the optimal demand for life insurance. Yet historically, asset allocation and life insurance decisions have consistently been analyzed separately both in theory and practice. In this paper, we develop a unified framework based on human capital in order to enable individual investors to make both decisions jointly. We investigate the impact of the magnitude of human capital, its volatility, and its correlation with other assets as well as bequest preferences and subjective survival probabilities on the optimal portfolio of life insurance and traditional asset classes. We do this through five case studies that implement our model. Indeed, our analysis validates some intuitive rules of thumb but provides additional results that are not immediately obvious.

[1] Peng Chen is Chief Investment Officer at Ibbotson Associates, Chicago. Roger G. Ibbotson is professor of finance at Yale School of Management, New Haven, Connecticut. Moshe Milevsky is Executive Director, The IFID Centre &Associate Professor of Finance at Schulich School of Business, York University, Toronto, Canda. Xingnong Zhu is Senior Research Consultant at Ibbotson Associates Chicago.

1. Introduction

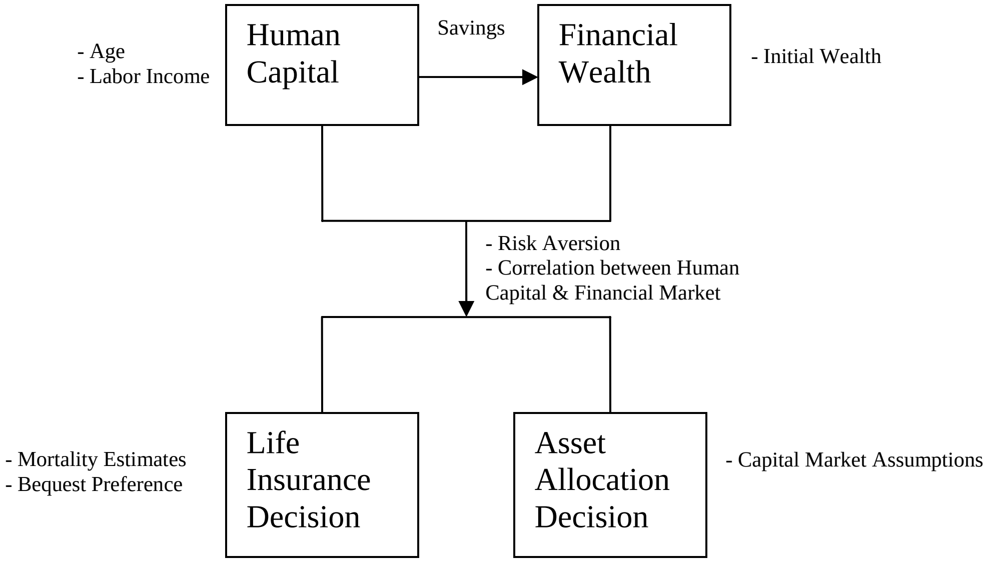
There is growing recognition amongst academics and practitioners that the risk and return characteristics of human capital—such as wage and salary profiles—should be taken into account when building portfolios for individual investors. Merton (2003) points out the importance of including the magnitude of human capital, its volatility, and its correlation with other assets into asset allocation decisions from a personal risk management perspective. The employees of Enron and WorldCom suffered an extreme example of this risk. Their labor income and their financial investment in the company provided no diversification, and they were heavily impacted by their company's collapse.[2]

One unique aspect of an investor's human capital is mortality risk, the loss of human capital in the unfortunate event of premature death. Life insurance has long been used to hedge against mortality risk. Typically, the greater the value of human capital, the more life insurance the family demands. Intuitively, human capital not only affects the optimal asset allocation, but also the optimal life insurance demand. However, these two important financial decisions—the demand for life insurance and the optimal asset allocation—have consistently been analyzed separately in theory and practice. We find few references in either the risk and insurance literature or the investment and finance literature on the importance of considering these decisions jointly, within the context of a life cycle model of consumption and investment. In other words, popular investment and financial planning advice regarding how much life insurance one should acquire is seldom framed in terms of the riskiness of one's human capital. And, conversely, the optimal asset allocation decision is only lately being framed in terms of risk characteristics of human capital, and rarely is it integrated with life insurance decisions.

Motivated by the need to integrate these two decisions, our paper merges these traditionally distinct lines of thought together in one framework. We argue that these two decisions must be determined jointly since they serve as risk substitutes when viewed from an individual investor's portfolio perspective. Life insurance is a perfect hedge for human capital in the event of death; i.e., term life insurance and human capital have a negative 100 percent correlation with each other in the live vs. dead states. If one pays off at the end of the year the other does not, and vice versa. Thus, the combination of the two provides great diversification to an investor's total portfolio. The diagram below illustrates the types of decisions the investor faces, along with the variables that impact the decisions.

[2] Benartzi (2001) showed that many investors invest heavily into the stock of the company they work for.

Diagram 1: Human Capital, Asset Allocation, and Life Insurance



We develop a unified model to provide practical guidelines for developing the optimal asset allocation and life insurance decisions for individual investors in their pre-retirement years (accumulation stage).[3] The remainder of this paper is organized as follows. The next section reviews the existing literature of asset allocation and human capital as well as the literature on the demand for life insurance over the human life cycle. Section 3 presents our model and the critical variables. Section 4 provides a number of case studies—or idealized scenarios—under which we illustrate various model allocations depending on income, age, and tolerance for financial risk. Section 5 provides summary and concluding remarks.

2. Human Capital and Financial Capital

An investor's total wealth consists of two parts. One is readily tradable financial assets; the other is human capital. Human capital is defined as the present value of an investor's future labor income.[4] Economic theory predicts that investors make asset allocation and life insurance purchase decisions to maximize their lifetime utilities of wealth and consumption. Both of these decisions are closely linked to human capital. Although human capital is not readily tradable, it is often the single largest asset an investor has. Typically, younger investors have far more human capital than financial capital. This is because younger investors have more years to work and they have had few years to save and accumulate financial wealth. Conversely, older investors tend to have more financial capital than human

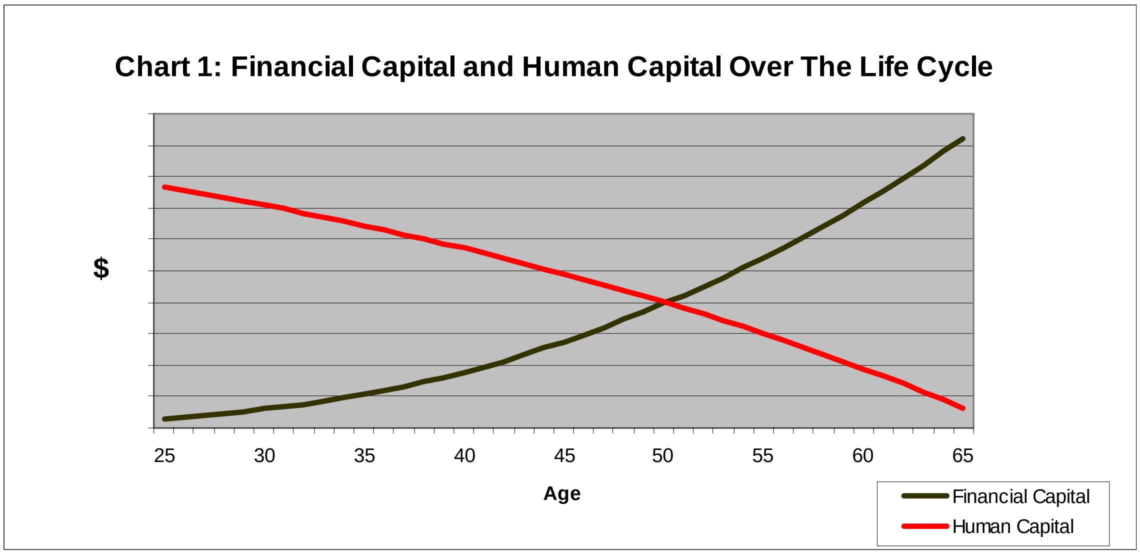
[3] How much an investor should consume or save is another important decision that is frequently tied to the concept of human capital. In this paper, we are concentrating only on the asset allocation and life insurance decisions; therefore, we simplify our model by assuming that the investor has already decided how much he will consume/save. Our numerical cases assume that the investor saves a constant 10 percent of their salary each year.

[4] The term "human capital" often conveys a number of different and at times conflicting concepts in the literature. In this paper, we define human capital to be the financial economic value of all future wages, which is a scalar quantity and dependent on a number of subjective or market equilibrium factors. The appendix provides a detailed explanation of human capital and discounted present value of future salaries.

capital, since they have fewer years ahead to work but have accumulated financial capital over a long career.

Chart 1 illustrates the amounts of financial capital and human capital over an investor's working years (pre-retirement) from age 25 to age 65. When the investor is young, his human capital far outweighs his financial capital. As the investor gets older, the investor will continue to make savings contributions and, with the returns from the existing financial portfolio, the amount of financial capital will increase.

Chart 1: Expected Financial Capital and Human Capital over the Life Cycle



2.1 Financial Asset Allocation and Human Capital

The changing mix of financial capital and human capital over the life cycle impacts financial asset allocation. In the late 1960s, economists established models that implied that individuals should optimally maintain constant portfolio weights throughout their life cycle (Samuelson 1969, Merton 1969). Those models assumed investors have no labor income (i.e., human capital). When labor income is included in the portfolio choice model, individuals will optimally change their allocation of financial assets in a pattern related to the life cycle. In other words, the optimal asset allocation depends on the risk-return characteristics and the flexibility of the labor income (such as how much or how long the investor works). In our model, the investor adjusts the financial portfolio to compensate for the non-tradable human capital risk exposures (e.g., Merton (1971), Bodie, Merton, and Samuelson (1992), Heaton and Lucas (1997), Jaganathan and Kocherlacota (1998), and Campbell and Viceira (2002)). The key theoretical implications are: 1) young investors will invest more in stocks than older investors; 2) investors with safe labor income (thus safe human capital) will invest more of their financial portfolio into stocks; 3) investors with labor income highly correlated with stock markets will invest their financial assets into less risky assets; and 4) the ability to adjust labor supply (i.e., higher flexibility) also increases an investor's allocation toward stocks. However, empirical studies show that most investors do not efficiently diversify their financial portfolio considering the risk of their human capital. Benartzi (2001) and Benartzi and Thaler (2001) showed that many investors use primitive methods to determine the asset allocation and many of them invest very heavily into the stock of the company they work for.[5]

2.2 Life Insurance and Human Capital

Many researchers have pointed out that the lifetime consumption and portfolio decision models[6] need to be expanded to take into account lifetime uncertainty (or mortality risk). Yaari (1965) is considered the first classical paper on this topic. Yarri pointed out ways of utilizing life insurance and life annuities to insure against lifetime uncertainty. He also derived conditions under which consumers would fully insure against lifetime uncertainty.[7]

Theoretical studies show a clear link between the demand for life insurance and the uncertainty of human capital. Campbell (1980) argues that for most households labor income uncertainty dominates financial capital income uncertainty. He further developed solutions to the optimal amount of insurance a household should purchase based on human capital uncertainty.[8] Buser and Smith (1983) model life insurance demand in a portfolio context using mean-variance analysis. They derive the optimal insurance demand and the optimal allocation between risky and risk-free assets. They find that the optimal amount of insurance depends on two components: the expected value of human capital and the risk-return characteristics of the insurance contract. Ostaszewski (2003) further states that life insurance is the business of human capital securitization—addressing the uncertainties and inadequacies of an individual's human capital. On the other hand, empirical studies on life insurance adequacy have shown that underinsurance is prevalent.[9] Gokhale and Kotlikoff (2002) argue that questionable financial advice, inertia, and the unpleasantness of thinking about one's death are the likely causes.

3. Description of the Model

In order to merge asset allocation and human capital with the optimal demand for life insurance, we need to have a solid understanding of the actuarial factors that impact the pricing of a life insurance contract. Note that although there are a number of life insurance product variations such as term life, whole life, or universal life—each worthy of their own financial analysis—we will focus exclusively on the most fundamental type: namely the *one-year renewable term* policy.[10] On a basic economic level, a one-year renewable term policy premium is paid at the beginning of the year—or on the individual's birthday—and protects the human capital of the insured for the duration of the year. If the insured person dies within that year, the insurance company pays the face value to the beneficiaries, soon after

---

[5] Heaton and Lucas (2000) showed that wealthy households with high and variable business income invest less in the stock market than other similar wealthy households, which is consistent with the theoretical prediction.
[6] E.g., Samuelson (1969), Merton (1969).
[7] Like Yaari, Fischer (1973) also pointed out that these earlier models either dealt with an infinite horizon or took the date of death to be known with certainty.
[8] Economides (1982) argued that under a corrected model, Campbell's approach underestimated the optimal amount of insurance coverage. Our model takes this correction into consideration.
[9] E.g., Auerbach and Kotlikoff (1991).
[10] One-year renewable term life insurance is used throughout this paper. The appendix provides a description of the pricing mechanism of the insurance policy. It is beyond the scope of this paper, but we believe that all other types of life insurance policies are financial combinations of term life insurance with investment accounts, added tax benefits, and embedded options.

the death or prior to the end of the year. Next year the contract is guaranteed to start anew with new premium payments made and protection received; hence the word *renewable*.

In following part of this section we provide a general overview on how to think about the joint determination of the optimal asset allocation and prudent life insurance holdings.[11] We assume there are two asset classes. The investor can allocate financial wealth between a risk-free and a risky asset (i.e., bond and stock). Also, the investor can purchase a term life insurance contract that is renewable each period. The investor's objective is to maximize the overall utility, which includes utility from the alive state and the dead state. The investor decides the life insurance demand (the face value of a term life insurance) and the asset allocation between risk-free and risky asset.[12] The optimization problem is expressed in detail with equation (4) in the appendix.

The model is inspired by Campbell (1980) and Buser and Smith (1983). We extend their framework in a number of important directions. First, we link the asset allocation decision with the life insurance purchase decision into one framework by incorporating human capital. Second, we specifically take into consideration the impact of the bequest motive on asset allocation and life insurance.[13] Third, we explicitly model the volatility of labor income and its correlation with the financial market. Fourth, we also model one's subjective survival probability.

Human capital is the central component that links both decisions. An investor's human capital can be viewed as a "stock" if both the correlation to a given financial market sub-index and the volatility of the labor income are high. It can be viewed as a "bond" if both the correlation and the volatility are low. In between these two extremes, human capital is a diversified portfolio of stocks and bonds, plus idiosyncratic risk.[14] We are quite cognizant of the difficulties involved in calibrating these variables—as pointed out by David and Willen (2000)—and we rely on some of their parameters for our case numerical examples in the following section.

There are several important implications from the model. First, the model clearly shows that both asset allocation and life insurance decisions affect an investor's overall utility, and they should be made jointly.[15] The model also shows that human capital is the central factor. The impacts of human capital on asset allocation and life insurance decisions are mostly consistent with the existing literature (e.g., Campbell and Viceira (2002) and Campbell

---

[11] The appendix contains a more detailed specification, which is the basis of our numerical examples and case studies in the subsequent section.

[12] We assume that the investor makes asset allocation and insurance purchase decisions at the start of each period. Labor income is also received at the beginning.

[13] Bernheim (1991) and Zietz (2003) show that the bequest motive has a significant impact on life insurance demand.

[14] Note that when we make statements such as: "This person's human capital is 40% long-term bonds, 30% financial services, and 30% utilities," we mean that the unpredictable shocks to future wages have a given correlation structure with the named sub-indices. Thus, for example, a tenured university professor could be considered to be a 100% real-return (inflation linked) bond, since shocks to wages—if there are any—would not be linked to any financial sub-index.

[15] The only scenarios in which the asset allocation and life insurance decisions are not linked are when the investor derives his/her utility 100% from consumption or 100% from bequest. Both are extreme scenarios, especially the 100% from bequest.

(1980)). One of our major enhancements is the explicit modeling of correlation between the shocks to labor income and financial market returns. The correlation between income and risky asset returns plays a very important role in both decisions. All else being equal, as the correlation term between shocks to income and risky asset increases, the optimal allocation to risky assets will decline and so will the optimal quantity of life insurance. While the former result might be intuitive from a portfolio theory perspective, we provide precise analytic guidance on how this should be implemented. Furthermore, and contrary to intuition, we show that a higher correlation with any given sub-index reduces the demand for life insurance. This is because the higher the correlation, the higher the discount rate used to compute human capital based on future income. A higher discount rate implies a lower human capital valuation; thus, less insurance demand.

Second, the asset allocation decision affects well-being in both the alive consumption state and the dead bequest state, while the life insurance decision mostly affects the bequest state. Bequest preference is arguably the most important factor other than human capital when evaluating the life insurance demand.[16] Investors who weight bequest more (higher D) are likely to purchase more life insurance. Another unique aspect of our model is the consideration of subjective survival probability ($1-\overline{q}$); it can be seen intuitively that investors with low subjective survival probability will tend to buy more life insurance. This adverse selection problem is well-documented in the insurance literature.[17]

Other implications are consistent with the existing literature. For example, our model implies that everything else being equal, the greater the financial wealth, the lower the life insurance demand. More financial wealth also indicates a more conservative portfolio when human capital is more like a "bond." When human capital is more like a "stock," more financial wealth indicates more aggressive portfolios. Naturally, risk tolerance also has a strong impact on the asset allocation decision. We find that investors with less risk tolerance will invest conservatively and buy more life insurance. These implications will be further illustrated in the case studies presented in the next section.

4. Case Studies

To understand the predictions of the model, we analyze the optimal asset allocation decision and the optimal life insurance coverage for five different cases. We solve the problem via simulation; the detailed solving process is presented in the appendix.

We assume there are two asset classes in which the investor can invest his/her financial asset. Table 1 provides the capital market assumptions used in all five cases. We also assume that the investor is male. His preference toward bequest is one-fourth of his preference toward consumption in the live state, $1-D=0.8$ and $D = 0.2$. He is agnostic about his relative health status, i.e., his subjective survival probability is equal to the objective actuarial survival probability. His income is expected to grow with inflation, and the volatility of the

[16] A well-designed questionnaire could help elicit the individual's attitude towards bequest, even though a precise estimate may be hard to obtain.

[17] The actuarial mortality tables can be taken as a starting point. Life insurance is already priced to take into account the adverse selection.

growth rate is 5 percent.[18] His real annual income is $50,000, and he saves 10 percent each year. He expects to receive a pension of $10,000 each year (in today's dollars) when he retires at age 65. His current financial wealth is $50,000. The investor is assumed to follow the constant relative risk aversion (CRRA) utility with a risk aversion coefficient (γ). Finally, the financial portfolio is assumed to be rebalanced and the term life insurance contract is renewed annually.[19] These assumptions remain the same for all cases. Other parameters such as initial wealth will be specified in each case.

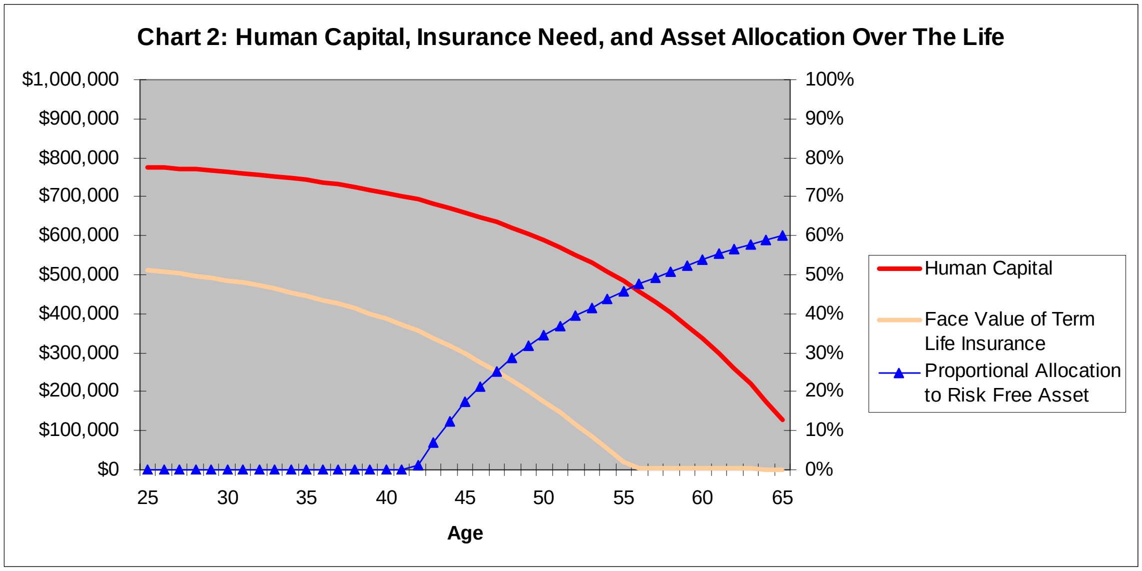
Table 1. Capital Market Return Assumptions

	Compounded Annual Return	Risk (Standard Deviation)
Risk-Free (Bonds)	5%	-
Risky (Stocks)	9%	20%
Inflation	3%	-

<u>Case #1: Human capital, financial asset allocation, and life insurance demand over lifetime</u>

In this case, we assume that the investor has a moderate risk aversion (relative risk aversion of 4). Also, the correlation between the investor's income and the market return (risky asset) is assumed to be 0.20.[20] For a given age, the amount of insurance the investor should purchase can be determined by his consumption/bequest preference, risk tolerance, and financial wealth. His expected financial wealth, human capital, and the derived optimal insurance demand over the investor's life (from age 25 to 65) are presented in Chart 2.

Chart 2: Human Capital, Insurance Demand, and Financial Asset Allocation over the Life



Several results are worth noting. First, human capital gradually decreases as the investor gets older and the remaining number of working years gets smaller. Second, the amount of

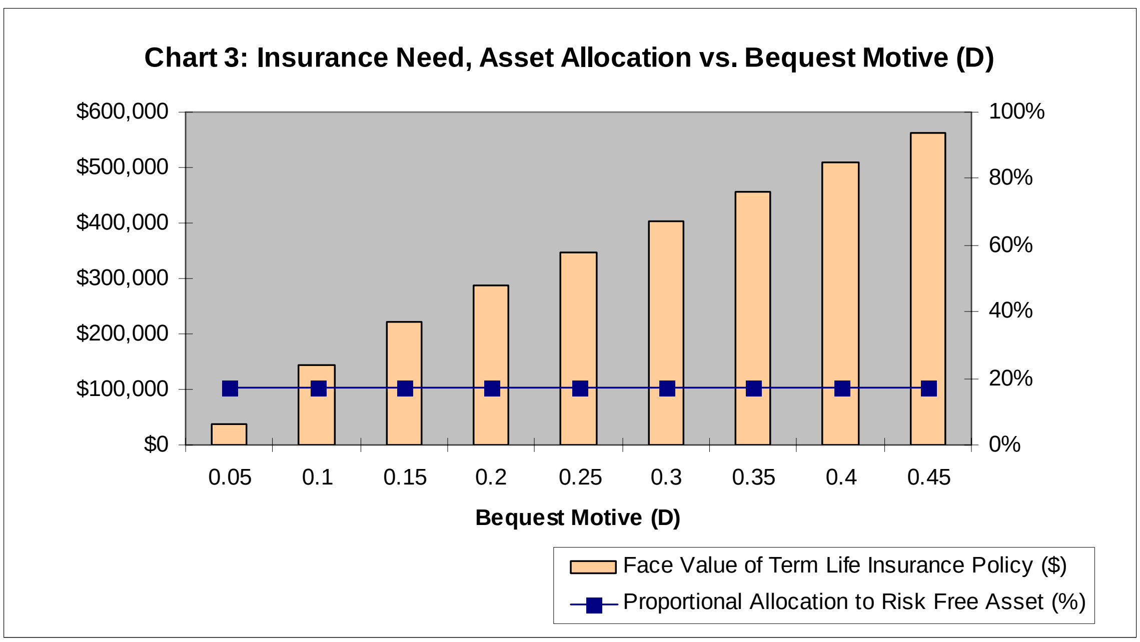
[18] The salary growth rate and the volatility are chosen mainly to show the implications of the model. They are not necessarily representative.
[19] The mortality and insurance loading is assumed to be 12.5%.
[20] Davis and Willen (2000) estimated the correlation between labor income and equity market returns using the Current Occupation Survey. They find that correlation between equity returns and labor income typically lies in the interval from –0.10 to 0.20.

financial capital increases as the investor ages; this is the result of growth of existing financial wealth and additional savings the investor makes each year. The allocation to risky asset decreases as the investor ages. This result is due to the dynamic between human capital and financial wealth over time. When an investor is young, the investor's total wealth is dominated by human capital. Since human capital in this case is less risky than the financial risky asset, young investors will invest more financial wealth into risky assets to offset the impact of human capital on the overall asset allocation. As the investor gets older, the allocation to risky assets is reduced as human capital gets smaller. Finally, the insurance demand decreases as the investor ages. This is not surprising, as the primary driver of the insurance demand is the human capital. The decrease in the human capital reduces the insurance demand. In the following cases, we will vary the investor's preference of bequest, risk preference, and existing financial wealth to illustrate their impact on the investor's optimal asset allocation and life insurance purchases.

Case #2: Strength of bequest motive

This case shows the impact of bequest motives on the optimal decisions on asset allocation and insurance demand. In the case, we assume the investor is at age 45 and has an accumulated financial wealth of $500,000. The investor has a moderate risk aversion coefficient of 4. The optimal allocations to the risk-free asset and the optimal insurance demands across various bequest levels are presented in Chart 3.

Chart 3: Optimal Insurance Demand and Asset Allocation across Strength of Bequest

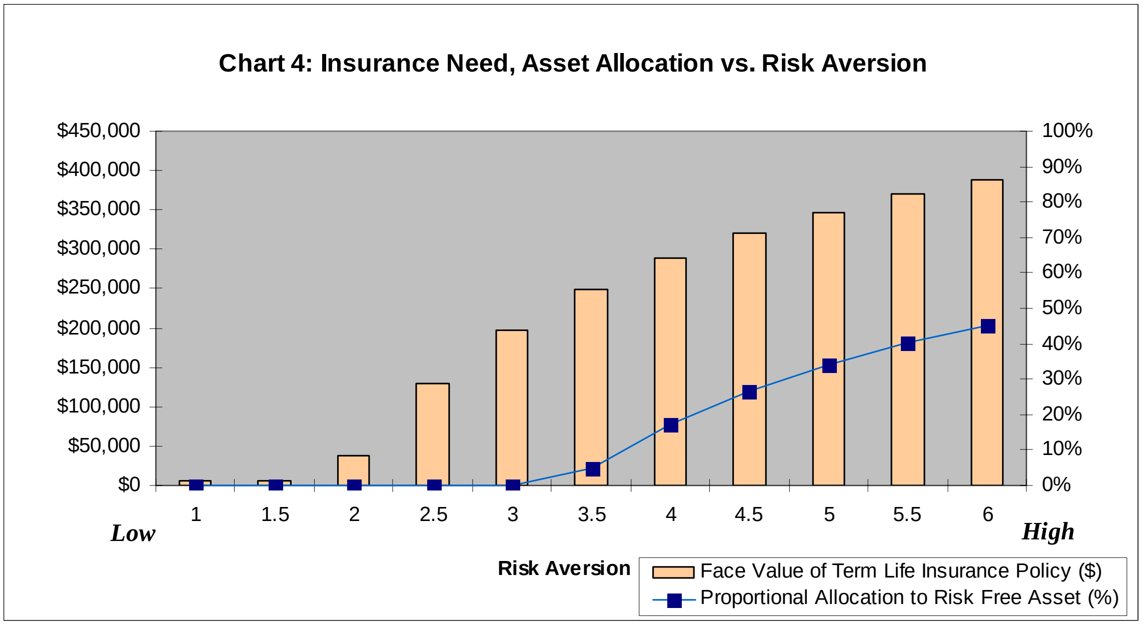


It can be seen that the insurance demand increases as the bequest motive gets stronger, i.e., the D gets larger. This result is expected because an investor with a stronger bequest motive is more concerned about his/her heirs and has the incentive to purchase a larger amount of insurance to hedge the loss of human capital. On the other hand, there is almost no change in the proportional allocation to risk-free asset at different strengths of bequest motive. This indicates that the asset allocation is primarily determined by risk tolerance, returns on risk-free and risky assets, and human capital. This case shows that bequest motive has a strong impact on insurance demand, but little impact on optimal asset allocation.[21]

Case #3: Risk tolerance

The purpose of this case is to show the impact of the different degrees of risk aversion on the optimal decisions on asset allocation and insurance demand. In this case, we again assume the investor is at age 45 and has accumulated a financial wealth of $500,000. The investor has a moderate bequest level, i.e., *D*=0.2. The optimal allocations to risk-free asset and the optimal insurance demands across various risk aversion levels are presented in Chart 4.

Chart 4: Optimal Insurance Demand and Asset Allocation at Different Risk Aversion Levels



As expected, the allocation to the risk-free asset increases with the investor's risk aversion level. This is the classical result in financial economics. Actually, the optimal portfolio is 100 percent in stocks for risk aversion levels less than 2.5. The optimal amount of life insurance has a very similar pattern. The optimal insurance demand increases with risk aversion. For a moderate investor (a CRRA risk aversion coefficient 4), the optimal insurance demand is about $290,000, which is roughly six times the current income of $50,000.[22] Therefore, conservative investors should invest more in risk-free assets and buy more life insurance, compared to aggressive investors.

Case #4: Financial wealth

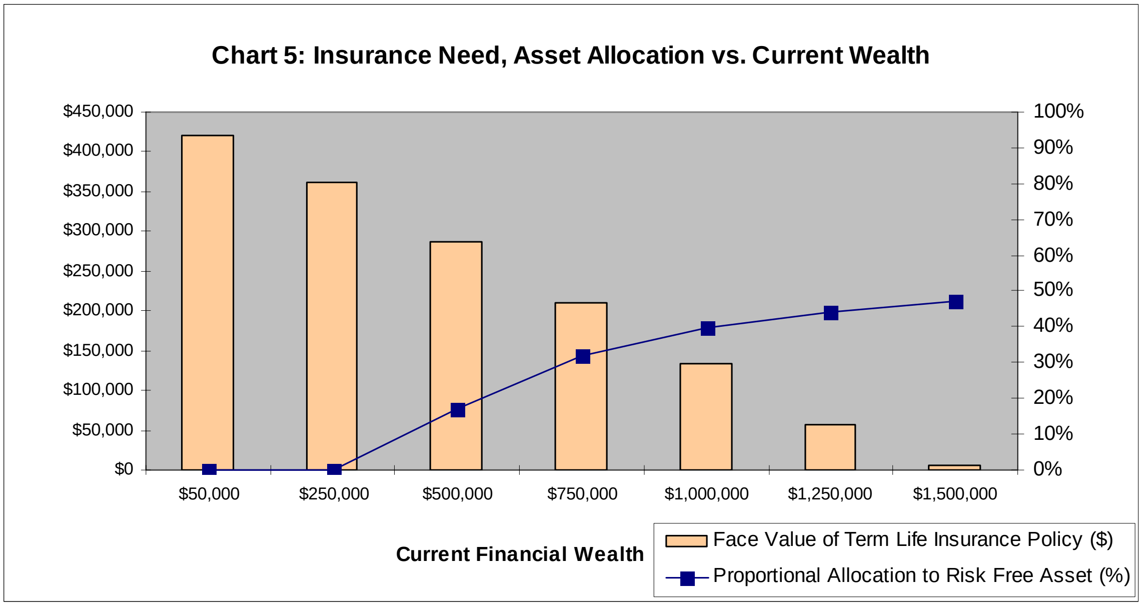
The purpose of this case is to show the impact of the different amounts of current financial wealth on the optimal asset allocation and insurance demand. We hold the investor's age at

[21] In this model, subjective survival probability has similar impact on the optimal insurance need and asset allocation as the bequest motive (D). When subjective survival probability is high, the investor will buy less insurance.

[22] This result is very close to the typical recommendation by financial planners; i.e., purchase a term life insurance policy that has a face value four to seven times one's current income. See, for example, Todd (2004).

45 and the risk preference and the bequest motive at the moderate levels (a CRRA risk aversion coefficient 4 and bequest level 0.2). The optimal asset allocations to risk-free asset and the optimal insurance demands for various financial wealth levels are presented in Chart 5.

Chart 5: Optimal Insurance Demand and Asset Allocation at Different Financial Wealth Levels



First, it can be seen that the optimal allocation to the risk-free asset increases with the initial wealth. This may seem inconsistent with the CRRA utility function, since the CRRA utility function implies the optimal asset allocation does not change with the amount of wealth the investor has. However, we need to note that the wealth includes both financial wealth and human capital. In fact, this is a classic example of the impact of human capital on the optimal asset allocation. An increase in financial wealth not only increases the total wealth, but also reduces the percentage of total wealth represented by human capital. In this case, human capital is less risky than the risky asset.[23] When the initial wealth is low, the human capital dominates the total wealth and the allocation. As a result, to achieve the target asset allocation of a moderate investor, say an allocation of 60 percent risk-free asset and 40 percent risky asset, the closest allocation is to invest 100 percent financial wealth in the risky asset, since the human capital is illiquid. With the increase in the initial wealth, the asset allocation is gradually adjusted to approach the target asset allocation a moderate risk-averse investor desires.

Second, the optimal insurance demand decreases with financial wealth. This result can be intuitively explained through the substitution effects between financial wealth and life insurance. In other words, with a large amount of wealth in hand, one has less demand for insurance, since the loss of human capital has a much lower impact on the well-being of one's heirs. The optimal amount of life insurance decreases from over $400,000, when the

[23] In this case, the income has a real growth rate of 0% and a standard deviation of 5%, yet the expected real return on stock is 8% and the standard deviation is 20%.

investor has little financial wealth, to almost zero, when the investor has $1.5 million in financial assets.

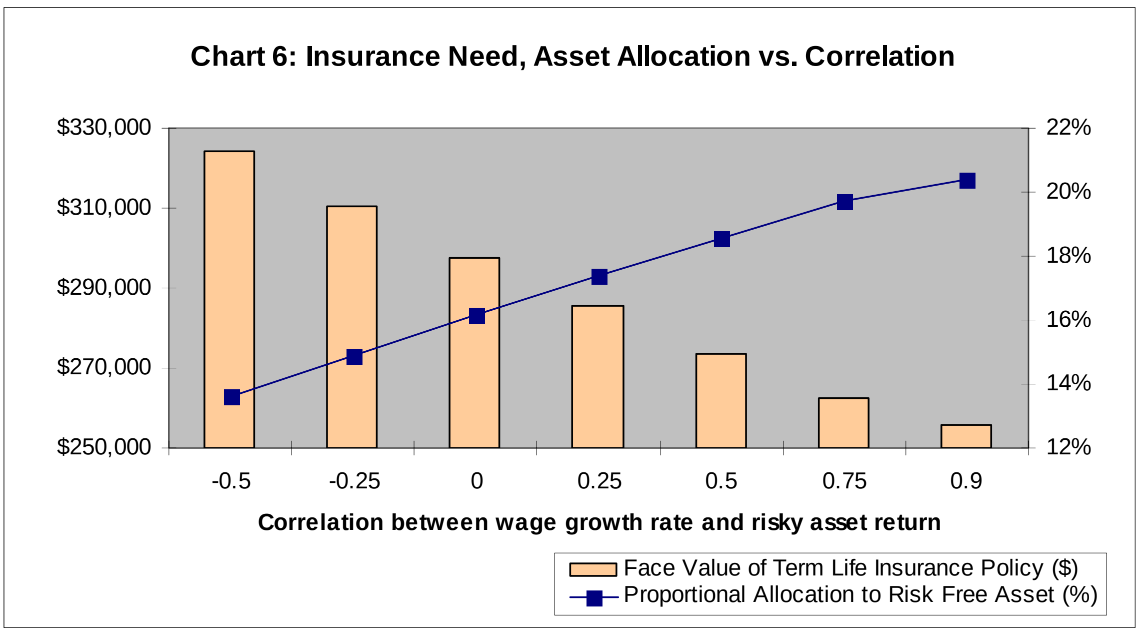
In summary, for a typical investor whose human capital is less risky compared to the stock market, the optimal asset allocation is more conservative and the life insurance demand is smaller for investors with more financial assets.

Case #5: Correlation between wage growth rate and stock returns

In this case, we examine the impact of the correlation between the shocks to wage income and the risky asset returns. In particular, we want to evaluate the life insurance and asset allocation decision for investors with highly correlated income and human capital. This can happen when the investor's income is closely linked to his employer's company stock performance, or where the investor's compensation is highly influenced by the financial market (e.g., the investor works in the financial industry).

Again, we hold the investor's age at 45 and the risk preference and the bequest motive at the moderate level. The optimal asset allocations to the risk-free asset and the optimal insurance demands for various financial wealth levels are presented in Chart 6 below.

Chart 6: Optimal Insurance Demand and Asset Allocation at Different Correlation Levels



The optimal allocation becomes more conservative (i.e., more allocation to risk-free asset) as the income and stock market return become more correlated. One way to look at this is that a higher correlation between the human capital and the stock market results in less diversification, thus a higher risk of the total portfolio (human capital plus financial capital). To reduce this risk, an investor will invest more financial wealth in the risk-free asset. The optimal insurance demand decreases as the correlation increases. Life insurance is purchased to protect human capital for the family and loved ones. As the correlation between the risky (stock) asset and the income flow increases, the *ex ante* value of the human capital to the surviving family becomes lower. Therefore, this lower human capital valuation induces a lower demand for insurance. Also, less money spent on life insurance also indirectly increases the amount of financial wealth the investor can invest. This also allows the investor to invest more in risk-free assets to reduce the risk associated with the total wealth.[24]

In summary, the optimal asset allocation becomes more conservative and the amount of life insurance becomes less, as wage income and the stock market returns become more correlated.

5. Summary and Conclusions

In this paper we have expanded on the Mertonian idea that human capital is a shadow asset class that is worth much more than financial capital early in life, and that it also has unique risk and return characteristics. Human capital—even though it is not traded and highly illiquid—should be treated as part of the endowed wealth that must be protected, diversified and hedged. We have demonstrated that the correlation between human capital and financial capital, i.e., whether you are closer to a bond or a stock, has a noticeable and immediate impact on the demand for life insurance as well as the usual portfolio considerations. Our main argument is that the two decisions—How much life insurance do I need? And where should I invest my money?—cannot be solved in isolation. Rather, they are different aspects of the same problem. For instance, a person whose income heavily relies on commissions should consider his human capital closer to a stock since the income is highly correlated with the market, which results in great uncertainty in his human capital. Consequently, he should purchase less insurance and invest more financial wealth in bonds. Conversely, a tenured university professor who considers her human capital closer to a bond, purchases more insurance, and invests more financial wealth in stocks. We develop a unified human capital-based framework to help individual investors with both decisions. There are several key results: 1) investors need to make asset allocation decisions and life insurance decisions jointly; 2) the magnitude of human capital, its volatility, and its correlation with other assets have a significant impact on the two decisions over the life cycle; 3) bequest preferences and the subjective survival probability have a significant impact on insurance demand, but little influence on optimal asset allocation; and 4) conservative investors should invest more in risk-free assets and buy more life insurance.

We presented five case studies to demonstrate the optimal decisions under different scenarios. Obviously, we have only traced out a rough sketch of the complete picture. More research remains to be done in order to make these decisions more suitable in practice. One possible next step along this holistic integration is to model the various competing types of life insurance as well as their unique tax-sheltered aspects within a unified asset allocation framework. Whole life insurance as well as other forms of variable life insurance can be viewed as a hedge against possible changes in systematic population mortality rates and hence might co-exist in an optimal portfolio with short-term life insurance. Another direction is to diverge from the traditional expected utility models to include other methods, such as minimizing shortfall probabilities, to determine the appropriate asset allocation and life insurance decision.

[24] See Case #3 for a detailed discussion on the wealth impact.

Appendix

In this appendix we describe human capital, the basic pricing mechanism of life insurance, and most importantly the detailed model underlying the numerical results and examples we provided.

1) Human Capital

If we let the symbol *w(i)* denote the random (real, after-tax) wage or salary that a person will receive during time period or year *i*, then the discounted value of this income flow at the current time zero would be represented mathematically by:

$$FHC := \sum_{i=1}^{n} \frac{E[w(i)]}{(1+r+v)^i}, \tag{1}$$

where the expectation in the numerator converts the random variables into a scalar. Note that in addition to taking expectations (under a physical real world measure), the denominator in equation (1) contains the term *v,* which accounts for risk, illiquidity, and other subjective factors that obviously reduce the time-zero value of the expression *FHC.*

And, depending on the investor's education and profession, he/she might be expected to earn the same exact $E[w(i)]$ at each time period *i*, the random shocks to wages: $w(i) - E[w(i)]$ might have very different statistical characteristics *vis a vis* the market portfolio, and thus each of these professions would induce a distinct "risk premium" value for *v* in equation (1) and thus lead to a lower or higher financial economic value for their specific human capital. In addition, the *v* in equation (1) would capture illiquidity and other market imperfections that will further increase the denominator and reduce the value of *FHC.*

Likewise, in the ensuing discussion when we focus attention on the correlation or covariance between human capital and other macro-economic or financial factors, we are of course referring to the correlation between shocks $w(i) - E[w(i)]$ and shocks or innovations to the return generating process in the market. This will induce a (quite complicated) dependence structure between *FHC* in equation (1) and the investor's financial portfolio.

2) The One-year Renewable Term Life Insurance Pricing Mechanism

The one-year renewable term *policy premium* is paid at the beginning of the year—or on the individual's birthday—and protects the human capital of the insured for the duration of the year. If the insured person dies within that year, the insurance company pays the *face value* to the beneficiaries, soon after the death or prior to the end of the year. Next year the contract is guaranteed to start anew with new premium payments made and protection received; hence the word *renewable.*

The policy premium is obviously an increasing function of the desired face value, and the two are related by the simple formula:

$$P = \frac{q}{(1+r)}\theta, \tag{2}$$

The premium *P* is calculated by multiplying the desired face value θ by the probability of death *q*, and then discounting by the interest rate factor *(1+r).* The theory behind equation (2) is the well-known *law of large numbers,* which guarantees that probabilities become percentages when individuals are aggregated. Note the implicit assumption in equation (2) is that although death can occur at any time during the year (or term), the premium payments are made at the beginning of the year and the face values are paid at the end of the year. From the insurance company's perspective, all of the premiums received from the group of N individuals with the same age (i.e., probability of death q) and face value θ, are co-mingled and invested in an *insurance reserve* earning a rate of interest r, so that at the end of the year $PN(1+r)$ is partitioned amongst the qN beneficiaries.

There is no savings component or investment component embedded within the premium defined by equation (2). Rather, at the end of the year the survivors lose any claim to the pool of accumulated premiums, since all funds go directly to the beneficiaries.

As the individual ages and the probability of death q_x increases (denoted by x), the same exact face amount of (face value) life insurance θ will cost more and will induce a higher premium P_x, as per equation (2). Note that in practice, the actual premium is *loaded* by an additional factor denoted by $(1+\lambda)$ to account for commissions, transaction costs, and profit margins and so the actual amount paid by the insured is closer to $P(1+\lambda)$, but the underlying pricing relationship driven by the law of large numbers remains the same.

Also, from a traditional financial planning perspective, the individual conducts a budgeting analysis to determine his or her life insurance demands, i.e., the amount the surviving family and beneficiaries need to replace the lost wages in present value terms. That quantity would be taken as the required face value in equation (2), which would then lead to a premium. Alternatively, one can think of a budget for life insurance purchases, and the face value would be determined by equation (2).

In our model and the ensuing discussion we will "solve" for the optimal age-varying amount of life insurance denoted by θ_x—which then induces an age-varying policy payment P_x—which maximizes the welfare of the family by taking into account its risk preferences and attitudes toward bequest.

3) Model Specification – Optimal Asset Allocation and Insurance Demands

We assume that the investor is currently age x and will retire at age Y. The term retirement is simply meant to indicate that the human capital income flow is terminated and the pension phase begins. We further assume that the financial portfolio will be rebalanced annually and that the life insurance—which is assumed to be of the one-year term variety—will be renewed annually as well. We do not consider tax in our models. The investor would like to know how much (i.e., the face value of term life) insurance he should purchase and what fraction of his financial wealth should be invested in a risky asset (stock).

In the model, an investor determines the amount of life insurance demand, θ_x—the face value of life insurance, a.k.a. the death benefit—together with the allocation α_x to risky asset to maximize the end year utility of total wealth (human capital plus financial wealth) weighted by the alive and dead states. The optimization problem can be expressed as:

$$\max_{\{\theta_x,\alpha_x\}} E\left\{(1-D)\times(1-\bar{q}_x)\times U_{alive}[W_{x+1}+H_{x+1}]+D\times(\bar{q}_x)\times U_{dead}[W_{x+1}+\theta_x]\right\} \quad (3)$$

subject to the budget constraint:

$$W_{x+1}=\left[W_x+h_x-(1+\lambda)\ q_x\theta_x e^{-r_f}-C_x\right]\left[\alpha_x e^{\mu_S-\frac{1}{2}\sigma_S^2+\sigma_S Z_S}+(1-\alpha_x)e^{r_f}\right] \quad (4)$$

$$\theta_0\le\theta_x\le\frac{(W_x+h_x-C_x)e^{r_f}}{(1+\lambda)\ q_x}, \quad (5)$$

and

$$0\le\alpha_x\le 1. \quad (6)$$

Equation (5) requires the cost (or price) of the term insurance policy to be less than the amount of current financial wealth the client has, and there is a minimum insurance amount ($\theta_0>0$) an investor is required to purchase in order to have a minimum protection from the loss of human capital. The symbols, notations, and terminology used in the optimal problem are explained below.

θ_x denotes the amount of life insurance.

α_x denotes the allocation to risky asset.

D denotes the relative strength of the utility of bequest. Individuals with no utility of bequest will have D = 0.

q_x denotes the objective probabilities of death at the end of the year *x*+1 conditional on being alive at age *x*. This probability is determined by a given population, i.e., mortality table.

$\bar{q}_x$ denotes the subjective probabilities of death at the end of the year *x*+1 conditional on being alive at age *x*. ($1-\bar{q}_x$) denotes the subjective probability of survival. Subjective probability of death may be different from the objective probability. In other words, a person might believe he or she is healthier (or less healthy) than average. This would impact the expected utility, but not the pricing of the life insurance, which is based on objective population survival probability.

λ denotes the fees and expenses (i.e., actuarial and insurance loading) imposed and charged on a typical life insurance policy.

W_t denotes the financial wealth at time *t*. We assume there are two assets in the market, one risky and one risk-free. This is consistent with the two-fund separation theorem that is consistent with traditional portfolio theory. Of course, this can always be expanded to multiple asset classes. The return on the risk-free asset is denoted by r_f. The value, S_t, of the risky asset follows a discrete version of a Geometric Brownian Motion.

$$S_{t+1}=S_t\exp\{\mu_S-\frac{1}{2}\sigma_S^2+\sigma_S Z_{S,t+1}). \quad (7)$$

Where, μ_S is the expected return and σ_S is the standard deviation of the return of the risky asset. $Z_{S,t}$ is an independent random variable and $Z_{S,t}\sim N(0,1)$.

h_t denotes the labor income. In our numerical cases, we assume that the income h_t follows a discrete stochastic process specified by

$$h_{t+1} = h_t \exp\{\mu_h + \sigma_h Z_{h,t+1}\}. \tag{8}$$

Where, $h_t > 0$. μ_h and σ_h are the annual growth rate and the annual standard deviation of the income process. $Z_{h,t}$ is an independent random variable and $Z_{h,t} \sim N(0,1)$. Based on equation (8), for a person at age *x*, his income at age *x+t* is determined by:

$$h_{x+t} = h_x \left(\prod_{k=1}^{t} \exp\{\mu_h + \sigma_h Z_{h,k}\} \right). \tag{9}$$

We further assume that correlation between the labor income innovation and the return of risky asset is ρ and

$$Z_h = \rho Z_S + \sqrt{1-\rho^2} Z \tag{10}$$

Where, Z is a standard Brownian motion independent of Z_S. That is,

$$Corr(Z_S, Z_h) = \rho. \tag{11}$$

H_t denotes the present value of future income from age $t+1$ to death. The income after retirement is the payment from pensions. Based on equation (9), for a person at age *x+t*, the present value of future income from age $x+t+1$ to the death is determined by:

$$H_{x+t} = \sum_{j=t+1}^{Y-x} \left[h_{x+j} \exp\{-(j-t)(r_f + \eta_h + \zeta_h)\} \right], \tag{12}$$

Where, η_h is the risk premium (discount rate) for the income process and captures the market risk of income. ζ_h is a discount factor in the human capital evaluation to account for the illiquidity risk associated with one's job. We assumed a 4 percent discount rate per year.[25]

Based on CAPM, the η_h can be evaluated by

$$\eta_h = \frac{Cov(Z_h, Z_S)}{Var(Z_S)} \left[\mu_S - (e^{r_f} - 1) \right] = \rho[\mu_S - (e^{r_f} - 1)] \frac{\sigma_h}{\sigma_S}. \tag{13}$$

Furthermore, we regard the expected value of H_t, i.e., $E[H_{x+t}]$, as the human capital one has at age $x+t+1$.

C_t denotes the consumption at year *t*. For simplicity, we assume that $C_t = C$, i.e., the constant consumption over time.

The power utility function (CRRA) is used in our numerical results. The function form of the utility function we used is,

[25] The 4 percent discount rate translates into about a 25 percent discount on the overall present value of human capital for a 45 year old with 20 years future salary. This 25 percent discount is consistent with empirical evidence on the discount factor between restricted stocks and their unrestricted counterparts (e.g., Amihud & Mendelson (1991)). Also, Longstaff (2002) reported that the liquidity premium for the longer-maturity Treasury bond is 10 to 15 percent of the value of the bond.

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma} \quad (14)$$

for $x > 0$ and $\gamma \neq 1$, and

$$U(x) = \ln(x) \quad (15)$$

for $x > 0$ and $\gamma = 1$. We use the power utility function for both $U_{alive}(\cdot)$ and $U_{dead}(\cdot)$, which are the utility functions associated with the alive and dead states, respectively.[26]

We solve the problem via simulation. We first simulate the values of the risky asset using equation (7). Then, we simulate Z_h through equation (10) to take into account the correlation between the income innovation and the return of financial market. Finally, we use equation (8) to generate income over the same period. Human capital, H_{x+t}, is calculated using equations (9) and (12). If wealth level at age *x*+1 is less than zero, we set the wealth equal to zero. That is, one does not have any remaining financial wealth. We simulate this process *N* times. The objective function is evaluated by:

$$\frac{1}{N}\sum_{n=1}^{N} U_{alive}[W_{x+1}(n) + H_{x+1}(n)] \quad (16)$$

$$\frac{1}{N}\sum_{n=1}^{N} U_{dead}[W_{x+1}(n) + \theta_x] \quad (17)$$

In the numerical examples, we set N = 20000.

[26] Stutzer (2004) pointed out the difficulties in applying the expected utility theories in practice and proposed the use of minimizing short-fall probability as an alternative approach. In this paper, we choose to follow the traditional expected utility model of linking asset allocation and portfolio decisions to individual risk aversion.

References

Ameriks, John and Stephen Zeldes (2001), "How Do household Portfolio Shares Vary with Age?" Working Paper, Columbia University.

Amihud, Yakov and Haim Mendelson (1991), "Liquidity, Asset Prices and financial Policy," *Financial Analysts Journal*, 47, 56-66.

Auerbach, Alan J. and Laurence Kotlikoff (1991), Life Insurance Inadequacy – Evidence From a Sample of Older Widows, National Bureau of Economic Research, Inc, NBER Working Papers: 3765 1991.

Benartzi, Shlomo (2001), "Excessive Extrapolation and the Allocation of 401(k) Accounts to Company Stock," *Journal of Finance* 56, 1747-64.

Benartzi, Shlomo and Richard H. Thaler (2001), "Naive Diversification Strategies in Defined Contribution Saving Plans," *American Economic Review* 91, 79-98.

Bernheim, B. Douglas (1991), "How Strong Are Bequest Motives? Evidence Based on Estimates of the Demand for Life Insurance and Annuities," *Journal of Political Economy* 99, 899-927.

Bodie, Z., R.C. Merton and W.F. Samuelson (1992), "Labor supply flexibility and portfolio choice in a life cycle model," *Journal of Economic Dynamics and Control*, Vol. 16, pp. 427-449.

Campbell, John and Luis Viceira (2002), "Strategic Asset Allocation – Portfolio Choice for Long-term Investors," Oxford University Press.

Campbell, R.A. (1980), "The Demand for Life Insurance: An Application of the Economics of Uncertainty," *The Journal of Finance*, Vol. 35, No. 5, pp. 1155-1172.

Davis, Stephen J. and Paul Willen (2000), "Occupation-Level Income Shocks and Asset Returns: Their Covariance and Implications for Portfolio Choice," University of Chicago Graduate School of Business, Working Paper.

Economides, Nicholas (1982), "Demand for Life Insurance: An Application of the Economics of Uncertainty: A Comment," *Journal of Finance* 37, 1305-09.

Fischer, S. (1973), "A Life Cycle Model of Life Insurance Purchases," *International Economic Review*, Vol. 14, No. 1, pp. 132-152.

Gokhale, Jagadeesh and L. J. Kotlikoff (2002), The Adequacy of Life Insurance, Research Dialogue, TIAA-CREF INSTITUTE, tiaa-crefinstitute.org, issue no. 72 July 2002.

Goldsmith, A. (1983), "Household Life Cycle Protection: Human Capital versus Life Insurance," *The Journal of Risk and Insurance*, Vol. 50, No. 3, pp. 473-486.

Heaton, John and Deborah Lucas (1997), Market Frictions, Savings Behavior, and Portfolio Choice, Macroeconomic-Dynamics. 1997; 1(1): 76-101.

Heaton, John and Deborah Lucas (2000), "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk," *Journal of Finance*. June 2000; 55(3): 1163-98.

Jagannathan, Ravi and N. R. Kocherlakota (1996), Why Should Older People Invest Less in Stocks Than Younger People? Federal Reserve Bank of Minneapolis Quarterly Review. Summer 1996; 20(3): 11-23.

Lewis, F.D. (1989), "Dependents and the Demand for Life Insurance," *The American Economic Review*, Vol. 79, No. 3, pp. 452-467.

Longstaff, Francis A. (2002), The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices, National Bureau of Economic Research, Inc, NBER Working Papers: 9312 2002.

Merton, Robert (1969), "Life Time Portfolio Selection Under Uncertainty: The Continuous Time Case," *Review of Economics and Statistics* 51, 247-257.

Merton, Robert (1971), "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," *Journal of Economic Theory*. Dec. 1971; 3(4): 373-413.

Merton, Robert C. (2003), "Thoughts on the Future: Theory and Practice in Investment Management," *Financial Analysts Journal*, Jan/Feb. 2003, 17-23.

Ostaszewski, K. (2003), "Is Life Insurance a Human Capital Derivatives Business?" *Journal of Insurance Issues*, 2003, 26, 1, pp. 1-14.

Samuelson, Paul (1969), "Life Time Portfolio Selection by Dynamic Stochastic Programming," *Review of Economics and Statistics,* Vol. 51, 239-246.

Smith, Michael L. and Stephen A. Buser (1983), Life insurance in a portfolio context, *Insurance: Mathematics and Economics 2*; 147-57.

Stutzer, Michael (2004), "Asset Allocation without Unobservable Parameters," *Financial Analysts Journal*, Sep/Oct. 2004, 38-51.

Viceira, Luis M. (2001), "Optimal Portfolio Choice for Long-Horizon Investors with Nontradable Labor Income," *Journal of Finance* 56, 433-70.

Yaari, M.E. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, Vol. 32, No. 2, pp. 137-150.

Zietz, Emily N. (2003), "An Examination of the Demand for life Insurance," *Risk Management and Insurance Review*, Vol. 6, No. 2, 159-191.