Who Should Pay for Bankruptcy Costs?

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Abstract

The fees of professionals (financial advisors, lawyers, accountants) are a substantial fraction of bankruptcy costs. Scholars have considered how best to reduce these costs, but have not considered how they should be allocated among creditors. The allocation issue is important because creditors can spend redistributionally (to violate or uphold absolute priority) and productively (to increase the value of the bankrupt firm). An efficient bankruptcy cost allocation scheme should discourage redistributional and encourage productive creditor spending.

We consider the desirability of various allocation schemes in a model in which senior and junior creditors can engage in both types of spending but the bankruptcy court cannot distinguish productive from rent seeking activities. We suppose that the senior claim is at or in the money. This implies that the seniors have an incentive to spend only to defend their position while the juniors have both good and bad incentives: to spend productively on value improvement because they are residual claimants and to spend redistributionally because they are partly or totally out of the money under absolute priority. A good bankruptcy cost allocation scheme thus should induce the seniors to spend more and the juniors to spend less.

We show: (i) The current US cost allocation system is unsatisfactory because the scheme partially reimburses junior expenses on professionals but does not reimburse senior expenses; (ii) Full reimbursement schemes that impose all costs on one set of parties, such as seniors, juniors or the government are dominated by partial reimbursement schemes, because these can be better tailored to encourage the right and discourage the wrong kind of spending; and (iii) A cost allocation scheme that approaches first best and is implementable would delegate the issue of professionals’ cost reimbursement to the debtor in possession. The incentive of Chapter 11 debtors to survive would induce them to partly reimburse senior spending but not to reimburse junior spending.

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I Introduction

The fees of professionals—lawyers, accountants, investment bankers, financial advisors and turnaround specialists—drain substantial resources from the estates of large bankrupts.\(^1\) The scholarly literature considers how total bankruptcy costs are best reduced, but devotes little attention to how the costs of professionals should be allocated. There are three candidates for bearing these costs: the creditors, the bankrupt estate, or the government. The U.S. Bankruptcy Code implicitly rules out the government but then is almost completely unilluminating with respect to the optimal allocation of costs between the parties and the estate. Instead, the Code confers a large discretion on the bankruptcy courts.\(^2\)

In this paper, we open the subject of how professional costs should be allocated. Creditors\(^3\) spend on professionals for two reasons: juniors to capture a larger part of the estate than the absolute priority rule would otherwise grant to them (seniors to fend off the redistributional efforts of others); and to increase the value of the estate. The cost allocation problem would be trivial if value-increasing and redistributive efforts were separate and courts could distinguish between them: courts then would reimburse only productive creditor efforts. We make the realistic assumption that courts cannot draw this distinction for two reasons. First, the same

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\(^1\)The magnitude of direct professional expenses in bankruptcy can be significant. Warner (1977) finds that the direct costs of bankruptcy—compensation provided to lawyers, accountants, consultants, and expert witnesses—are about 4 percent of the market value of the firm one year prior to the default. Altman (1984) calculates these costs to be about 7.5 percent of firm value using a broader sample of 19 bankrupt companies from 1974–1978. In a sample of 22 firms from 1994, Lubben (2000) calculates the cost of legal counsel in Chapter 11 bankruptcy to represent 1.8 percent of the distressed firm’s total assets, with percentages above 5 in some cases. In the average case, the debtor spends $500,000 on lawyers, and creditors spend $230,000. LoPucki and Doherty (2004) study a sample of 48 cases from 1998 to 2002, mostly from Delaware and New York cases, and report that professional fees were 1.4 percent of the debtor’s total assets at the beginning of the bankruptcy case. Evidence from administrative fees from 105 Chapter 11 cases from the Western District of Oklahoma in Ang, Chua, and McConnell (1982) suggests that administrative fees are about 7.5 percent of the total liquidating value of the bankrupt corporation’s assets. Weiss (1990) and Betker (1997) have similar estimates.

\(^2\)Subsection 330(a)(1) of the Code authorizes the bankruptcy court to compensate professionals by awarding “reasonable compensation for actual, necessary services.” The court, in making its award, is to consider “the nature, the extent and the value of such services taking into account all relevant factors.” These factors include time spent, rates charged, comparable fees for non-bankruptcy cases, and “whether the services were necessary to the administration of, or beneficial...toward the completion of, a case...”

\(^3\)The equity have the same incentives as do junior creditors. Hence, we refer only to juniors and seniors for convenience.
actions can have both productive and redistributitional effects. For example, business continuation may increase firm value, but also can redistribute wealth in favor of the juniors and against the seniors. Continuation increases the value of the juniors’ call option on the firm while liquidation may fully pay off senior claims. Also, efforts by juniors to establish the firm’s value can be productive, because an accurate valuation is necessary to create a viable business plan, but the juniors have an incentive to inflate value to increase their payout. Second, courts usually cannot observe the parties’ production functions for effort, and therefore cannot assess the optimality of the parties’ actions. For these reasons, when bankruptcy courts cannot tell the difference, the public policy problem is to allocate the costs of professionals in such fashion as to reward productive and to dampen redistributional creditor efforts. This turns out to be a difficult problem, and our early effort here tries to isolate the relevant considerations.

The incentive problem that bankruptcy law has to solve is created by the parties’ priority rankings. The seniors have too little incentive to spend on productive activities because, at the inception of a bankruptcy, their claim is partly or totally in the money. The seniors’ primary motive to spend thus is defensive—to fend off the redistributional efforts of the juniors. The juniors, in contrast, have an incentive to spend too much on professionals. Because the juniors are the residual claimants, they have an incentive to spend optimally on value increasing activities; but because junior efforts also decrease the probability that absolute priority will be followed (as we assume), the juniors have an additional incentive to spend redistributionally. The policy task thus is to allocate the costs of professionals such that the seniors spend more, some of which would be productive, and the juniors spend less to curb their socially inefficient rent-seeking.

We show that the three corner solutions are not optimal:

- It is inefficient for the government to pay for all professional expenses because that would subsidize spending on both value increasing activities and redistributional activities. This would increase firm value but reduce social welfare, because subsidies induce excessive professional representation.

- It is inefficient for the firm to pay for all professional expenses because that would permit the seniors to spend the juniors’ money to prevent redistributional efforts by juniors. This is socially wasteful. On the other hand, subsidized senior spending would likely preserve absolute priority, thereby making
the juniors purely residual claimants. Under APR, juniors then invest optimally in value increasing activities.

It is inefficient for creditors to pay their own professional expenses. The seniors’ claim would be partly or totally in the money so they would spend primarily to fend off the redistributational efforts of the juniors, but not enough to increase value. The juniors, on the other hand, would spend both to increase value and to undermine absolute priority. Thus, the seniors would spend too little and the juniors would spend too much.

The failure of the three corner solutions to achieve efficiency suggests that partial reimbursement schemes are best. Any such scheme should partially reimburse only the seniors. This would increase their incentive to invest in value increasing activities. Also, because increased senior spending increases the probability that absolute priority will be followed, subsidizing the seniors would discourage junior redistributational efforts. Regrettably, the precise partial percentage for a senior subsidy to get efficiency is parameter specific; and the parameters likely would be difficult for an administrator, such as a bankruptcy judge, to recover. Thus, the policy problem is to choose among second best schemes.

The current US Bankruptcy Code is perverse, because it is a scheme that does not reimburse seniors but sometimes reimburses juniors. This cost allocation fails

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4The Bankruptcy Code does not authorize the bankruptcy court to compensate the expenses of creditors whom it defines as “senior”. Section 506(b), however, provides that “there shall be allowed to the holder” of a secured claim that is in the money “any reasonable fees, costs, or charges provided for under the agreement under which such claim arose.” The relevant contract is an Article 9 security agreement that is regulated under the Uniform Commercial Code. Section 9-615(a)(1) of the UCC permits the secured party to recover out of “the cash proceeds of disposition under Section 9-610” “the reasonable expenses of retaking, holding, preparing for disposition, processing and disposing, and to the extent provided for by agreement and not prohibited by law, reasonable attorney’s fees and legal expenses incurred by the secured party.” Consistent with the UCC, the leading practical guide to secured lending advises creditor attorneys to ensure that the security agreement creates a “right to recover fees, costs and charges arising from any and all collection efforts...[These include] actions to foreclose on encumbered property, to object to or assist in a trustee’s sale ..., or any other actions to protect its security interest.” Ruda (2003), §16.12(5)(d)(iii) at 16-168.6. The Article 9 security agreement that the Bankruptcy Code enforces thus is not relevant to the concerns of this article because the agreement permits the secured party only to recover legal costs incurred in the course of liquidating the collateral. This article focuses on the Chapter 11 context, in which the collateral remains in the firm, and the article analyzes expert costs generally, not just legal expert costs. Some bankruptcy courts approve reorganization plans that pay a portion of a senior creditor’s expenses if the creditor participates in the reorganization. The extent of this practice is unknown, and the payments cannot influence incentives unless they are foreseen. Regarding junior creditors, a Chapter 11 creditors’ committee is composed of
to encourage the seniors to spend on increasing value and encourages the juniors to spend even more on rent seeking.

Our paper suggests a partial reimbursement scheme that is plainly better. Under our scheme, the debtor in possession is given discretion to reimburse creditor spending on professionals. To the extent that the debtor in possession can be incentivized to maximize the value of the firm, the management would have no incentive to subsidize junior redistributal spending. Any dollar spent by managers on reimbursing creditor professionals would provide a greater increase in value if given to the seniors instead. As a consequence, our scheme does much better than current law at reducing strategic spending by the juniors. But managers will purchase (subsidize) productive senior professional spending to the extent that it enhances firm value. Thus, our scheme also does better than current law at enlisting the seniors in the task of value maximization.

Our scheme will not produce first best for three reasons. First, the managers of a debtor may attempt to maximize private benefits rather than firm value. Fortunately, bad managers are often replaced in bankruptcy, and compensation contracts can create incentives for managers to lead firms out of Chapter 11 (Skeel [2003]). Second, other asymmetric information and agency problems may prevent the firm managers from making first best contracts to secure professional services. Third, the juniors may spend their own money on redistributal efforts, which the managers cannot prevent.

Nevertheless, our scheme responds directly to the actual incentive problems that professional spending in bankruptcy poses, and thus should materially improve efficiency. Also, though our proposal is radical in one way (taking professional reimbursement decisions largely out of the hands of bankruptcy judges), it is traditional in another. The debtor in possession today already has discretion to make many business decisions. We add to this discretion only the ability to take charge of creditors "that hold the seven largest claims against the debtor of the kinds represented on such committee." Bankruptcy Code §1102(b)(1). A creditors' committee "with the court's approval ... may select and authorize the employment of one or more accountants, attorneys, or other agents, to represent or perform services for such committee." These services may include the investigation of "the acts, conduct, assets, liabilities and financial condition of the debtor, the operation of the debtor's business, and the desirability of such business, and any other matter relevant to the case or the formulation of a plan." §§1103(b), 1103(C)(2). The bankruptcy court is authorized to award "reasonable compensation for actual, necessary services rendered by [a] ... professional person" who had been employed under §1103. §330(a)(1)(A) and (B). The Bankruptcy Code thus permits the court to reimburse a substantial portion of the expert expenses that juniors incur.
the creditors’ professional reimbursement process, which will result in choices that enlist professionals more efficiently in the task of value maximization.

Our paper proceeds as follows: Section II provides a numerical example of our results. Section III sets out the model. Section IV derives the first best solution and Section V introduces the influence component. Sections VI and VII analyze the three corner solutions and the current US system. Section VIII works out our proposed solution. Section IX considers a utopian omniscient government financed partial reimbursement scheme and Section X concludes.

II  Numerical Illustration

Suppose the value of a firm is $100 without professional effort. The firm has two creditors: a senior creditor who is owed $40, and a junior creditor who is owed $80. Each creditor can spend a maximum of 20 hours on professionals. Professionals play two roles: they may tilt the court’s final decision on how to split the $100 in their favor; and they may potentially increase the value of the firm.

The professionals’ efforts sway the court in a particular way: if the senior creditor expends enough professional time, APR is upheld;\(^5\) if the senior creditor spends nothing on professionals, then the parties are paid pro-rata; and if both parties spend moderately on professionals, the allocation will be between these two choices.

The extent to which APR is upheld is represented by the parameter $\theta$.

\[
\begin{array}{c|c|c}
\text{PPR:} & \theta = 0 & \text{APR:} \\
S = $33.33 (= 1/3). & S = $40.00. \\
J = $66.67 (= 2/3). & J = $60.00. \\
\end{array}
\]

\[
\theta(s, j) = 1 - \left(\frac{j}{20}\right) \cdot \left[1 - \left(\frac{s}{20}\right)\right], \tag{1}
\]

\(^5\)The basis for this assumption is that the Bankruptcy Code requires the court to follow absolute priority, so calling the court’s attention to its legal duty is likely to be effective.
where 20 is a maximum number of professional hours that a creditor can hire. This function states that if the senior hires \( s = 20 \) hours of professionals, then APR obtains \((\theta = 1)\), regardless of what the junior does. If the senior hires fewer than 20 hours of professionals, then the junior’s professionals can influence the court. For example, if the senior hires zero professionals, and the junior hires 20 hours of professionals, PPR obtains \((\theta = 0)\). If the senior hires 10 hours of professionals, and the junior hires 20 hours of professionals, an outcome halfway between PPR and APR comes about: the senior receives $36.67 and the senior receives the remaining $63.33, as indicated by the small arrow in the figure.

Professionals also can increase the value of the estate. Our professionals increase the value of the firm by \(2 \cdot \sqrt{x}\) dollars if they spend \(x\) hours at work.\(^6\) Professionals are expensive and charge 50 cents per hour. We can now consider the costs and benefits of different professional hiring schemes, summarized in Table 1.

**In the Social First-Best Outcome**, it would not be optimal for a creditor to hire, e.g., 20 hours of professional work, because this costs \(20 \cdot $0.50 = $10.00\). This outweighs the benefit of \(2 \cdot \sqrt{20} = $8.94\). Instead, society would want each creditor to hire 4 hours worth of professionals. Each creditor increases the firm value by $4 \((2 \cdot \sqrt{4})\) at a cost of $2, for a net benefit of $2 per creditor.

**If the Government Reimburses Professionals**, but they are hired by the creditors, each creditor would spend the maximum of 20 hours at a cost of $10. This would improve firm value by $8.94 each. Hence, a full government reimbursement scheme would induce creditors to spend too much. Professionals would reduce social welfare by $2.12.

**If the Firm Reimburses Professionals**, the senior will spend 20 hours, because this ensures that the court will uphold APR and her claim will be fully satisfied. The junior then becomes the residual claimant and fully internalizes both costs and benefits of any professionals he hires. Thus, he will spend the optimal 4 hours of his professionals’ time. The total value of the professionals will be \(2 \cdot \sqrt{4} + 2 \cdot \sqrt{20} = $12.94\), but at a cost of \(4 \cdot $0.50 + 20 \cdot $0.50 = $12\). The professionals’ total social contribution is a positive \(+$0.94\)—which is not as high as the social optimum, but

\(^6\)We assume that it is more efficient for creditors to spend individually, than it is for them to pool their resources and spend jointly, at least on the types of expenses we are considering. We motivate our assumption in more detail below.
higher than if the government reimburses professionals.

**IF CREDITORS PAY FOR THEIR OWN PROFESSIONALS,** the junior will overspend on professionals in order to sway the courts away from APR, thus “attacking” the senior’s claim. Some professional benefits would accrue to the senior because, in equilibrium (see below), the court’s allocation ends up between the absolute priority rule and a pro-rata priority rule. In addition to this participation in the social gain, the senior expenses help defend her claim against the junior’s professionals. The senior’s objective function is $\theta(s, j) \cdot $40 + $[1 - \theta(s, j)] \cdot [1/3 \cdot ($100 + $2\cdot\sqrt{s} + $2\cdot\sqrt{j})] - s \cdot $0.50.

The junior receives relatively more of the residual claim than the senior. Also, each dollar of expense on professionals shifts the court more in his favor. This means that the junior is more eager to hire professionals than the senior. The junior's objective function is $\theta(s, j) \cdot [($100 + $2\cdot\sqrt{s} + $2\cdot\sqrt{j}) - $40] + [1 - \theta(s, j)] \cdot [2/3 \cdot ($100 + $2\cdot\sqrt{s} + $2\cdot\sqrt{j})] - j \cdot $0.50.

In the unique Nash equilibrium, the senior thus hires 0.14 hours of advice; the junior hires 9.11 hours of advice. As a consequence, the court chooses 54.8% APR, 45.2% PPR; the senior receives $37.94, and the junior receives $64.23. The net social professional contribution is therefore $2.16. This is the highest that can be achieved in a non-partial reimbursement system. Still, the problem remains that senior creditors underspend, while junior creditors overspend.

**IN THE CURRENT U.S. SYSTEM,** the court reimburses only juniors.⁷ A naïve and incorrect intuition is that this case is similar to the previous case, because the junior is the residual claimant (so firm reimbursement or self-reimbursement is the same for the junior), and the senior pays for her own expenses. This is incorrect, because when the junior spends a lot on professional advice (and in equilibrium, he does) and the senior does not spend a lot on professional advice (and in equilibrium, she does not), then the court leans away from APR. Thus, under the current U.S. system, junior creditors “morph” from being residual claimants into becoming more pro-rata claimants.⁸

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⁷In the U.S., juniors first request reimbursement, which the court may or may not grant. LoPucki and Doherty (2004) find that, on average, courts reimburse 97.91 percent of the fees and professional expenses sought by creditors.

⁸If the junior can commit himself *ex-ante* not to seek court reimbursement in exchange for better loan terms, the U.S. case becomes like the previous case, in which creditors pay for their own professionals. Table 1 shows that this is an improvement, but not a major one.
In this equilibrium, the junior spends up to the limit, 20 hours, because his own professional expenses are fully reimbursed by the firm. The junior’s objective function is \( \theta(s, j) \cdot \left[ (100 + 2 \cdot \sqrt{s} + 2 \cdot \sqrt{j} - j \cdot 0.50) - 40 \right] + \left[ 1 - \theta(s, j) \right] \cdot \left\{ \frac{2}{3} \cdot (100 + 2 \cdot \sqrt{s} + 2 \cdot \sqrt{j} - j \cdot 0.50) \right\} \). The senior maximizes its payoff with respect to the amount of professional spending it chooses to purchase, holding fixed the level of junior spending. Because the subsidized junior spends the maximum 20 hours, the senior’s objective function becomes \( \theta(s, j) \cdot \left[ 40 + \left[ 1 - \theta(s, j) \right] \cdot \left\{ \frac{1}{3} \cdot (100 + 2 \cdot \sqrt{s} + 2 \cdot \sqrt{j} - j \cdot 0.50) \right\} - j \cdot 0.50 \right] \). The senior thus spends only 2.22 hours (at a cost of $1.11) on professionals, which increases only slightly the probability that APR will be followed.

In equilibrium, the court chooses 11.1% APR, 88.9% PPR; the senior receives $33.53, and the junior receives $67.28. The social net professional benefit from professionals is +$0.82. This allocation system is worse than when creditors self-pay, because it is counterproductive to subsidize the junior’s professionals.

There are three potential improvements to the U.S. system.

1. **The simplest improvement to the U.S. system**, would be to reverse the U.S. reimbursement system, i.e., to have the court reimburse the seniors instead of the juniors. In our model, from the senior’s perspective, this is equivalent to having the firm pay all: a senior is the full first claimant in equilibrium (spending 20 hours). The junior remains the residual claimant and spends first best.\(^9\) The net social professional benefit is +$0.94.

2. **In our first improved partial reimbursement scheme**, the firm can commit in its lending agreement to reimburse the creditors in a manner that maximizes professional benefits. In our model, managers are assumed to want to maximize firm value, but are not interested in how much or little creditors spend themselves. They will want to calibrate their reimbursement policy to subsidize the senior, but not excessively so. They want the senior to hire more professional hours than when the senior has to pay herself (in which case the senior does not contribute enough, only 0.14 hours) and fewer professional hours than when the firm gives the senior

\(^9\)This result follows from our assumption that APR is protected with certainty when the senior spends, and may not hold under more general assumptions. The more general insight is that senior creditors would overspend in this regime.
a blank check (in which case the senior overcontributes, the full 20 hours). In our example, the firm is best off if it reimburses 84.6 percent of the senior’s professionals. In equilibrium, the senior then spends 4.54 hours on professionals. Her objective function is $\theta \cdot 40 + (1 - \theta) \cdot [1/3 \cdot \{($100 + 2\cdot\sqrt{s} + 2\cdot\sqrt{j}) - 84.6\% \cdot (s\cdot$0.50)\}] - 15.4\% \cdot (s\cdot$0.50). The junior spends 7.09 hours. His objective function is $\theta \cdot \{(($100 + 2\cdot\sqrt{s} + 2\cdot\sqrt{j}) - 84.6\% \cdot (s\cdot$0.50)\} - 40\} + (1 - \theta) \cdot [2/3 \cdot \{(($100 + 2\cdot\sqrt{s} + 2\cdot\sqrt{j}) - 84.6\% \cdot (s\cdot$0.50)\} - 40\} - j\cdot$0.50. Both professionals overspend, a direct consequence of the firm’s intent to maximize its value ex-ante with its reimbursement policy.

It would not be optimal for the firm to reduce the senior reimbursement proportion. Doing so would encourage the senior to reduce her professional hours toward her socially optimal professional choice of 4 hours. But it would also induce the junior to spend more than 7.09 hours on professionals. (In equilibrium, junior and senior professional engagements are substitutes.) In equilibrium, the senior creditor does not enjoy full APR, but she receives a higher share of the firm than when she has to pay for her own expenses. In this equilibrium, professionals contribute a social gain of +$3.77.

This scheme relies on ex-ante professional reimbursement contracts. However, first, the firm may not know the appropriate model parameter values at the time that it borrows money. Second, ex-ante contracts that constrain the power of the bankruptcy court to manage the process are seldom legally enforcible. And third, the firm will not have social maximization as its objective after bankruptcy will have occurred. Instead, the firm will want to maximize its value at bankruptcy time regardless of any precommitment it may have made. (Naturally, it will still not take into account any professional expenses not charged to the firm.) This leads to the following suggested scheme:

3. **In our second improved partial reimbursement scheme,** the firm itself makes compensation decisions, subject to the approval of the court, ex-post. Compared to the previous case, the firm is a bit more reluctant to reimburse the senior creditor. Thus, if the firm reimburses 81.5% of the senior’s expenses (rather than 84.6%), the senior will choose to reduce her professionals’ value enhancement to the firm somewhat. Although this slight reduction is socially undesirable, from the managers’ perspective, it is offset by the lower sum the firm now pays out in total expenses. Managers do not mind junior professional spending: even if these
professionals’ cost is socially excessive, management welcomes any professional contributions, because they increase firm value. After all, the firm does not reimburse the junior creditor.

In equilibrium, the senior hires 3.16 hours of professional advice, which is below the social optimum of 4 hours. Her objective function is $\theta \cdot \$40 + (1-\theta) \cdot [\frac{1}{3} \cdot (\$100 + \$2 \cdot \sqrt{s} + \$2 \cdot \sqrt{j}) - 81.5 \% \cdot (s \cdot \$0.50)] - 18.5 \% \cdot (s \cdot \$0.50)$. Induced by the senior’s smaller professional retention, the junior now spends 7.46 hours. His objective function is $\theta \cdot \{(\$100 + \$2 \cdot \sqrt{s} + \$2 \cdot \sqrt{j}) - 81.5 \% \cdot (s \cdot \$0.50)\} - j \cdot \$0.50$.

Compared to the case in which the firm can commit itself to the socially optimal reimbursement, the professionals’ contribution to social welfare at $3.71 is lower than the earlier $3.77, because the managers cannot precommit to a higher reimbursement rate. And, the social value improvement is not as great as in the social optimum, because this scheme suffers from the same difficulties as the ex-ante scheme: the firm also does not have the socially correct incentives, but wants to maximize its own value. On the other hand, although the firm may be unable to observe all of the economic parameters ex-post, these parameters are more likely to be observable after insolvency. In our model, the firm makes one upfront hiring/reimbursement choice. In real life, management can continuously evaluate creditor professionals for their contributions to firm value and terminate professionals when they cease their usefulness. Thus, our reform proposal may work better in practice than the model implies.

Both of our two partial reimbursement schemes ignore that management has just run the firm into bankruptcy, and therefore may be either incapable or too conflicted to make good choices. However, old management is often replaced with new management in Chapter 11. Our solutions also ignore that management may prefer to continue firm operation, even if liquidation is optimal. (We ignore this by assuming that management wants to maximize firm value, regardless of continuation or termination.) In a more general model, this would lead management not to adequately reimburse senior creditors for professionals, which might establish socially optimal termination. (Recall that senior creditors will almost invariably argue in favor of termination!) However, the current U.S. system suffers from the same problem: senior creditor expenses to convince the court that termination is optimal must already be borne by senior creditors themselves.
Appendix A completes our numerical examples by illustrating a utopian second-best scheme, in which an omniscient government can finetune reimbursement.

III The Firm and Professionals

A The Firm

The firm is financed with equity and debt. Debt claims can be either “senior” (for example, secured) or “junior” (unsecured) and have a face value of $S$ and $J$, respectively. (All variables are summarized in Table 2.) The securities are issued to finance a risky project. If the project succeeds, all creditors are fully repaid without financial distress and without any professional representation. Our model is concerned with the state in which the project fails and debt cannot be paid off, i.e., $V < S + J$, and we normalize the claims to add up to 100% ($S + J \equiv 1$). We assume that project proceeds are insufficient to repay the total debt, but they are enough to repay either the senior creditors ($V > S$) or the junior creditors ($V > J$).

B Professionals’ Contributions To Firm Value

Senior creditors and junior creditors engage $s$ and $j$ units\(^\text{10}\) of professional representation, respectively. We assume $0 \leq s \leq \bar{s}$, $0 \leq j \leq \bar{j}$, where $\bar{s} \equiv \bar{j} \equiv 1$. Creditors’ professionals increase the value of the firm in a linear fashion. The expected value of the firm post-enhancement but pre-professional-payout (if applicable) is $V(s, j) \equiv V + h \cdot \sqrt{s} + h \cdot \sqrt{j}$, where $h > 0$. Thus $h$ is a parameter measuring professional effectiveness.

\(^{10}\)These units are measured in terms of time in our model, but they also could be measured in terms of quality. For example, they could equally well represent hiring a better and more expensive professional.
The cost of professional representation is parameterized by \( c \ (c > 0) \). Senior and junior creditors pay \( c \cdot s \) and \( c \cdot j \) for professionals, respectively. We work exclusively with the monotonic transformation \( s = \sqrt{s} \) and \( j = \sqrt{j} \), so the value function is 
\[
V(s, j) \equiv V + h \cdot s + h \cdot j \href{#2}{\text{and professional costs are } c \cdot s^2 \text{ and } c \cdot j^2 \text{ for creditors, respectively. That is, our model with its linear improvements and quadratic costs is identical to a model with square-root improvements and linear costs.}}
\]

Our specification deliberately assumes that it is not optimal for creditors to pool resources and hire all professional advice jointly.

\[
h \cdot (s + j) - c \cdot (s + j)^2 < h \cdot s + h \cdot j - c \cdot s^2 - c \cdot j^2 \cdot \tag{2}
\]

This is largely a domain assumption. Conflict among creditors, the possible possession by creditors of different private information about the effect of creditor efforts on the firm’s prospects, and different types of creditor expertise will make creditor specialization efficient some of the time. When specialization would be inefficient, the firm, representing all creditor classes, can hire its own professionals (as firms sometimes do). In sum, we assume that there are reasons for having professionals on both sides, rather than a single dedicated professional team working jointly on behalf of the creditors. (If this were not so, the current U.S. system would be utterly non-sensical.)

We make four assumptions regarding the costs and benefits of hiring professionals:

\textbf{Assumption 1, } \( V + 2 \cdot h - 2 \cdot c > S \): This ensures that, if APR is satisfied, senior creditors always fully recover their claim \( S \). In other words, if both groups of creditors spend \( s = j = 1 \) in professionals, and the firm reimburses their expenses, the value of the firm is sufficient to pay the senior creditors’ claim \( S \).

We motivate this assumption as follows: if the senior claim is substantially out of the money, junior claims will obviously be worthless. In such cases, the senior will be allowed to take its collateral and the firm will be liquidated in Chapter 7. Our paper focuses on Chapter 11 reorganizations, where both types of creditors can engage in the value-increasing and redistributional activities we model.
**Assumption 2**, \( V + 3 \cdot h < S + J \equiv 1 \): This restriction states that even with exceptional professional contributions, the firm remains in bankruptcy. This includes the less restrictive condition that the firm remains in default even with maximal professional participation, \( V + 2 \cdot h < S + J = 1 \).

**Assumption 3**, \( h < 2 \cdot c \): There is an unconstrained optimum for creditors’ professional expenses.

**Assumption 4**, \( V - 2 \cdot c > 0 \): Creditor expenses can be reimbursed out of the firm’s assets.

### IV The Socially Optimal Professional Expenses

The socially optimal spending on professionals trades off increases in firm value against the costs of professional representation without regard to redistributional properties. This is the solution to

\[
\max_{s,j} W \equiv h \cdot (s + j) - c \cdot (s^2 + j^2) .
\]  

(3)

**Proposition 1**  
*The first order conditions show that it is socially optimal for both senior and junior creditors to hire professional advice of*

\[
s^\ast = \frac{h}{2 \cdot c}, \quad j^\ast = \frac{h}{2 \cdot c} .
\]  

(4)

*The resulting welfare \( W \) is*

\[
W^\ast = \frac{h^2}{2 \cdot c} .
\]  

(5)

We use a star throughout the paper to denote solutions under the socially first-best case. Our model specification was chosen to produce these closed-form interior

\[\text{11 Although we assume that professionals of both groups of creditors are equally efficient, we have solved the case where professionals are differentially efficient. The results are qualitatively the same.}\]
solutions. The optimal expenses are such that, for every creditor, the marginal revenue of using professionals $h$ equals its marginal costs $2 \cdot c \cdot j$ and $2 \cdot c \cdot s$.

If such first-best equilibrium behavior could be forced upon the two creditors, we can determine how their behavior changes with the two key parameters, the professionals’ costs $c$ and the professionals’ benefits $h$. The optimal choice of professionals increases with benefit and decreases with cost, both in terms of units (quantity or quality) and in terms of expense.

![Equilibrium professional retentions, $s^*$ (solid) and $j^*$ (dashed).](image1)

![Total professional costs ($c \cdot (s^2 + j^2)$), gross improvements ($h \cdot (s + j)$), net improvements ($h \cdot (s + j) - c \cdot (s^2 + j^2)$).](image2)

Although this is only a hypothetical benchmark, in the numerical illustration above and the model below we posit a function for how senior and junior professionals sway the equilibrium between two extremes: a pro-rata allocation ($\theta = 0$)
and an absolute priority allocation ($\theta = 1$). In this benchmark, the in-equilibrium APR violation probabilities are highest for intermediate values of professional costs: With very low costs, senior creditors fully defend their claim. With very high costs, junior creditors “attack” less.

Equilibrium APR violation $1 - \theta$.

The creditors’ relative payoffs are determined by both the cost-benefit contribution of professionals and the implicit APR violation. Senior creditors end up being better off with less professional participation, junior creditors with more. Thus, senior creditors are better off in equilibria with high professional costs and low professional benefits.

Junior ($P_J / J$) and Senior Relative Payoffs ($P_S / S$)

15
V  The Influence Component

The current U.S. bankruptcy code suffers from a confusion. Its intent is appropriate enough, as in §330(a)4A: \textit{The court shall not allow compensation for [...] that were not reasonably likely to benefit the debtor’s estate [...]}. Unfortunately, it defines an admissible reimbursable value enhancement to be any professional activity which enhances the unsecured creditor’s estate. But the junior estate could increase not only if the total estate increases, but also if the junior can redistribute wealth from the senior. This creates an incentive for the junior to engage in redistribuional activities, such as discovering minor flaws in senior liens. The statute thus should define benefit to the estate as actual increases in the value of the insolvent firm. This naturally brings us to the core of our paper: the redistributinal nature of some professional activities.

Professionals not only enhance firm value but also try to convince the court to take a more favorable view towards their clients. A professional hired by the junior creditor thus may advocate that the firm has a higher value than previously thought, so that junior creditors deserve a larger fraction of the recapitalized claims. Professionals hired by the senior creditor will respond by advocating that the firm has a lower value. If the court overestimates firm value, then it in effect violates APR in favor of junior creditors.

In our model, courts allocate the residual value of the firm in a fashion that is influenced by professional activities. To be precise, courts allocate either according to APR, or pro-rata, or somewhere in between. If the court is “APR-oriented,” the proceeds fully satisfy the senior creditors first, and then repay the junior creditors with the remainder. The payoffs to senior creditors are $P_S = S$; the payoffs to junior creditors are $P_J = V - S$, where $P_S$ and $P_J$ denote the allocation to senior and junior creditors, respectively. If the court is “anti-APR oriented,” the firm’s cashflows are distributed proportionally, i.e., $P_S = \left(\frac{S}{S+J}\right) \cdot V = S \cdot V$, and $P_J = \left(\frac{J}{S+J}\right) \cdot V = J \cdot V = (1 - S) \cdot V$.

The parameter $\theta \in [0, 1]$ determines the extent to which one or the other of these allocation schemes is followed. $\theta$ can be interpreted as the probability that the court upholds APR, or as a compromise fractional allocation between APR and non-APR, or both. Both senior and junior creditors’ professionals influence the
reorganization parameter $\theta$. We specify the outcome of the senior-junior conflict as

$$\theta(j, s) = 1 - j \cdot (1 - s) .$$

(6)

This functional form implies that

1. If the junior creditors do not spend money on professionals, the court is APR-oriented irrespective of the senior creditors’ behavior ($j = 0 \Rightarrow \theta = 1$). This assumption is consistent with the law and practice in the United States.

2. The more resources that junior creditors spend on professionals, the less APR-oriented the court is: $\partial \theta / \partial j = -(1 - s) \leq 0$.

3. Senior creditors can mitigate ($\partial \theta / \partial s = j \geq 0$) or even completely undo the juniors’ efforts by spending on professionals ($s = 1 \Rightarrow \theta = 1$).

VI Full Payment Systems

Before we proceed to systems in which professional costs are shared by participants, it is instructive to compare schemes in which all professional fees are paid for by one party. This highlights the incentive issues arising later. The next sections consider partial reimbursement regimes.

A Government Pays Professionals

If the government fully pays for all professionals retained by creditors (GP), the senior and junior creditors would solve, respectively,

$$\max_s P_S \equiv \theta(s, j) \cdot S + [1 - \theta(s, j)] \cdot \left(\frac{S}{S + J}\right) \cdot V(s, j) ,$$

(7)

$$\max_j P_J \equiv \theta(s, j) \cdot [V(s, j) - S] + [1 - \theta(s, j)] \cdot V(s, j) \cdot \left(\frac{J}{S + J}\right) .$$

(8)

Because we have exogenously imposed a limit on expenditures in our model,

$$s^{GP} = \bar{s} \equiv 1 , \quad j^{GP} = \bar{j} \equiv 1 .$$

(9)
Relative to the first-best solution (5), both creditors overspend on representation. The court is APR-oriented, so $P_S = S$, $P_J = V + 2 \cdot h - S$. Social welfare is a low $2 \cdot h - 2 \cdot c < 0$. Fortunately, although it may be realistic to presume that the government carries some expenses caused by professionals (e.g., in maintaining a court system), it is neither realistic nor desirable to have the government pay for all creditor professionals.

B  Firm Pays Professionals

If professionals’ fees are reimbursed at the court’s discretion out of firm value, senior creditors solve:

$$\max_s P_S = \theta(s, j) \cdot \min [S, V(s, j) - c \cdot s^2 - c \cdot j^2]$$

$$+ [1 - \theta(s, j)] \cdot [V(s, j) - c \cdot s^2 - c \cdot j^2] \cdot \left(\frac{S}{S + J}\right). \quad (10)$$

This equation assumes that even if the maximum amount is spent on professionals ($s = \bar{s}, j = \bar{j}$), creditors still receive some value. Similarly

$$\max_j P_J = \theta(s, j) \cdot \max [V(s, j) - c \cdot s^2 - c \cdot j^2 - S, 0]$$

$$+ [1 - \theta(s, j)] \cdot [V(s, j) - c \cdot s^2 - c \cdot j^2] \cdot \left(\frac{J}{\bar{s} + J}\right). \quad (12)$$

**Proposition 2**  If the firm fully pays for creditors’ expenses (FP), senior creditors overspend on representation, and junior creditors spend optimally:

$$s^{FP} = \bar{s}, \quad j^{FP} = j^*,$$

where $\bar{s}$ is the maximum expense possible. Further, $W^{FP} > W^{GP}$.

**Proof:** See Appendix, page 36.

Because the senior spends a lot, APR is followed, so the junior becomes a pure residual claimant.
C Creditors Pay Professionals

If each creditor pays for his own professionals, senior creditors solve

\[
\max_s P_s \equiv \theta(s, j) \cdot S + [1 - \theta(s, j)] \cdot \left( \frac{S}{S + J} \right) \cdot V(s, j) - c \cdot s^2
\]  

(14)

and junior creditors solve

\[
\max_j P_j \equiv \theta(s, j) \cdot [V(s, j) - S] + [1 - \theta(s, j)] \cdot \left( \frac{J}{S + J} \right) \cdot V(s, j) - c \cdot j^2
\]  

(15)

**Proposition 3** If creditors pay for their own expenses, junior creditors overspend on representation,

\[
j^{CP} > j^*.
\]  

(16)

Senior creditors underspend on professional advice

\[
s^{CP} < s^* \quad \text{if } S < S^M,
\]  

(17)

If the senior claim is less than

\[
S^M \equiv \frac{(2c)^{\text{h}}}{[1-V+h-(S \cdot h^2)/(2c)]}.
\]  

This is a (mild) sufficient, but not necessary condition.

Further, \(W^{CP} > W^{FP}\).

**Proof:** See Appendix, page 36.

When APR is violated, senior creditors share in the residual claim. Hence, they are better off when firm value increases, and will contribute some improvement, though too little from a social perspective. Among all full-pay schemes, this regime, in which creditors pay for their own expenses, is best. Unlike the earlier schemes, it does not induce either creditor to spend to the max.
VII  Asymmetric Full Payment Systems

A  The Current U.S. System: Full Junior Creditor Reimbursement

In the U.S., senior creditors are not reimbursed but junior creditors are. We analyze here the effects of this system on expenses. Our results are similar to Welch (1997). When the junior claim is out-of-the-money, junior creditors are indifferent between bearing costs themselves, or being reimbursed by the court. However, if spending on professionals tilts the court against APR, senior creditors ultimately bear part of the juniors’ professional expenses. The next proposition shows that junior creditors overspend with respect to the case where costs are paid for by the creditors. Additionally, senior creditors will overspend, in order to overcome the juniors’ efforts.

**Proposition 4**  Under the U.S. system, both creditors spend more than when neither is reimbursed. Also, the senior underspends and the junior overspends relative to the social optimum.

\[
\begin{align*}
    \bar{s}_{CP} & < s^{US} < \bar{s}^*, \\
    j^* & < j_{CP} < j^{US}.
\end{align*}
\]

(Social welfare in this case is not higher than when creditors pay themselves \(W^{US} \leq W^{CP}\).)

**Proof:** See Appendix, page 37.

The intuition is that the junior creditor attempts to cause APR violations, knowing that the senior creditor will typically not find it in her interest to hire enough professionals to fully defend APR. Still, unlike in the self-financing expenses case, senior creditors spend more in this system than when creditors self-reimburse. This is because when the junior is subsidized, he will overspend, thereby increasing the likelihood that the senior will be partly a residual claimant. The senior creditor then becomes more concerned about junior expenses that reduce the value of the firm and her own claim. Her optimal response is to increase spending.
B Another Poor Alternative: Full Senior Creditor Reimbursement

As our numerical illustration showed, reversing the current U.S. reimbursement system (so that senior creditors are reimbursed rather than junior creditors) is not a panacea. Indeed, because APR is fully upheld, the solution is the same as when the firm fully reimburses professionals from both parties. In our model, we can show
\[ W^{\text{US}} < W^{\text{reverse U.S.}} = W^{\text{FP}} < W^{\text{CP}} \quad (19) \]

This system can be better than the current U.S. system, because it at least attempts to correct reimbursement in the right direction. However, beyond the context of our model, it will depend on how bad excessive senior spending in such a system would be relatively to how bad excessive junior spending is in the current U.S. system.

VIII Systems With Partial Reimbursement By Firms

Unlike earlier schemes, we now allow society to set more flexible rules on who pays for professional expenses. These schemes may permit firm managers the discretion to decide how creditors are reimbursed for their professionals' contributions. Any unreimbursed expenses must be carried by the creditors themselves.

We consider two schemes: In the first, the firm can set reimbursement rules at the time of loan origination. In the second, the firm can set reimbursement rules only at the time of bankruptcy. In both cases, the debtor's managers are presumed to maximize the value of the firm.
A Socially Optimal Partial Reimbursement By Firm

Proposition 5 Managers that can commit to reimburse creditors’ expenses will contract for a scheme that maximizes firm-value ex-ante (at loan origination). In this scheme, the firm agrees to partially reimburse senior creditors, but will not reimburse junior creditors. Both creditors overspend in equilibrium:

\[ s^{\text{FRA}} > s^*, \quad j^{\text{FRA}} > j^* \]  

(20)

Social welfare in this case is higher than when creditors pay for their own expenses \((W^{\text{FRA}} > W^{\text{CP}})\).

Proof: See Appendix, page 37.

When the firm borrows, it will fully internalize the costs and benefits of any allocation scheme. As we have seen, it is optimal for the firm to reimburse some senior expenses, but no junior expenses. Maximizing managers will offer creditors the optimal contract, because this gives the lowest cost of capital.

In numerical experiments, this scheme yields solutions that are usually close to the first-best situation when compared with the current U.S. bankruptcy regime. To see why, recall that the social problem in bankruptcy is that senior creditors contribute too little, because most residual benefits accrue to the junior. The junior creditors spend too much, because they not only internalize most of the professional’s residual value enhancement, but they also receive the additional redistributional part of the pie from the senior creditors’ claims. The social goal is thus to subsidize the senior creditors in order to enhance their incentives to hire professionals while dampening junior spending. The trick is to avoid subsidizing the senior too much, i.e., not to reimburse all senior expenses. If the firm reimburses senior creditors, it will find it in its interest to suitably constrain the senior’s reimbursement. In equilibrium, the junior creditor still chooses to hire too much representation, because the senior creditor does not spend enough to fully defend APR. The junior creditor thus should not be subsidized by the estate.

Although this system is better than all single-payor systems and the current U.S. bankruptcy system, it still fails to reach the first-best solution. The problem is that
firms can only subsidize and not tax professionals’ participation. In our model, in equilibrium, senior and junior professionals are substitutes: more spending by one leads to less spending by the other. Because the junior cannot be further restrained from seeking redistribution, the best reimbursement system induces the senior creditor to overspend a little on professional representation, in order to reign in the junior creditor’s rent-seeking.

B Self-Seeking Partial Reimbursement By Firm

We have explained above the difficulties that plague any compensation scheme. Therefore, we consider an alternative scheme in which managers are not only free to contract ex-ante for reimbursement schemes, but may choose a reimbursement scheme in bankruptcy that maximizes firm value then. Such an ex-post firm value maximizing system is time-consistent, relatively simple to implement, and robust with respect to different parameter values. Although this scheme is inferior to an ex-ante contracted for firm value maximizing system, the difference is often surprisingly small, e.g., as in our numerical illustration.

Proposition 6 Managers that are permitted to reimburse creditors’ expenses ex-post will not maximize benefits because managers fail to internalize enough of the senior professional costs. Their objective leads them to partially reimburse senior creditors (less than in the FRA case), but not to reimburse junior creditors.

In this scheme, both creditors overspend in equilibrium. Compared to the professional-benefit maximizing choices (case FRA), senior creditors receive less reimbursement and thus hire less professional advice; and junior creditors spend more, taking advantage of less senior resistance.

\[ s^{\text{FRP}} < s^* < s^{\text{FRP}}, \quad j^{\text{FRP}} > j^{\text{FRA}} > j^* \]  \hspace{1cm} (21)

Social welfare is lower when the firm maximizes ex-post than when it can maximize ex-ante, \( W^{\text{FRP}} < W^{\text{FRA}} \).

\[^{12}\text{Because the economic intuition is solid, this system is also robust with respect to many other functional specifications.}\]
Proof: See Appendix, page 38.

C Discussion

Because our results suggest that it is appealing to allow the managers of insolvent firms to choose how to compensate creditors in bankruptcy—for such reasons as simplicity, decentralized decision making, and relative social outcome—our paper suggests closer consideration of such a scheme.

Our first figure plots the contribution of professionals to the social good and to the value of the firm as a proportion of senior expenses that are reimbursed by the firm.

![Graph](image)

Contribution of professionals to social value and to firm value as a proportion of senior expenses that are reimbursed by the firm.

The figure shows that there is a modest, but steady improvement in professionals’ contribution, both to firm value and to social value, over a wide range of reimbursement fractions. As our propositions and figure show, the social gain is strictly higher when firms are permitted discretion in reimbursement than when one party fully pays for the expenses. If reimbursement is 0, this case reverts to “creditors self-pay”—a modestly bad scheme. If reimbursement is 100%, this case reverts to “firm fully reimburses creditors,” in which case senior creditors overspend dramatically—a very bad scheme.
The figure also shows that there is little difference in the optimal choice of reimbursement between the case when the firm sets reimbursement rates at the time the contract is written or at the time of bankruptcy. When the firm “shaves” the reimbursement proportion, it has to pay a little less to the senior creditor, but it also gains less from the senior creditor’s value improvement. Because all of the value improvement provided by senior creditors is internalized in the firm manager’s objective function, the difference between the optimal reimbursement fractions when set \textit{ex-ante} (which is 72%) and \textit{ex-post} (which is 69%) is small.

The remaining figures below consider the comparative statics of the model when managers choose the optimal reimbursement to senior creditors at the time of bankruptcy.\footnote{The figures are practically identical if the consideration for senior professionals is decided \textit{ex-ante} (at the time of loan initiation). When plotting as a function of \(c\) (left-side figures), we hold \(V = 0.9, h = 0.05,\) and \(S = 0.4.\) When plotting as a function of \(h\) (right-side figures), we hold \(V = 0.9, c = 0.05,\) and \(S = 0.4.\)}

Our first figure shows that our scheme has the desirable characteristic that professional retention in equilibrium decreases with cost \(c\) and increases with value effectiveness \(h.\) For low enough professional costs, senior professionals should be 100\% reimbursed. (Junior creditors will similarly hire 100\% professionals for very low \(c.\))

![](Equilibrium.png)

Equilibrium professional retentions, \(s^*\) (solid) and \(j^*\) (dashed).

The next figure shows how social benefits decrease with \(c\) and increase with \(h.\) There is a kink in professional expenses because for low enough costs \(c,\) senior creditors are 100\% reimbursed and thus max out on professional advice. Any increase in
costs just increases the total cost. Beyond such very small $c$, professional retention decreases in $c$, and with it total professional expenses in equilibrium. However, the professionals’ net contribution to both social and firm value always declines with higher costs.

Total professional costs ($c \cdot (s^2 + j^2)$), gross professional improvements ($h \cdot (s + j)$), net professional improvements ($h \cdot (s + j) - c \cdot (s^2 + j^2)$), and firm value improvement $h \cdot (s + j) - \gamma^* \cdot c \cdot s^2$, where $\gamma^*$ is the optimal firm reimbursement of senior professional expenses.

When senior creditors do not max out on their professionals (the optimal reimbursement fraction is below 100%), APR is violated in equilibrium with positive probability.
Putting together professional expenses and APR violations in equilibrium, we can compute the relative satisfaction of senior and junior claims. For very low costs $c$ and very high value improvement $h$, the professionals provide sufficiently valuable improvements in equilibrium to keep both creditors’ recovery rates relatively high. For medium costs $c$ and medium value improvement $h$, junior creditors succeed in capturing some value from senior creditors. Still, both creditors are worse off. If costs are very high and improvement very low, junior creditors find it less worthwhile to influence the court. They are thus worse off. Senior creditors, however, benefit from the reduced aggressive behavior of junior creditors, and are thus better off.

![Graph](graph.png)

**Junior ($P_J/J$) and Senior Relative Payoffs ($P_S/S$)**

Even though our analysis suggests that it is desirable to allow value-maximizing firms to set creditor reimbursement fractions, caveats are in order. Our assumptions that managers are competent and objective value-maximizers with knowledge of the parameters may not always be satisfied. It seems reasonable to presume that even if managers do not have full knowledge of professionals’ effectiveness, their knowledge of the effects of professional advice on firm value (the parameters) is likely to be better than that of a court or the government. Further, in this solution, in contrast to a theoretically better system in which a court or government maximizes social value (see below), the firm needs to judge only how reimbursement influences its own value. The firm is buying a service and does not have to know creditor costs in order to make its reimbursement decision.\footnote{Debtors in possession today sometimes retain professionals themselves, and are compensated by the court. Our suggestion that the debtor be permitted to reimburse senior creditor expenses...}
A more serious concern relates to our assumption that the managers of a debtor in possession want to maximize the value of the insolvent firm. Managers sometimes have a continuation bias. This bias could yield two types of bad behavior. First, the firm could choose professionals who are good at delay or who will help to propose overly optimistic reorganization plans. Second, if matters look bleak, the managers could enlist professionals in a collusive scheme with junior claimants to cause the firm to over invest. We nevertheless prefer the firm over the court as principal decision maker. Bankruptcy courts also have a continuation bias, so letting the firm decide is unlikely to make matters worse. Of greater importance, that the managers who brought the firm into bankruptcy sometimes have poor incentives now is widely recognized. Creditors who are residual claimants thus have an incentive, more frequently acted on than heretofore, to replace the old managers with new ones, whose compensation and reputations are tied to success at reorganization. These new managers should act to promote efficient and to dampen bad creditor spending.

IX Systems Permitting Partial Reimbursement System By Government

We still have one degree of freedom that we have not exploited: What if the government is also allowed to pay a part of professionals’ costs? Suppose that both the government and the firm can reimburse different proportions of senior and junior creditors’ claims. The remainder, unreimbursed by government and firm, is carried by creditors themselves. Further, we assume that not only the firm, but also the government, has zero administrative cost, is benevolent, and is omniscient about the model parameters. Call this case “GR.”
Proposition 7  If both the government and the firm can reimburse creditors, the optimal solution is for the government to reimburse a part of the senior creditor’s expense, and nothing of the junior creditor’s expense. The firm should not reimburse either creditor. The remainder of the expenses should be borne by the creditors themselves.

Even in this scheme, both creditors overspend on professional representation

\[ s^{GR} > s^* \quad , \quad j^{GR} > j^* \quad . \tag{22} \]

The professionals’ welfare contribution is higher in this scheme than in schemes considered earlier, inferior only to the first-best solution.

Proof: See Appendix, page 39.

Senior creditors need to be encouraged. Either the firm or government reimbursement schemes can be tuned to achieve an almost socially optimal first-best senior professional participation. The remaining problem is how to limit junior creditors from over-retaining professionals. The problem if the firm reimburses is that the junior creditors’ residual claim becomes smaller, inducing them to become more “aggressive” in seeking the larger senior share.

Naturally, with the extra degrees of freedom, this scheme can approach the social optimum better than previous schemes. However, even four degrees of freedom are not enough to fine-tune the choices so that junior creditors do not overspend. First-best could only be achieved if one could “tax” the junior creditors’ professional expenses.\(^{15}\)

We consider such optimal government reimbursement schemes to be academic. It is doubtful that governments can fine-tune their reimbursements appropriately to attain this second-best solution.\(^{16}\) The optimal reimbursement fraction depends on the professionals’ production function \((c\text{ and }h)\), which is not only model-specific, but unlikely to be accessible either to legislator or courts.\(^{17}\) Otherwise, a simpler system that achieves the first best would be one where the court (or the government)

\(^{15}\)Note that this would need to be a tax on junior creditor’s expenses, which is different from forcing junior creditors to subsidize senior creditor's expenses.

\(^{16}\)In the real world, the government subsidizes the court system, so we have something of a partial reimbursement system now.

\(^{17}\)Simple comparative statics, which can be derived, are not sufficient for guidance.
fully reimburses creditors’ expenses when they spend exactly the social optimum, and zero otherwise”. Thus, we are reluctant to advocate such government involvement in the bankruptcy reimbursement system.

X Conclusion

Our model describes the tradeoffs faced in designing an optimal professional reimbursement process in bankruptcy: Good solutions must balance the value increasing benefits of professionals against the value decreasing distortions caused by creditors’ rent-seeking activities.

When the government fully reimburses creditors’ professional expenses, both types of creditor spend too much money (in rent-seeking) from a social perspective. When the firm fully reimburses both creditors’ expenses, senior creditors’ expenses are effectively subsidized by junior creditors (the residual claimants), and both parties end up spending excessively on professionals. When creditors fully pay for their own professionals’ expenses, senior creditors fail to provide sufficient contribution in the corporate value-enhancement process.

Partial reimbursement systems work better. A robust characterization is that some, but not all, possible senior creditors’ expenses should be reimbursed, while junior creditors should not receive any subsidy. We close our paper with our judgment on which solutions can reasonably be implemented:

1. The first-best solution can only be obtained if redistributional activities of all production function parameters are observable by the parties and the court. However, if redistributional activities can remain hidden, or production functions remain unobservable, first best is not attainable. A second-best solution, in which the government pays part of the senior creditors’ professional expenses, and does so without introducing other distortionary incentives into the process, similarly suffers from an inability to observe the relevant parameters.

2. A third-best solution, often nearly as good as the first-best, can be obtained if managers are permitted to reimburse creditors, either ex-ante or ex-post. Firm-value maximizing managers would choose to partially reimburse senior
creditors, but not junior creditors. This scheme can be implemented by a simple amendment to the bankruptcy code. The amendment would provide that creditors do not apply to the court for reimbursement, but rather apply to the firm. The court then would review firm decisions only to ensure that they are not made in an arbitrary fashion. This scheme not only increases the amount of productive professional spending, but, by substantially dampening redistributional spending, will also reduce the total amount of professional spending. Our two proposed schemes strike us as feasible and efficient bankruptcy reimbursement systems.

3. The current U.S. practice, in which junior creditors are reimbursed from the estate and senior creditors are not, is counterproductive. Junior creditors end up seeking redistributional rents (a bigger share of the estate), and senior creditors end up not spending enough on professional advice.

(Also, within the current U.S. system, it would make sense to change the definition of redistributional activities to encompass the entire estate, not just the junior lien.)

We conclude by noting three promising areas for future research. First, the functional form we use for relating expenses to the probability that the court will follow APR is deterministic: hours spent on professionals affect the probability in a predefined, quantified way. Creditors, however, may be able to direct spending relatively more towards productive or rent-seeking activities. This could in turn endogenize our theta function. Second, although creditor committees coalesce multiple junior creditors into one party, it would be interesting to allow for more creditors (Bris and Welch [1994]), and allow creditors to form coalitions. Third, our proposed reform should dampen creditor rent seeking, and this will increase the likelihood that courts will follow APR. A bankruptcy procedure that awards the equity nothing, however, may worsen pre-bankruptcy investment incentives (Povel [1999]). Thus, a more complete treatment should ask whether the reform we advocate would be an improvement, all things considered.18

18Schwartz (2004) analyzes the tradeoffs between the ex-ante efficiency effects of following absolute priority strictly, and the possible ex-post inefficiency effects of discouraging use of the bankruptcy process.
<table>
<thead>
<tr>
<th>Benchmark: First Best Solution</th>
<th>Senior</th>
<th>Junior</th>
<th>APR Allocation</th>
<th>Senior Payoff</th>
<th>Junior Payoff</th>
<th>Net Welfare</th>
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<td>Cost</td>
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<td>Cost</td>
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<td>$10.00</td>
<td>$8.94</td>
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<td>$8.94</td>
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<tr>
<td>Firm reimburses all professionals.</td>
<td>20.00</td>
<td>$10.00</td>
<td>$8.94</td>
<td>4.00</td>
<td>$2.00</td>
<td>$4.00</td>
<td>100.0%</td>
</tr>
<tr>
<td>Creditors pay own professionals.</td>
<td>0.14</td>
<td>$0.07</td>
<td>$0.75</td>
<td>9.11</td>
<td>$4.56</td>
<td>$6.04</td>
<td>54.8%</td>
</tr>
<tr>
<td>Firm reimburses only junior creditor. (Current U.S. System.)</td>
<td>2.22</td>
<td>$1.11</td>
<td>$2.98</td>
<td>20.00</td>
<td>$10.00</td>
<td>$8.94</td>
<td>11.1%</td>
</tr>
<tr>
<td>Firm reimburses only senior creditor. (Outcome like “Firm Pays All.”)</td>
<td>20.00</td>
<td>$10.00</td>
<td>$8.94</td>
<td>4.00</td>
<td>$2.00</td>
<td>$4.00</td>
<td>100.0%</td>
</tr>
<tr>
<td>Partial Reimbursement System Ex-Ante: Firm reimburses senior to maximize welfare, 84.6% of senior’s expenses.</td>
<td>4.54</td>
<td>$2.27</td>
<td>$4.26</td>
<td>7.09</td>
<td>$3.55</td>
<td>$5.33</td>
<td>72.6%</td>
</tr>
<tr>
<td>Partial Reimbursement System Ex-Post: Firm reimburses senior to maximize firm value, 81.5% of senior’s expenses.</td>
<td>3.16</td>
<td>$1.58</td>
<td>$3.56</td>
<td>7.46</td>
<td>$3.73</td>
<td>$5.46</td>
<td>68.6%</td>
</tr>
<tr>
<td>Optimal scheme: Government carries 81.0% of senior’s expense.</td>
<td>4.60</td>
<td>$2.30</td>
<td>$4.29</td>
<td>6.39</td>
<td>$3.19</td>
<td>$5.06</td>
<td>75.4%</td>
</tr>
</tbody>
</table>

The per-hour cost of professional advice is $0.50. Each creditor can accumulate no more than 20 hours of such advice. Firm value improvement due to professionals are \$2 \cdot \sqrt{s} + 2 \cdot \sqrt{j}\), where \(s\) and \(j\) are the number of professionals’ hours (units) hired by senior (\(S\)) or junior (\(J\)) creditors. Given this value improvement function, for a given number of hours, benefits are highest if they are split equally between creditors. Courts are swayed by professionals in a way that depends on senior and junior expenses, \(\theta(s, j) = 1 - (j/20) \cdot (1 - s/20)\). A theta of 1 implies absolute priority allocation, a theta of 0 implies face-value pro-rata allocation.
Table 2. Variables and Conditions

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>Face Value of Claim of Senior Creditor. Exogenous.</td>
</tr>
<tr>
<td>$J$</td>
<td>Face Value of Claim of Junior Creditor. Exogenous.</td>
</tr>
<tr>
<td>$V$</td>
<td>Value of the Firm Before Professional Activity. Exogenous.</td>
</tr>
<tr>
<td>$P_S$</td>
<td>Payoff to Senior Claimant. Exogenous.</td>
</tr>
<tr>
<td>$P_J$</td>
<td>Payoff to Junior Claimant. Exogenous.</td>
</tr>
<tr>
<td>$s$</td>
<td>units of professional representation purchased by senior creditor. Endogenous.</td>
</tr>
<tr>
<td>$j$</td>
<td>units of professional representation purchased by junior creditor. Endogenous.</td>
</tr>
<tr>
<td>$h$</td>
<td>per-unit firm value enhancement. Exogenous.</td>
</tr>
<tr>
<td>$c$</td>
<td>parameterization of cost of professional representation. Exogenous.</td>
</tr>
<tr>
<td>$\theta = \theta(s, j)$</td>
<td>APR adherence of the Court, specified in (6): $\theta(j, s) = 1 - j \cdot (1 - s)$. Exogenous.</td>
</tr>
<tr>
<td>$W$</td>
<td>Social welfare (value enhancement net of professional expenses) $W = h \cdot (s + j) + c \cdot (s^2 + j^2)$. Derived.</td>
</tr>
</tbody>
</table>

Policy Choices

Model With Optimal Reimbursement, Section IX

1-$\gamma_S$ | Fraction of senior's costs paid by government to senior creditor. |
1-$\gamma_J$ | Fraction of junior's costs paid by government to junior creditor. |
$\delta_S$ | (Post-government $\gamma_S$): Fraction of remaining costs paid by senior creditor himself. |
$\delta_J$ | (Post-government $\gamma_J$): Fraction of remaining costs paid by junior creditor himself. |
* | “star” designates a best policy |

Imposed Conditions.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S + J = 1$</td>
<td>Normalization.</td>
</tr>
<tr>
<td>$0 \leq s, j \leq 1$</td>
<td>Limited professional expense.</td>
</tr>
<tr>
<td>$V + 2 \cdot h &lt; 1$</td>
<td>Firm still in distress after maximum value enhancement.</td>
</tr>
<tr>
<td>$V - 2 \cdot c &gt; 0$</td>
<td>Firm still worth something after maximum professional expense.</td>
</tr>
<tr>
<td>$V + 2 \cdot h - 2 \cdot c &gt; S$</td>
<td>Secured creditors’ claim is satisfied even after maximum professional expenditures.</td>
</tr>
</tbody>
</table>
A Numerical Example Continuation

For completeness, we now extend the numerical illustration of Section II to an even better partial (senior) reimbursement scheme, in which a “benevolent omniscient government” maximizes social welfare. We do not suggest this to be a feasible improvement.

In this scheme, the government would reimburse the senior for 81.0 percent of the senior’s expense, and none of the junior’s expense. The firm would reimburse nothing, and a creditor who would want to spend more on professionals would have to fund such professional hours herself.

This case has an interesting intuition. Recall the case in which the firm’s intention was to maximize professional contribution (case A). If the senior creditor had been reimbursed for 83.6% of her expenses, the senior would have spent the social optimum of 4 hours, but the junior would have hired about 7.23 hours. Increasing the senior reimbursement from 83.6% to 84.6% meant that the senior would hire a little bit more than the optimal 4 hours, but at a social cost of second order importance (guaranteed by the envelope theorem). Because the junior and senior expenses are substitutes in equilibrium, this reduces the junior’s professionals from 7.5 to 7.09 hours, which raises welfare.

Still, the problem in equilibrium remains that both creditors overspend. But, reducing the firm reimbursement to senior creditors increases the junior’s incentive to rent-seek. The reason is of course that when the firm reimburses some senior expenses, the pro-rata payoffs become worse for the junior and rent-seeking becomes relatively more attractive. If instead of the firm the government reimburses the senior creditor, the junior is less eager to assail the senior claim and will spend less (socially inefficiently) on professionals. Thus, a better equilibrium can be achieved when the government reimburses creditors than when the firm itself reimburses creditors.

The optimal government reimbursement proportion for seniors is 81.0 percent, similar to the proportions when the firm reimburses creditors. The senior spends 4.6 hours. Her objective function is \( \theta \cdot 40 + (1 - \theta) \cdot \left\{ \frac{1}{3} \cdot \left( (100 + 2 \cdot \sqrt{s} + 2 \cdot \sqrt{j}) \right) \right\} - 20.7\% \cdot (s \cdot 0.5) \). The junior still finds it optimal to attack the senior claim and spends 6.39 hours. His objective function is \( \theta \cdot \left( (100 + 2 \cdot \sqrt{s} + 2 \cdot \sqrt{j}) - 40 \right) + (1 - \theta) \cdot \left( \frac{2}{3} \cdot \left( (100 + 2 \cdot \sqrt{s} + 2 \cdot \sqrt{j}) \right) - j \cdot 0.5 \). Thus, both creditors overspend on representation. In this system, the court chooses 75.4% APR, 24.6% PPR; and the senior receives $38.69, the junior receives $67.02.

Therefore, in the case, the social net professional benefit is +$3.85. Unfortunately, although this is the best partial reimbursement system, an optimal government reimbursement is probably not feasible. It relies on governmental knowledge of parameters and unbiased, uninfluencable decisions—abilities and tendencies that we were not willing to attribute to courts.
B Numerical Background for Figures

This section computes the base values for the figures in the text. The base parameters are:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of the Firm before Enhancement</td>
<td>$V$</td>
</tr>
<tr>
<td>Senior Creditor Claim</td>
<td>$S$</td>
</tr>
<tr>
<td>Junior Creditor Claim</td>
<td>$J$</td>
</tr>
<tr>
<td>Per-unit Professional Value Enhancement</td>
<td>$h$</td>
</tr>
<tr>
<td>Cost of Professionals</td>
<td>$c$</td>
</tr>
</tbody>
</table>

Because the professional fees are subtracted from the firm value, $V + 2 \cdot h - 2 \cdot c = 0.9 < 1$. If APR were to hold (when $s = 1$, $j = 0$, for instance), firm value would be $V + h - c = 0.9$, and junior creditors would receive only $0.9 - 0.5 = 66.66\%$ of their claim. If instead APR is completely violated, junior creditors could be able to recover $0.6 \cdot \left( \frac{0.9}{0.9 + 0.05} \right) = 90\%$ of their claim. Both junior and senior creditors can improve firm value up to $0.9 + 0.05 + 0.05 = 1$. Note that this is only possible if $s = j = 1$, in which case the probability of APR violation is $j \cdot (1 - s) = 0$, senior creditors receive $0.4$ (100 percent of their claim), and junior creditors receive $1 - 0.4 = 0.6$ (100 percent of their claim).

With these parameters,

**Socially Optimal** The optimal first-best solution entails $s^* = j^* = 0.5$, and total welfare $W^* = 0.025$. APR is violated with a 25% probability.

**Government Pays** If the government assumes all the professional expenses, $s = j = 1$, and social welfare is zero, $W = 0$ because too much is spent in professionals. APR is always upheld.

**Creditors Pay** If creditors finance their own expenses, then $s = 0.23$, $j = 0.59$, social welfare is $W = 0.021$, and probability of APR violation is 45%.

**Firm Pays** If the firm finances creditors’ expenses, then $s = 1.0$, $j = 0.50$, and social welfare is $W = 0.0125$. The probability of violating APR is 0%.

**U.S. System** If the firm finances junior creditors’ expenses only, and seniors finance their own expenses, then $s = 0.33$, $j = 0.75$, and social welfare is $W = 0.0203$. The probability of violating APR is 49.26%.

**Partial Senior Reimbursement: Socially Optimal** The second-best solution with only firm reimbursement of senior expenses, yields second-best values of $s = 0.505$, $j = 0.556$, and welfare $W = 0.0248$. In the optimal reimbursement policy, the firm reimburses a fraction $1 - \delta_S = 0.72$ of the senior professional expenses, APR is violated with probability 28%.

**Partial Senior Reimbursement: Managerial Firm-Value Maximization** The second-best solution, with partial senior reimbursement to maximize firm value, yields second-best values of $s = 0.47$, $j = 0.56$, and welfare $W = 0.0247$. In the optimal reimbursement policy, the firm reimburses senior creditors a percentage $1 - \delta = 69.1\%$ of their professional expenses, and the rest is financed by the creditors themselves. Junior creditors carry their own expenses. APR is violated with probability 29.4%.
Government Partial Asymmetric Reimbursement System The second-best solution with multipayer reimbursement and asymmetric creditor treatment, yields second-best values of \( s = 0.51 \), \( j = 0.54 \), and welfare \( W = 0.0249 \). In the optimal reimbursement policy, the government reimburses fraction \( 1 - y_S = 0.31 \) of the senior professional expenses, APR is violated with probability 27%.

C Proofs

Proof of Proposition 2. If senior creditors do not pay for their professionals, they maximize their profits by spending \( s = 1 \), because in that case APR is upheld for sure and they receive \( S \). Consequently, junior creditors maximize

\[
\max_j P_j = V + (s + j) \cdot h - c \cdot s^2 - c \cdot j^2 - S
\]

which is maximized when \( j > \left( \frac{h}{2 \cdot c} \right) \). Finally, because \( s^{GP} = j^{GP} = 1 \), then \( W^{GP} = \arg \min_{s,j} W \). Hence \( W^{GP} < W^{FP} \).

Proof of Proposition 3. Differentiating eq. 15 with respect to \( j \) yields

\[
\frac{\partial P_j}{\partial j} = S \cdot (1 - s) \cdot [1 - V - s \cdot h - 2 \cdot h \cdot q] + h - 2 \cdot c \cdot j
\]

The first term in the previous expression is positive if \( V + 3 \cdot h < 1 \). Hence it must be \( h - 2 \cdot c \cdot j^* < 0 \) for \( \frac{\partial P_j}{\partial j} = 0 \). Therefore, \( j > \left( \frac{h}{2 \cdot c} \right) \). Solving for \( s \) in the partial derivative \( \frac{\partial P_s}{\partial s} = 0 \), yields

\[
s = \frac{j \cdot S \cdot [1 - V + h \cdot (1 - j)]}{2 \cdot j \cdot S \cdot h + 2 \cdot c}
\]

Note that this previous expression is decreasing in \( j \). Therefore, because \( j > \left( \frac{h}{2 \cdot c} \right) \),

\[
s = \frac{j \cdot S \cdot [1 - V + h \cdot (1 - j)]}{2 \cdot j \cdot S \cdot h + 2 \cdot c} \quad < \quad \left( \frac{h}{2 \cdot c} \right) \cdot \left( \frac{S \cdot [1 - V + h \cdot (1 - j)]}{\frac{S \cdot h^2}{c} + 2 \cdot c} \right) < \left( \frac{h}{2 \cdot c} \right)
\]

when \( \frac{S \cdot [1 - V + h \cdot (1 - j)]}{\left( \frac{S \cdot h^2}{c} + 2 \cdot c \right)} < 1 \). Because \( j > \left( \frac{h}{2 \cdot c} \right) \), the condition is equivalent to \( S < \left( \frac{2 \cdot c}{1 - V + h - \frac{3 \cdot h^2}{2 \cdot c}} \right) \).

To prove that \( W^{CP} > W^{FP} \), note that \( \frac{\partial W^{CP}}{\partial s} > 0 \), \( \frac{\partial W^{CP}}{\partial j} < 0 \). Therefore, using the envelope theorem \( \frac{\partial W^{CP}}{\partial s} < 0 \), for all \( j^{CP}, s > \left( \frac{h}{2 \cdot c} \right) \), and \( \frac{\partial W^{CP}}{\partial j} < 0 \), for all \( s^{CP}, j > \left( \frac{h}{2 \cdot c} \right) \). Hence, because \( j^{FP} = \left( \frac{h}{2 \cdot c} \right), s^{FP} = 1 \), it follows \( W^{FP} < W^{CP} \).
Proof of Proposition 4. In Proposition 3, \( \frac{\partial J^C}{\partial s} < 0 \), because \( \frac{\partial^2 p_s}{\partial s^2} = 2 \cdot S \cdot h \cdot s - S \cdot h + S \cdot V + 2 \cdot j \cdot S \cdot h - S < 0 \). Additionally, \( \frac{\partial J^C}{\partial j} > 0 \), because \( \frac{\partial^2 p_s}{\partial s \partial j} = S \cdot - S \cdot [V + (s + j) \cdot h] - j \cdot S \cdot h + (1 - s) \cdot S \cdot h > 0 \). Under a system where the firm reimburses junior creditors only, and senior creditors pay for their own expenses,

\[
P_s = \theta(s, j) \cdot S + [1 - \theta(s, j)] \cdot S \cdot [V(s, j) - c \cdot j^2] - c \cdot s^2, \quad (27)
\]

\[
P_j = \theta(s, j) \cdot [V(s, j) - c \cdot j^2 - S] + [1 - \theta(s, j)] \cdot (1 - S) \cdot [V(s, j) - c \cdot j^2] \quad , \quad (28)
\]

which implies \( s^{US} = \frac{j^{US} - [1 - V + h \cdot (1 - j) + c \cdot j^2]}{2 \cdot J \cdot S \cdot h + 2 \cdot c} \).

From Proposition 3, \( s^{CP} = \frac{j^{CP} \cdot S \cdot [1 - V + h \cdot (1 - j)]}{2 \cdot J \cdot S \cdot h + 2 \cdot c} \). Therefore \( s^{US} > s^{CP} \) for a given \( j \). Suppose now that \( s \) is given. In equation (28), it means that \( j^{US} > j^{CP} \), for a given \( s \). This is so because \( q \) increases as more \( j \) is financed by the firm. Therefore, because \( \frac{\partial J^C}{\partial s} < 0 \), \( \frac{\partial J^C}{\partial j} > 0 \), and because \( s^{US} > s^{CP} \) for a given \( j \), \( j^{US} > j^{CP} \) for a given \( s \), it must be \( s^{US} > s^{CP} \). Moreover, differentiating (28),

\[
\frac{\partial P_j}{\partial j} = (h - 2 \cdot c \cdot j) \cdot (1 - S \cdot [1 - \theta(s, j)]) + (1 - s) \cdot (1 - S) \cdot [V(s, j) - c \cdot j^2 - S] = 0, \quad (29)
\]

which implies \( q > \left( \frac{h}{2 \cdot c} \right) \) because the second term in the previous expression is positive, and \( S \cdot [1 - \theta(s, j)] < 1 \). To prove \( W^{US} \leq W^{CR} \), and using Propositions 3 and 2, \( \frac{\partial J^C}{\partial j} > 0, \frac{\partial J^C}{\partial s} < 0 \). Therefore, \( \frac{\partial W^{CP}}{\partial s} > 0 \) for for all \( j^{CP}, s > \left( \frac{h}{2 \cdot c} \right) \). Similarly, \( \frac{\partial W^{CP}}{\partial s} < 0 \) for for all \( s^{CP}, j > \left( \frac{h}{2 \cdot c} \right) \). Hence, because \( j^{US} > \left( \frac{h}{2 \cdot c} \right), s^{US} \leq 1, \) then \( W^{US} \leq W^{CP} \).

Proof of Proposition 5. Let \( \delta_S \) and \( \delta_J \) be the percentage of creditors expenses that is paid by themselves, the rest is paid by the company. From Propositions 3 and 2, \( j^{CP} > \left( \frac{h}{2 \cdot c} \right) \) and \( j^{CP} = \left( \frac{h}{2 \cdot c} \right) \). Therefore, \( j \) is increasing in \( \delta_S \). From the proof of Proposition 7, \( j \) is also increasing in \( \delta_J \). Therefore, the optimal constrained reimbursement policy is \( 0 < \delta_S < 1, \delta_J = 1 \).

From the Proof of Proposition 7, it must be \( y_S \cdot (1 - \delta_S) = 0 \) for \( s^* = \left( \frac{h}{2 \cdot c} \right) \). If the government cannot subsidized creditors, then \( y_S = 1 \), hence it must be \( \delta_S = \delta_J = 1 \) for \( j^* \) to be minimum. However, for \( \delta_S = \delta_J = 1, s^* = 1, j^* = \left( \frac{h}{2 \cdot c} \right) \), which can never be optimal. Therefore it cannot be \( \delta_S = 1 \) when \( y_S = 1 \). Therefore \( \delta_S < 1 \). However, this implies \( s^* > \left( \frac{h}{2 \cdot c} \right) \) because \( s \) is decreasing in \( \delta_S \). Finally, for \( 0 < \delta_S < 1, \delta_J = 1, j^* > \left( \frac{h}{2 \cdot c} \right) \) from Propositions 4 and 7. Finally, to prove that \( W^{FRA} > W^{CP} \), note that the \( FRA \) policy is the optimal reimbursement policy among those where the firm reimburses creditors, including the one where the firm reimburses nothing (the CP policy).
Proof of Proposition 6. If the firm reimburses a percentage $y$ of the senior expenses, then the firm’s objective function is

$$ F = h \cdot (s + j) - c \cdot y \cdot s^2 \quad . \quad (30) $$

Similarly, senior and junior creditors maximize respectively

$$ P_s = \theta \cdot s + (1 - \theta) \cdot s \cdot \left[ V + h \cdot (s + j) - c \cdot y \cdot s^2 \right] - c \cdot (1 - y) \cdot s^2 \quad , \quad (31) $$

$$ P_j = \theta \cdot \left[ V + h \cdot (s + j) - c \cdot y \cdot s^2 - S \right] + (1 - \theta) \cdot (1 - S) \cdot \left[ V + h \cdot (s + j) - c \cdot y \cdot s^2 \right] - c \cdot j^2 \quad . \quad (32) $$

The first order conditions are

$$ \frac{\partial F}{\partial y} = (h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA}) \cdot \frac{\partial \delta_s}{\partial y} - c \cdot s_{FRA}^2 + h \cdot \frac{\partial j}{\partial y} = 0 \quad , $$

$$ \frac{\partial P_s}{\partial s} = j_{FRA} \cdot s - j_{FRA} \cdot S \cdot \Theta + j_{FRA} \cdot (1 - s_{FRA}) \cdot S \cdot (h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA}) - 2 \cdot c \cdot (1 - y_{FRA}) \cdot s_{FRA} = 0 \quad , $$

$$ \frac{\partial P_j}{\partial j} = S \cdot (1 - s_{FRA}) \cdot (1 - \Theta) + [1 - j_{FRA} \cdot S \cdot (1 - s_{FRA})] \cdot h - 2 \cdot c \cdot j_{FRA} = 0 \quad , \quad (33) $$

where $\Theta = V + h \cdot (s_{FRA} + j_{FRA}) - c \cdot y_{FRA} \cdot s_{FRA}^2$. From (33), it cannot be $h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA} < 0$, because, using the envelope theorem, and because $\frac{\partial j}{\partial s} < 0$ from the second line in (33), $F$ always increases by reducing $s$ so that $h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA} > 0$.

Therefore

$$ \frac{\partial^2 P_s}{\partial s^2} = S \cdot (1 - \Theta) - j_{FRA} \cdot S \cdot h + \left(1 - s_{FRA}\right) \cdot S \cdot (h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA}) \left[ \frac{\partial \delta_s}{\partial y} \right] $$

$$ - (1 - \Theta) - S \cdot (1 - s_{FRA}) \cdot \left[(h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA}) + j_{FRA} \cdot S \cdot h\right] \frac{\partial^2 j_{FRA}}{\partial j^2} $$

$$ = - S \cdot c \cdot j_{FRA} \cdot s_{FRA} - 2 \cdot c \cdot s_{FRA}^2 \cdot j_{FRA} \cdot (1 - s_{FRA}) \cdot S + 2 \cdot c \cdot s_{FRA} > 0 \quad , \quad (34) $$

Note that it must be $\frac{\partial^2 \delta_s}{\partial y^2} < 0$ to satisfy the second order conditions. Moreover $S \cdot c \cdot j_{FRA} \cdot s_{FRA} - 2 \cdot c \cdot s_{FRA} \cdot j_{FRA} \cdot (1 - s_{FRA}) \cdot S + 2 \cdot c \cdot s_{FRA} > 0$. Since $h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA} > 0$, then $S \cdot (1 - \Theta) - j_{FRA} \cdot S \cdot h + (1 - s_{FRA}) \cdot S \cdot (h - 2 \cdot c \cdot y_{FRA} \cdot s_{FRA})$, and using (33), it must be $\frac{\partial \delta_s}{\partial y} > 0 \Rightarrow \frac{\partial j}{\partial y} < 0$.

Finally, from (30) it must be $y_{FRA} < y_{FRA}$, and using the previous result, this implies $s_{FRA} < s_{FRA}^*$, $j_{FRA} > j_{FRA}^*$. Moreover, because the FRA-policy maximizes social welfare for all policies where the firm reimburses creditors, then $W_{FRA} < W_{FRA}^*$.

To show that $s_{FRA} < \left( \frac{h}{2 \cdot c} \right)$, we can rewrite the firm's objective function as follows:

$$ F = h \cdot (s + j) - c \cdot y \cdot s^2 = W + c \cdot j^2 + c \cdot (1 - y) \cdot s^2 \quad (35) $$
Because $W$ is maximized for $s = \left( \frac{h}{2c} \right)$, then differentiating $F$ around $s = \left( \frac{h}{2c} \right)$, and using the envelope theorem:

$$\frac{\partial F}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} = \frac{\partial [c \cdot (1 - Y) \cdot s^2]}{\partial s} + 2 \cdot c \cdot j \cdot \frac{\partial j}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} = 2 \cdot s \cdot c \cdot (1 - Y) \cdot \frac{\partial s}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} - c \cdot s^2 + 2 \cdot c \cdot j \cdot \frac{\partial j}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} = 2 \cdot s \cdot c \cdot \left( \frac{h}{2c} \right) \cdot \frac{\partial s}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)}$$

(36)

(37)

Suppose $s^{FRP} \geq \left( \frac{h}{2c} \right)$. Then it must be $\frac{\partial F}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} > 0$, which implies, from the previous equation, $\frac{\partial s}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} \geq \frac{\frac{h}{2c} \cdot s}{c \cdot (1 - Y)}$, because $\frac{\partial j}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} < 0$. Now, differentiating $F$ with respect to $s$ around $s = \left( \frac{h}{2c} \right)$, from (35), yields:

$$\frac{\partial F}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} (h - 2 \cdot c \cdot Y \cdot s) \cdot \frac{\partial s}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} + h \cdot \frac{\partial j}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} - c \cdot s^2 = 0$$

(38)

for $s = \left( \frac{h}{2c} \right)$, which is greater than or equal to zero for $s^{FRP} \geq \left( \frac{h}{2c} \right)$. Finally, using the condition that $\frac{\partial s}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} \geq \frac{\frac{h}{2c} \cdot s}{c \cdot (1 - Y)}$, results:

$$(h - 2 \cdot c \cdot Y \cdot s) \cdot \frac{s}{2 \cdot (1 - Y)} + h \cdot \frac{\partial j}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} - c \cdot s^2 > 0$$

(39)

which implies:

$$(h - 2 \cdot c \cdot Y \cdot s) \cdot \frac{s}{2 \cdot (1 - Y)} - c \cdot s^2 > 0$$

(40)

because $\frac{\partial j}{\partial Y} \bigg|_{s=\left( \frac{h}{2c} \right)} < 0$. This is equivalent to $s < \left( \frac{h}{2c} \right)$, which contradicts the initial assumption.

Therefore, it must be $s^{FRP} < \left( \frac{h}{2c} \right)$. ■

**Proof of Proposition 7.** Let $1 - Y_S$ and $1 - Y_J$ be the percentage of creditors expenses that is paid by the government, the rest is paid by either the creditors or the company. Similarly, let $\delta_S$ and $\delta_J$ be the percentage of the remaining expenses that is paid by the creditors themselves.

1. The maximization program for this problem is

$$\max_{Y, \delta} G = h \cdot (s^* + j^*) - c \cdot (s^{*2} + j^{*2})$$

(41)
where
\[
\begin{align*}
 s^* &\in \arg\max_{s} P_S(s) = \theta \cdot S + (1 - \theta) \cdot S \cdot \left[ V + s \cdot h + j^* h - c \cdot y_s \cdot (1 - \delta_s) \cdot s^2 - c \cdot y_j \cdot (1 - \delta_j) \cdot j^* \right] - y_s \cdot \delta_s \cdot c \cdot s^2 \\
 j^* &\in \arg\max_{j} P_J(j) = \theta \cdot \left[ V + s^* \cdot h + j \cdot h - c \cdot y_s \cdot (1 - \delta_s) \cdot s^* \cdot j^* - c \cdot y_j \cdot (1 - \delta_j) \cdot j^* \right] - y_j \cdot \delta_j \cdot c \cdot j^*.
\end{align*}
\]

First note that \( j^* = \left( \frac{h}{2 \cdot c} \right) \) only when \( s^* = 1 \). From Propositions 3 and 2, \( j > \left( \frac{h}{2 \cdot c} \right) \) for any \( y_s, \delta_s, \) and \( s^* \neq 1 \), and it is closest to \( \left( \frac{h}{2 \cdot c} \right) \) when \( y_j^* = \delta_j^* = 1 \). To see that \( j \) is minimized for \( y_j^* = \delta_j^* = 1 \), note that \( j^* \) is decreasing in \( y_j^* \), so it must be \( y_j^* = 1 \). Let \( P_J^c \) be the value of the junior creditors’ claim when professional fees are paid for by the creditors. Similarly, \( P_J^c \) is the value of the junior debt when professional fees are financed by the firm. It is straightforward to show that \( \frac{\partial^2 P_J^f}{\partial j \partial s} < 0 \), \( \frac{\partial^2 P_J^c}{\partial j \partial s} < 0 \). Besides, \( P_J^f \) and \( P_J^c \) satisfy
\[
\frac{\partial P_J^f}{\partial j} = \frac{\partial P_J^c}{\partial j} + c \cdot (1 - s) \cdot (1 - \delta) \cdot (s^2 + 3 \cdot j^2),
\]
for all \( s \) and \( j \). Suppose \( s^f = s^c \). Then, because \( c \cdot (1 - s) \cdot (1 - \delta) \cdot (s^2 + 3 \cdot j^2) > 0 \), and \( \frac{\partial P_J^c}{\partial j} = 0 \) for \( j = j^c \), \( s = s^c = s^f \), it must be \( j^f < j^c \). Because \( s^f > s^c \), it is still true that \( j^f < j^c \), because \( j^c \) is decreasing in \( s^c \). Therefore it must be \( \delta_j^* = 1 \).

Substituting these values in the first order condition for the junior creditors, yields
\[
\frac{\partial P_J^f}{\partial j} = (1 - s^{GR}) \cdot S \cdot (1 - \Lambda) + h \cdot \left[ 1 - j^{GR} \cdot (1 - s^{GR}) \cdot S \right] - 2 \cdot c \cdot j^{GR} = 0,
\]
where \( \Lambda = V + s \cdot h + j \cdot h - c \cdot y_s \cdot (1 - \delta_s) \cdot s^2 \), and GR is the “government partially reimburses” scheme.

Because \( \Lambda \) is decreasing in \( y_s \cdot (1 - \delta_s) \), then using the second order conditions, it must be \( \frac{\partial^2 P_J^f}{\partial j^2} > 0 \). Given \( s^{GR} \), \( G \) is maximized with the lowest \( j \), so it must be \( y_s \cdot (1 - \delta_s) = 0 \), which implies either \( \delta_s = 1 \) or \( y_s = 0 \).

If \( \delta_s^{GR} = 1 \), then \( j^{GR} \) and \( y_s^{GR} \) solve:
\[
\frac{\partial P_c}{\partial s} = j^{GR} \cdot S - j^{GR} \cdot S \cdot \left[ V + 2 cs^{GR} + j^{GR} \cdot h \right] + j^{GR} \cdot (1 - s^{GR}) \cdot S \cdot h - 2 c s^{GR} y_s^{GR} = 0 \quad (46)
\]
\[
\frac{\partial P_J^f}{\partial j} = (1 - s^{GR}) \cdot S \cdot \left[ 1 - \left( V + 2 cs^{GR} + 2 \cdot j^{GR} \cdot h \right) \right] + h - 2 \cdot c \cdot j^{GR} = 0 \quad (47)
\]
Note that $j_{GR} > \frac{h}{c}$ because the first term in the previous expression is positive. Then, from (46)

$$
\gamma^*_S = \frac{j_{GR} \cdot S - j^*_{GR} \cdot S \cdot \left[ V + 2c s^* + j^*_{GR} \cdot h \right] + j^*_{GR} \cdot (1 - s^*_{GR}) \cdot S \cdot h}{2 \cdot c \cdot s}.
$$

(48)

• If $\gamma^*_S = 0$ then $\frac{\partial P_S}{\partial S} = 0$ and $\frac{\partial P_J}{\partial J} = 0$ define a system of equations where only $j$ is unknown, and therefore leads to an absurd.

Therefore $G$ is maximum where $\delta^*_S = \gamma^*_J = \delta^*_J = 1$, and $\gamma^*_S > 0$.

• To prove that $s^*_{GR} > \left( \frac{h}{c} \right)$, let us assume that $s^* = \left( \frac{h}{c} \right)$. Using the envelope theorem, an infinitesimal decrease $\epsilon$ in $s^*_{GR}$ increases $j^*_{GR}$ because, from Proposition 4, $\frac{\partial j^*_{GR}}{\partial s^*_{GR}} < 0$.

Therefore $G = h \cdot (s^*_{GR} + j^*_{GR}) - c \cdot (s^* + j^*_{GR})$ decreases. However, for an infinitesimal increase $\epsilon$ in $s^*_{GR}$, $j^*_{GR}$ increases and therefore $G$ increases. Therefore the optimal $\gamma^*_S > 0$ must be such that $s^*_{GR} > \left( \frac{h}{c} \right)$, $j^*_{GR} > \left( \frac{h}{c} \right)$.  

•
References


