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Comments Welcome

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Economic and Statistical Measures of Diversification Benefits

Abstract

In this paper, we develop a utility based economic measure for diversification benefits, calculated as the maximum premium that an investor is willing to pay for holding a more diversified portfolio. The utility based economic measure allows one to evaluate the expansion of the investment opportunity set by combining the information in both risk and return properties. It also offers a flexible framework to examine how investors with different tolerances for risk may respond to the expansion of the investment opportunity set. This measure is contrasted with the results of mean-variance spanning tests. Empirical analysis shows that investors enjoy substantial diversification benefits by adding emerging stock markets and major bond markets to the existing portfolio of G7 stock markets. Investors' risk tolerance affects their evaluation of new assets. Short-sale constraints reduce, but do not eliminate, diversification benefits.

Keywords: Measuring Diversification Benefits, Asset Allocation, Short Sale Constraint

JEL Codes: G11, G15

Contents

1 Introduction

The past twenty years has seen an increasing interest among investors to diversify across borders and across different asset classes. The trend of diversification is matched by the increasing accessibility to international capital markets and a greater variety of investment vehicles. A natural question that investors seek to answer is whether these assets indeed provide them with significant diversification benefits, and if so, how we can measure these benefits.

Huberman and Kandel (1987) are the first to provide a formal tool to analyze these questions. They develop a regression based test for whether the introduction of some new assets significantly expands the mean-variance frontier spanned by existing assets. Kan and Zhou (2001) provide a comprehensive review of this literature and refine the regression based mean-variance spanning test by deriving both the asymptotic and small sample properties of this test. Motivated by the duality between mean-variance frontiers of asset returns and Hansen-Jagannathan (1991) bounds on stochastic discount factors, Ferson, Forester, and Keim (1993), and DeSantis(1995) propose a series of GMM tests for mean-variance spanning. Bekaert and Urias (1996) show the equivalence between these two types of tests and apply the latter to test the diversification benefits of closed-end country funds. More recently, Hentschel and Long (2002) have developed a numeraire portfolio test of market integration based on non-arbitrage principle and have applied it to evaluate the diversification benefits of emerging markets.

The first two types of tests are statistical tests in nature. They are not sufficient to answer the questions that we pose earlier because they do not measure the economic value of adding new assets to the existing investment universe. Besides, measuring diversification benefits involves evaluating the changes in both risk and return as the investment universe changes. Evaluating the trade-off between risk and return depends on investors' risk tolerance, which is not present in the framework of these statistical tests. Moreover, it is also difficult to incorporate various practical investment constraints in this framework. The numeraire portfolio approach, however, is very computationally intensive as it searches the entire investment universe for the existence of one particular portfolio.

This paper contributes to the existing literature by developing a utility based economic measure of diversification benefits. This measure evaluates the economic significance of adding new assets by calculating the maximum entry fee that the accessability of the new assets warrants. It is a function of the statistical properties of the new and existing assets, as well as the risk tolerance of investors. This makes it particularly suitable for illustrating the link between investors' attitudes towards risk and their diversification decisions. Moreover, short-sale constraints can be conveniently incorporated in this framework.

The usefulness of this measurement is illustrated in two cases. In the first case, investors attempt the diversification benefits of adding three regional emerging markets (EM3 Stocks) to their existing investment universe of G7 stock markets. This case is related to the vast literature of Home Bias, whose central debate is whether investors can benefit from investing in international financial markets, especially the emerging markets. [2] In the second case, we evaluate the diversification benefits of adding four major long-term government bonds (G4 Bonds) to the G7 stock portfolio. Campbell and Viceira (2002) investigate the dynamic asset allocation of stocks, bonds and cash. However, they only use U.S. data and do not examine international evidence. Empirical results of these two cases show that the introduction of the major bond markets and the emerging markets helps improve investors' asset allocation. The diversification benefits are substantial and should not be ignored. Short-sale constraints reduce, but do not eliminate diversification benefits. The more risk averse the investors are, the smaller the impact of short sale constraint constraint.

The remaining part of this paper is organized as follows: Section 2 briefly reviews the literature of testing and measuring diversification benefits; Sec-

[2] The contemporary *Home Bias* literature is originated by Gruber (1968), and Levy and Sarnat (1970). Other notable works include Grauer and Hakansson (1987), and French and Poterba (1991). The recent papers related to emerging markets include Bekaert and Harvey (1995), Baxter and Jermann (1997), Cooper and Kaplanis (1994), DeSantis (1994), and Harvey (1995).

tion 3 explains the methodology and various robustness tests; Section 4 discusses the data used in this paper; Section 5 evaluates the diversification benefits of three regional emerging equity markets and G4 bond markets respectively for the investors who already have access to G7 stock markets; Section 6 concludes.

2 Literature Review

The existing literature on testing diversification benefits generally falls under three categories: regression based mean-variance spanning tests, stochastic discount factor (SDF) based GMM tests, and numeraire portfolio tests based on non-arbitrage principles. In this section, we briefly review these tests and discuss the inadequacy of using their results as the measure for diversification benefits. In Section 5, we contrast the results of the new measure developed in this paper with the results of the first two types of test. The comparisons offer additional insights into the determining factors of our results.

2.1 Regression Based Spanning Tests

Huberman and Kandel (1987) introduce a simple regression based test for the hypothesis that the introduction of new assets (test assets) expands the mean-variance frontier spanned by the existing assets (benchmark assets):

$$r = \alpha + BR + e \tag{1}$$

where r is the $NX1$ return vector of test assets and R is the $KX1$ return vector of benchmark assets. They show that this hypothesis is equivalent to imposing the following restrictions on the Equation 1:

$$\begin{aligned} \alpha &= 0 \\ B \times \iota_K &= \iota_N \end{aligned} \tag{2}$$

They show that these restrictions can be tested using a likelihood ratio (LR) test. Jobson and Korkie (1989) extend this test to a multivariate setting. Kan and Zhou (2001) point out an error in Huberman and Kandel (1987) test, as well as the lack of power of the LR test in multivariate cases. To resolve the problem with the power of the LR test, they suggest that one should look at the results of all other equivalent tests, i.e. Lagrangian multiplier test (LM) and Wald test. Moreover, they derive both the asymptotic and small sample test statistics for all three equivalent tests. They also show that regression based mean-variance spanning tests perform similarly with SDF based GMM tests when the underlying asset return are normally distributed. However, the former tests show superior properties to the latter when the asset returns distribution deviates from normality.

2.2 SDF Based GMM Tests

Ferson, Forester, and Keim (1993), DeSantis(1995), and Bekaert and Urias (1996) propose a series of tests for mean-variance spanning by exploiting the duality between the mean-variance frontiers of asset returns and the Hansen-Jagannathan (1991) bounds on stochastic discount factors. Under Hansen-Jagannathan (1991) framework, the conditional asset pricing restriction can be expressed as follows:

$$E\left(\left(R_{t+1}+\iota\right)m_{t+1}|\Phi_t\right)=\iota \tag{3}$$

where R_{t+1} is the vector of asset returns, m_{t+1} is the stochastic discount factor, and Φ_t is the information set at time t. Define m^{α}_{t+1} as the stochastic discount factor formed from the projection of m_{t+1} onto asset returns R_{t+1},

$$m^{\alpha}_{t+1}\equiv\alpha+\left[R_{t+1}-E\left(R_{t+1}\right)\right]'\beta^{(\alpha)} \tag{4}$$

Let $\beta^{(\alpha)}_N$ be the portion of $\beta^{(\alpha)}$ corresponding to the test assets, then the spanning restrictions in the Hansen-Jagannathan (1991) framework are:

$$\begin{aligned}\beta^{(\alpha)}_N &= 0_N \\ E\left(R_{t+1}m^{\alpha}_{t+1}\right)+E\left(m^{\alpha}_{t+1}\right)\iota &= \iota\end{aligned} \tag{5}$$

Bekaert and Urias (1996) explicitly show that the restrictions above are equivalent to those in Huberman and Kandel (1987) as in Equation 2, and these restrictions can be tested using a GMM test. However, since GMM tests generally rely on the validity of its asymptotic distribution, they do not perform well in small samples. Bekaert and Urias (1996) compare the LR test of Huberman and Kandel (1987) with the GMM test. Using various simulations, they find a lack of power of the latter test, especially in small sample.

2.3 Numeraire Portfolio Tests

Long (1990) shows that, when trading is restricted to a set of assets, the non-arbitrage condition is equivalent to the existence of a numeraire portfolio of these assets that satisfy the following conditions:

$$\begin{aligned} Prob_t\left[1 + r_{N,t+1} > 0\right] &= 1 \\ E_t\left[\frac{1 + r_{i,t+1}}{1 + r_{N,t+1}}\right] &= 1 \end{aligned} \tag{6}$$

where $r_{i,t+1}$ represents any asset return, and $r_{N,t+1}$ is a numeraire portfolio.[3] Based on this concept, Hentschel and Long (2002) develop a test of market integration by searching for such a numeraire portfolio. They also extend this approach to the context of measuring diversification benefits.

2.4 Discussion about Mean-Variance Spanning Tests

The first two types of mean-variance spanning tests are based on statistical properties of asset returns, rather than any economic assumption. Aside from the stability and validity issues of these tests, there are four problems associated with using these statistical measures to evaluate the diversification benefits when test assets are included:

[3] Chen and Knez (1995) develop a concept of market integration similar to Long's definition of numeraire portfolio.

First, these statistical tests do not address the magnitude of efficiency improvement associated with the introduction of test assets. Both the regression based tests and the SDF based tests only test the hypothesis that the new mean-variance frontier coincides with the frontier of the benchmark assets. Even when this hypothesis can be safely rejected, the test statistics still do not indicate how much the investors can benefit from access to new assets. Kan and Zhou (2001) also argue that the economic significance and statistical significance can reverse order.

Second, mean-variance frontier is important only if the asset returns are normally distributed or the investors have quadratic utility. It is well-known that these two assumptions are neither theoretically nor empirically valid: asset return is bounded from below by -100%, which makes it impossible to be normally distributed; quadratic utility implies that utility does not monotonically increase with wealth (or consumption).

Third, it is conceivable that investors with different risk aversion attitudes are likely to evaluate the same set of new assets differently. This is because the addition of new assets provides investors with different combinations of risk reduction and return enhancement. However, evaluating the trade-off between risk and return entirely depends on investors' risk tolerance. Empirical results to be presented in Section 5 lend strong support to this argument. The statistical measures do not allow for the role played by investors' attitudes towards risk.

Fourth, it is difficult to incorporate some practical constraints of asset allocation, such as transaction cost and short-sale constraints, in these statistical tests. Recently, De Roon, Nijman, and Werker (2001) develop a test for mean-variance spanning with short-sale constraints. However, their test produces counter-intuitive results, which indicates that, as the number of markets and/or constraints increases, there is a quick loss of power of the test. Li, Sarkar, and Wang (2001) attempt to tackle this problem using a Bayesian approach.

A lot of effort has been devoted to the development of sound economic measures for diversification benefits to complement the statistical measures. Goetzmann, Li, and Rouwenhorst (2001) evaluate diversification benefits by measuring how much risk can be diversified away in an equally-weighted portfolio. However, their approach only focuses on risk reduction and ignores return enhancement. Similar to Kandel, McCulloch, and Stambaugh (1995), Kan and Zhou (2001) propose a two-step approach by looking at expected return increase and risk reduction separately, which does not yield a unified measure. Bekaert and Urias (1996) use Sharpe Ratios to measure the improvement of diversification benefits when investors invest in closed-end country funds. However, as Goetzmann, Ingersoll, Spiegel, and Welch (2002) point out, Sharpe Ratio can be a misleading indicator, especially when the asset returns are not normally distributed.

In this paper, we use certainty equivalence (CE) between benchmark and new asset universes as the economic measure of diversification benefits.[4] This approach is similar to the transaction cost measure proposed by Hentschel and Long (2001). In a sense, we refine their measure by deriving the closed-form solution of indirect utility function under log-normal assumption. As a special case, this intermediate step helps us reduce the dependence of our results on computer simulation and optimization procedures.

3 Methodology

3.1 Certainty Equivalence

The certainty equivalence can be considered as the maximum percentage fee that an investor is willing to pay in order to gain access to a set of new (test) assets. We define it as follows:

$$CE = \sup \left[\delta | U\left(R_{bm}\right) \leq U\left(R_{aa} - \delta\right) \right] \tag{7}$$

where $U\left(\cdot\right)$ is the utility function of the investor, R_{bm} is the return vector of benchmark assets, and R_{aa} is the return vector of all assets, including both benchmark and test assets. This approach has the following features

[4] Certainty Equivalence (CE) has been extensively used in the literature of test portfolio efficiency, e.g., Stambaugh and Pastor (2000).

that complement the statistical tests reviewed in the previous section: CE is designed to measure the economic value of adding test assets to the existing investment universe of benchmark assets. It is a function of investors' risk tolerance and can also conveniently incorporate portfolio constraints, such as no short-sale constraints and asymmetric transaction cost. Although this approach is applicable to any form of utility function and any distribution of asset returns, we show below that, as a special case, greater computation accuracy of CE can be achieved under the assumptions of power utility function and log-normal asset returns.

3.2 Indirect Utility Function

Let us assume that investors have power utility function over one-period terminal wealth $U(W) = \frac{W^{1-\gamma}}{1-\gamma}$. Then their investment objective is to maximize one-period terminal expected utility:

$$\max E\left(\frac{W^{1-\gamma}}{1-\gamma}\right) \tag{8}$$

Define that $r_w = d\log W_t$ as the growth rate of wealth. Under the log-normality assumption of asset returns, Equation (8) is equivalent to:

$$\max \ E(r_w) + \frac{1}{2}(1-\gamma)Var(r_w) \tag{9}$$

this transformation holds exactly for log-normal asset returns. It holds approximately in small time interval if the distribution of asset returns deviates from log-normality. Let us now turn to the representation of the return of wealth. We can represent the vector of asset prices as P_t and the log asset return vector, $d\log P_t$, as

$$d\log P_t = udt + \sigma dw_t \tag{10}$$

Then the vector of arithmetic returns assumes the following form:

$$\frac{dP_t}{P_t} = \left(\mu + \frac{1}{2}\left[\sigma_i\sigma_i'\right]\right)dt + \sigma dw_t \tag{11}$$

where $[\sigma_i\sigma_i']$ is a vector whose ith element is σ_i^2

The wealth process follows

$$\frac{dW_t}{W_t} = \alpha' \frac{dP_t}{P_t} = \alpha' \left(\mu + \frac{1}{2}\left[\sigma_i\sigma_i'\right]\right) dt + \alpha'\sigma dw_t$$

which leads to

$$\left(\frac{dW_t}{W_t}\right)^2 = \alpha'\sigma\sigma'\alpha dt$$

Applying Ito's lemma, the log wealth process is:

$$\begin{aligned} d\log W_t &= \frac{dW_t}{W_t} - \frac{1}{2}\left(\frac{dW_t}{W_t}\right)^2 \\ &= \alpha'\left(\mu + \frac{1}{2}\left[\sigma_i\sigma_i'\right]\right) dt - \frac{1}{2}\alpha'\sigma\sigma'\alpha dt + \alpha'\sigma dw_t \\ &= \alpha' \cdot d\log P_t + \frac{1}{2}\left(\alpha'\left[\sigma_i\sigma_i'\right] - \frac{1}{2}\alpha'\sigma\sigma'\alpha\right) dt_t \end{aligned}$$

note that $\alpha' \cdot d\log P_t \neq d\log W_t$. let $dt = 1; \sigma^2 = [\sigma_i\sigma_i']; \Sigma = \sigma\sigma'$, we arrive at the following equations

$$E\left(d\log W_t\right) = \alpha'\left(\mu + \frac{1}{2}\sigma^2\right) - \frac{1}{2}\alpha'\Sigma\alpha \qquad (12)$$

$$Var\left(d\log W_t\right) = \alpha'\sigma\sigma'\alpha \qquad (13)$$

Applying (12) and (13) to (9), investors'optimization problem can be transformed as the following:

$$\begin{aligned} &\max_{\{\alpha\}} \alpha'\left(\mu + \frac{1}{2}\sigma^2\right) - \frac{1}{2}\gamma\alpha'\Sigma\alpha \\ s.t. \quad &: \quad \alpha'\iota = 1 \end{aligned} \qquad (14)$$

We summarize the optimization problem in the following proposition.

Proposition 1 *Investors who seek to solve the optimization problem as in*

(14) *will choose the following portfolio* α^**:*

$$\alpha^* = \frac{1}{\gamma}\Sigma^{-1}\left[\left(\mu + \frac{1}{2}\sigma^2\right) - \left(\frac{i'\Sigma^{-1}\mu + \frac{1}{2}i'\Sigma^{-1}\sigma^2 - \gamma}{i'\Sigma^{-1}i}\right)i\right] \tag{15}$$

Their maximal utility can be represented as:

$$U\left(\alpha^*\right) = U\left(\overline{\mu}, \Sigma\right) = \frac{1}{2\gamma}\left[\overline{\mu}'\Sigma^{-1}\overline{\mu} - \frac{1}{i'\Sigma^{-1}i}\left(\overline{\mu}'\Sigma^{-1}i - \gamma\right)^2\right] \tag{16}$$

where $\overline{\mu} = \left(\mu + \frac{\sigma^2}{2}\right)$

Proof. See Appendix ■

This proposition shows that, with these two assumptions, one can derive the closed-form solution of the optimal portfolio weights, and therefore the indirect maximal utility as a function of the first two moments of the asset returns. Substituting (16) into (7) allows us to transform CE to

$$CE = \sup\left[\delta | U^*\left(\overline{\mu}_{bm}, \Sigma_{bm}\right) \leq U^*\left(\overline{\mu}_{aa} - \delta, \Sigma_{aa}\right)\right] \tag{17}$$

where $U^*\left(\mu, \Sigma\right)$ is the maximal utility given first two moments of asset returns, $\left(\overline{\mu}_{bm}, \Sigma_{bm}\right)$ are the first two moments of the benchmark assets and $\left(\overline{\mu}_{aa}, \Sigma_{aa}\right)$ are the first two moments of all assets. These two assumptions are plausible since power utility function implies constant relative risk aversion, and log-normality of asset returns conforms to the empirical distribution better than normality. Compared with Equation (7), Equation (17) involves only one step of searching for δ as the step of searching for optimal portfolio weights is already imbed in Equation 16. This procedure increases the accuracy of our results and reduces the reliance on computer optimizations.

3.3 Robustness Validations

It is worth noting that CE is bounded below by zero as investors cannot be hurt by the expansion of investment opportunity set. Therefore, the crucial question is weather the certainty equivalence computed in (17) is indeed greater than zero. We attempt to answer this question using two methods: bootstrapping under the alternative hypothesis and the null hypothesis respectively.

Method I is a straightforward exercise of bootstrapping under the alternative hypothesis that new assets significantly expand the investment opportunity set. We randomly draw a subset of data sample along the time dimension while preserving its cross sectional structure. Then for each draw, we compute the CE for the sub-sample according to (17). This gives us an empirical distribution of CE generated under the alternative hypothesis. We then compute the standard deviation of the bootstrapped $CE's$ and present them along with the $CE's$ for the full sample.

Method II is a bootstrapping exercise under the null hypothesis that the test assets are redundant assets with respect to the investment universe of the existing benchmark assets. We first look for a set of replicating portfolios of the benchmark assets that satisfy the following conditions: 1)they are on the mean-variance frontier spanned only by the benchmark assets; 2)they have the same mean returns as the test assets. The existence of these replicating portfolios is guaranteed by the continuous property of the mean-variance frontier. In the second step, we randomly generate a set of simulating portfolios as the mean-preserving spread of the replicating portfolios. Clearly, the simulating portfolios also have the same expected returns as the test assets, but are strictly dominated by the mean-variance frontier spanned by the benchmark assets. The CE in this case should be zero. We repeat the second step for a number of times, which gives us the empirical distribution of CE under the null hypothesis. The following sections, we report the percentage of the simulated $CE's$ that exceeds the $CE's$ computed for the full sample as its $P-value$.

3.4 Short-Sale Constraints

The major mutual funds and pension funds are not allowed to hold short positions in assets. The practice of short-sale is often either outlawed or discouraged in many countries (Bris, Goetzmann, and Zhu, 2002). Short-sale constraints probably represent one of the most important deviations of practical portfolio optimization problem from its theoretical counterpart. Sharpe (1991), De Roon, Nijman, and Werker (2001), Li, Sarkar, and Wang (2002) consider the impact of the short-sale constraints on diversification decisions under various frameworks.

When short-sale constraints are imposed, the closed-form solution of indirect maximal utility (16) is not obtainable. In this case, investors' optimization problem is (14) augmented with additional restrictions on non-negative portfolio weights.

$$\begin{aligned} & \max_{\{\alpha\}} \alpha' \left(\mu + \frac{1}{2}\sigma^2 \right) - \frac{1}{2}\gamma\alpha'\Sigma\alpha \\ s.t. \quad : \quad & \alpha'\iota = 1 \text{ and } \alpha_i \geq 0 \text{ for any } i \end{aligned} \tag{18}$$

Here, we search for the optimal solution for CE in (7) in conjunction with the above optimization problem. Unfortunately, the aforementioned Method II of robustness check is not applicable in this case. This is because the mean-variance frontier spanned by the benchmark assets under short-sale constraints is no longer continuous. There is no guarantee that we can find the frontier portfolios that have the same mean return as the test assets.

4 Data

In this paper, we use the total return indexes of G7 stock markets (U.S., U.K., France, Germany, Japan, Canada, Italy), G4 long-term government bond indexes (U.S., U.K., Germany, Japan), and 3 regional emerging market indexes. All data series are monthly and converted to U.S. dollar returns.

The G7 stock indexes are from MSCI (Morgan Stanley Capital Indexes). MSCI country stock indexes are market value-weighted indexes. The value-weight indexes are less prone to the nonsynchronous trading problem than the equally-weighted indexes. These indexes are from January 1970 to 2002.

The G4 long-term government bond indexes are from IFC-IMF database.

They are computed using long-term (usually 10 years) benchmark government bond indexes. The benchmark government bonds are those with larger market value and better liquidity compared to most of the bonds with similar maturity. We choose only these four bond markets because the government bond markets are much larger in these four countries than in other countries. These indexes are also from January 1970 to 2002.

The three regional stock indexes are Latin American index (including Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela); Far East index (including China, Hong Kong, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan and Thailand); and emerging European and Mid-East index (including Czech Republic, Hungary, Poland and Russia, Israel, Jordan, Turkey). These indexes are from January 1970 to 2002 and are significantly shorter than the other G7 stocks and G4 bonds. Moreover, the compositions of these three regional indexes also change as China, Russia, and East European countries are only included in these indexes from 1990 onwards. It is important to note that, although each individual country's stock index is value-weighted, the regional indexes are the equally-weighted across countries.

Table 1 presents the sample statistics of the returns computed using these indexes. The average stock returns of G7 markets are generally greater than their bond returns for both the 1970-2002 and 1988-2002 intervals, with the noticeable exception of Japan. Japanese average stock returns has been merely above zero in the 1988-2002 period, much lower than its average bond return of 5.09% during this period. It is rather surprising that Far East and EUME (Emerging Europe and Mid East) stock indexes do not seem to outperform those of G7 countries, although their volatility is higher. Table 2 shows the first 6-lag autocorrelation of asset returns. The evidence of stock returns being autocorrelated is very weak, whereas U.K. and German government bonds returns seem to be strongly autocorrelated.

5 Empirical Analysis

5.1 Case I: Adding EM3 Stocks to G7 Stocks

In this subsection, we evaluate the diversification benefits of EM3 regional emerging stock markets for investors who already have access to G7 stock markets. This exercise is limited to the 1988-2002 period when the regional emerging market indexes are available.

Panel A of Table 3 presents the diversification benefits associated with this expansion of investment opportunity set, measured as the certainty equivalence between the old and the new investment universes when investors are not subject to any portfolio constraints. We allow the relative risk aversion coefficient (γ) to range from 1.1 to 20.[5] Higher γ indicates lower risk tolerance. This table shows that depending on their risk tolerance, the entry fee that investors are willing to pay for access to regional emerging markets ranges from almost zero, for the most risk averse investor, to as high as 17%, for the least averse investor. The certainty equivalence decreases monotonically with the constant relative risk aversion coefficient, γ, which indicates that it is the less risk averse investors who take stronger interest in investing in emerging markets. The results of bootstrapping under the alternative hypothesis (validation method I) indicate that the standard deviation of the CE is relatively small compared to their level, leaving us little statistical doubt that the CE is significantly above 0. Similarly, the results of bootstrapping under the null hypothesis (validation method II) confirm those of method I. The low P-values that we observe in Panel A suggest that, if the regional emerging markets are redundant assets to investors given their access to the G7 stock markets, then it is very unlikely that the actual CE can be as large as what we observe in this table.

Table 4 presents the results of other mean-variance spanning tests. Panel A shows both the asymptotic and small sample test results of regression based tests (see Kan and Zhou, 2001, for details). Panel B is based on the GMM test proposed by Bekaert and Urias (1996). The regression based

[5] We do not compute the $\gamma = 1$ case because, although power utility function equals log utility function in limit $\gamma = 1$, it does impose computational problem.

tests all fail to reject at 5% significance level the hypothesis that the mean-variance frontiers of the new and the old investment universe coincide. The GMM test strongly favors the null hypothesis. This seems to be at odds with the previous table, which indicates significant diversification benefits. Two factor may have contributed to this discrepancy. First, it is well known that the power of these statistical tests tends to be low, especially the GMM test, which relies on the validity of asymptotic distribution in a relatively small sample. Second, it is possible that the difference between the old and new asset universes is indeed too small for statistical tests to detect. However, the less risk averse investors are able to achieve high CE by forming very risky portfolios to exploit the small opportunity.

Figure 1 clearly illustrates this point. The mean-variance frontier of the new asset universe narrowly expands that of the old universe. The portfolios of risk averse investors cluster around the part of the frontier where the two asset universes do not differ much. In contrast, the portfolios of less risk averse investors expand along the upper part of the frontier very rapidly.

5.1.1 Imposing Short-Sale Constraints

When investors are restricted from short selling, as in the case of large asset management institutions, investors' CE decreases sharply for all investors. Panel B of Table 3 presents the measure of diversification benefits when short sale constraints are imposed. In particular, the CE of the least risk averse investor ($\gamma = 1.1$) decreases sharply from 17.35% to 10.21% when short-sale constraints are imposed. This provides the evidence that many of the large $CE's$ that these investors have are achieved by short sale. Figure 2 shows the frontiers with short-sale constraints. When short-sale constraints are imposed, mean-variance frontier is no longer continuous and is also limited by the asset with highest expected return. It is clear from the picture that the availability of regional emerging markets greatly expands the opportunity set of investors by offering them, especially the less risk averse ones, assets with higher mean returns. However, the more risk averse investors shy away these assets because they are also more risky.

Figure 3 compares the $CE's$ with and without short-sale constraints as functions of risk tolerance. Overall, the access to regional emerging markets is much more valuable to less risk averse investors. Although the value of this access diminishes when short-sale constraints are imposed, it does not completely disappear. As γ increases, the impact of short-sale constraints becomes much smaller.

Table 5 shows the optimal weights of G7 plus EM3 stock portfolios for investors with different risk tolerances, with and without short-sale constraints. It further explains how imposing short-sale constraints affects investors' portfolio decisions. When there are no short-sale constraints, U.S., France and Latin America are the favorite *long* markets, and EUME, Japan, and Canada are the favorite *short* markets. As risk aversion increases, investors place most of their investment in U.S. and U.K. markets. These investors show hardly any interest in short-selling. When short-sale constraints are imposed, the most aggressive investors choose to invest everything in Latin American Markets, while the most risk averse investors stay with U.S. and U.K. markets. It is rather remarkable that U.S. stocks market accounts for more than 50% of investors' most desirable portfolios, regardless of their attitudes towards risk and whether short-sale constraints are imposed. The attractiveness of U.S. stock market can be attributed to its usual performance during the 1990s, which is unlikely to be repeated in the future.

5.2 Case II: G7 Stocks Plus G4 Bonds

In this section, we evaluate the diversification benefits of G4 bond markets for investors who already have access to G7 stock markets for the period of 1970-2002.

Including G4 bond markets in G7 stock portfolio yields some interesting results. Panel A of Table 6 show that the relationship between CE and risk tolerance is no longer monotonic. CE initially decreases with γ, then the direction changes for $\gamma = 5$. This indicates that investors at either end of the risk tolerance value the access of G4 bond markets higher than the investors in the middle. However, the most risk averse investors enjoy much higher CE than the least risk averse ones. The results of both validation methods strongly indicate that the inclusion of G4 bond markets significantly improve the G7 stock portfolio. Table 7 present the results of the regression based mean-variance test and the GMM test. These results are consistent with Table 6 and reject the hypothesis that the old and the new frontiers coincide.

Figure 4 shows that the the greater expansion of the frontier with the inclusion of G4 bond markets lies at the lower part of the frontier. This indicates that the primary benefit of investing in major bond markets lies in risk reduction, rather than return enhancement.

5.2.1 Imposing Short-Sale Constraints

From Panel B of Table 6, we can see that the relationship between CE and γ is again monotonic when short-sale constraints are imposed. This explains why the least risk averse investors also value G4 bonds – they take more aggressive portfolios in G7 stocks by shorting bonds. Figure 5 shows that, with short-sale constraints, the expansion of the investment opportunity set happens only at the lower part of the efficiency frontier. Therefore, the diversification benefits are appreciated only by the more risk risk averse investors.

Figure 6 helps to summarize the effect of including G4 bond markets: short-sale constraints only affect the aggressive investors, whose CE dramatically decreases to 0 when the constraints are imposed. The more risk averse investors are the primary beneficiaries of adding G4 bond markets. CE increases from 0 to about 10% for $\gamma = 20$.

Table 8 shows the optimal weights of G7 stock plus G4 bond portfolios for investors with different risk tolerances, with and without short-sale constraints. When there are no short-sale constraints, stocks of the U.S., the U.K., France, Germany, and Japan are the favorite *long* markets. Stocks of Canada and Italy, and bonds of the U.S. and Germany are the favorite *short* markets. As risk aversion increases, investors place most of their investment in U.S. stock market and G4 bond markets. Again, these investors show little interest in short-selling. When short-sale constraints are imposed, the most aggressive investors choose to concentrate on stocks of U.K., France and Japan, while the most risk averse investors stay with U.S. stock market and the G4 bond markets. U.S. stock and bond combined accounts for more than 50% of the optimal portfolios of the more risk averse investors.

6 Conclusion

In this paper, we develop a utility based economic measure for diversification benefits, calculated as the maximum premium that an investor is willing to pay for holding a more diversified portfolio. This measure is contrasted with the regression based mean-variance spanning tests and the GMM tests based on Hansen and Jagannathan (1991) duality. The utility based economic measure allows one to evaluate the expansion of the investment opportunity set by combining the information in both risk and return properties. It also offers a flexible framework in which to examine how investors with different tolerances for risk may respond to the expansion of the investment opportunity set. Using this measure, we show that the introduction of the major bond markets and the emerging markets still helps to reduce the overall risk and increase the expected return. The diversification benefits are substantial and should not be ignored. In both cases, U.S. financial markets account for the majority of the optimal portfolios of strongly risk averse investors.

Appendix

Proof of Proposition 1.

$$\begin{aligned} & \max_{\{\alpha\}} \alpha' \left(\mu + \frac{1}{2}\sigma^2\right) - \frac{1}{2}\gamma\alpha'\Sigma\alpha \\ s.t. \quad : \quad & \alpha'\iota = 1 \end{aligned} \tag{19}$$

is equivalent to the following lagrangian equation

$$\max \left\{ \alpha' \left(\mu + \frac{1}{2}\sigma^2\right) - \frac{1}{2}\gamma\alpha'\Sigma\alpha + \lambda \left[1 - \alpha' i\right] \right\} \tag{20}$$

The two first order conditions are:

$$\left(\mu + \frac{1}{2}\sigma^2\right) - \gamma\Sigma\alpha - \lambda i = 0 \;\text{ and } 1 = \alpha' i \tag{21}$$

This allows to solve the optimal portfolio choice

$$\alpha^* = \frac{1}{\gamma}\Sigma^{-1}\left[\left(\mu + \frac{1}{2}\sigma^2\right) - \left(\frac{i'\Sigma^{-1}\mu + \frac{1}{2}i'\Sigma^{-1}\sigma^2 - \gamma}{i'\Sigma^{-1}i}\right) i\right] \tag{22}$$

The maximal utility the investors can achieve is:

$$U\left(\alpha^*\right) = \alpha^{*\prime}\left(\mu + \frac{1}{2}\sigma^2\right) - \frac{1}{2}\gamma\alpha^{*\prime}\Sigma\alpha^* \tag{23}$$

Let $A = \mu'\Sigma^{-1}\mu$; $B = \sigma^{2\prime}\Sigma^{-1}\mu$; $C = i'\Sigma^{-1}i$; $D = \mu'\Sigma^{-1}i$; $E = \sigma^{2\prime}\Sigma^{-1}i$; $F = \sigma^{2\prime}\Sigma^{-1}\sigma^2$; then

$$\alpha^{*\prime}\mu = \frac{1}{\gamma}\left[A + B - \frac{D}{C}\left(D + \frac{1}{2}E - \gamma\right)\right] \tag{24}$$

$$\alpha^{*\prime}\frac{1}{2}\sigma^2 = \frac{1}{2\gamma}\left[B + F - \frac{E}{C}\left(D + \frac{1}{2}E - \gamma\right)\right] \tag{25}$$

$$-\frac{1}{2}\gamma\alpha^{*\prime}\Sigma\alpha^* = -\frac{1}{2\gamma}\left[A + B + \frac{1}{4}F - \frac{1}{C}\left(D + \frac{1}{2}E\right)^2 + \frac{\gamma^2}{C}\right] \tag{26}$$

Substituting $(24), (25), (26)$ into (23), the indirect utility function is:

$$\begin{aligned} U(\alpha^*) &= \frac{1}{2\gamma}(A + B + \frac{1}{4}F - \frac{1}{C}\left(D + \frac{1}{2}E - \gamma\right)^2) \\ &= \frac{1}{2\gamma}\left[\left(\mu + \frac{\sigma^2}{2}\right)' \Sigma^{-1}\left(\mu + \frac{\sigma^2}{2}\right) - \frac{1}{i'\Sigma^{-1}i}\left(\left(\mu + \frac{\sigma^2}{2}\right)' \Sigma^{-1} i - \gamma\right)^2\right] \\ &= \frac{1}{2\gamma}\left[\overline{\mu}'\Sigma^{-1}\overline{\mu} - \frac{1}{i'\Sigma^{-1}i}\left(\overline{\mu}'\Sigma^{-1}i - \gamma\right)^2\right] \end{aligned} \tag{27}$$

where $\overline{\mu} = \left(\mu + \frac{\sigma^2}{2}\right)$ ■

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Tables and Figures

Table 1: Sample Statistics

This table presents annualized sample statistics of G7 stock returns, G4 long-term government bond returns, and EM3 emerging market regional stock returns. G7 (U.S., U.K., France, Germany, Japan, Canada, Italy) stock returns are computed using MSCI market capitalization-weighted stock indexes for each country. G4 (U.S., U.K., Germany, Japan) bond returns are calculated using IMF-IFS long-term government bond indexes. The EM3 emerging market regional stock returns are Far East (including China, Hong Kong, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan and Thailand), EUME (East Europe and Middle East, including Czech Republic, Hungary, Poland, Russia, Israel, Jordan, Turkey), and Latin America (Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela). They are calculated using MSCI regional stock indexes. MSCI regional stock indexes are equally-weighted across countries, although the country indexes are market capitalization weighted. All returns are monthly and converted to U.S. returns.

Panel A: G7 stock	U.S.	U.K.	France	Germany	Japan	Canada	Italy
(1988-2002)							
Mean (%)	13.87	10.45	13.11	11.76	0.47	9.77	8.61
S.D. (%)	14.20	15.97	19.72	21.10	24.51	17.70	24.14
(1970-2002)							
Mean (%)	11.91	13.86	13.66	12.31	13.24	10.98	9.54
S.D. (%)	15.54	23.50	22.82	20.65	22.82	19.41	25.68
Panel B: G4 Bonds	U.S.	U.K.	Germany	Japan			
(1988-2002)							
Mean (%)	9.81	9.85	5.03	5.09			
S.D. (%)	8.30	13.49	11.76	13.95			
(1970-2002)							
Mean (%)	9.28	10.64	10.03	10.68			
S.D. (%)	10.44	15.79	12.84	14.07			
Panel C: EM Equity	Far East	EUME	Latin America				
(1988-2002)							
Mean (%)	11.48	10.02	24.59				
S.D. (%)	26.50	30.14	33.17				

Table 2: Autocorrelation

This table examines the autocorrelation structure of G7 (U.S., U.K., France, Germany, Japan, Canada, Italy) stock returns, G4 (U.S., U.K., Germany, Japan) bond returns, and EM3 emerging market regional stock returns (Far East, East Europe and Middle East, Latin America. * indicates significant at 5% level.

Panel A: G7 Stock	U.S.	U.K.	France	Germany	Japan	Canada	Italy
1-Lag	-0.002	0.079	0.054	-0.017	0.088	0.035	0.067
2-Lag	-0.040	-0.101*	-0.020	-0.012	-0.010	-0.083	-0.035
3-Lag	0.020	0.063*	0.115	0.050	0.112	0.048	0.098
4-Lag	-0.010	0.029	0.031	0.073	0.041	-0.051	0.078
5-Lag	0.088	-0.129*	0.000	-0.101	0.053	0.056	0.014
6-Lag	-0.060	-0.052*	0.025	0.043	-0.006	0.061	0.122*
Panel B: G4 Bonds	U.S.	U.K.	Germany	Japan			
1-Lag	0.058	0.236*	0.170*	0.116*			
2-Lag	-0.021	-0.053*	0.038*	0.022			
3-Lag	-0.121	-0.077*	0.004*	0.037			
4-Lag	0.021	-0.048*	-0.033*	0.014			
5-Lag	0.038	-0.026*	-0.038*	0.013			
6-Lag	0.026	-0.089*	-0.056*	-0.075			
Panel C: EM Equity	Far East	EUME	Latin				
1-Lag	0.112	0.118	0.026				
2-Lag	0.065	-0.022	0.003				
3-Lag	-0.069	0.028	-0.058				
4-Lag	-0.101	0.027	-0.089				
5-Lag	-0.008	-0.026	0.039				
6-Lag	-0.044	-0.044	-0.084				

Table 3: Certainty Equivalence of Expanding from G7 stocks to EM3 Regional Stocks (1988-2002)

This table evaluates the diversification benefits of adding EM3 emerging market regional stocks (Far East, East Europe and Middle East, and Latin America) to G7 stock markets for investors with different risk tolerance. γ is the relative risk aversion coefficient. Larger γ stands for greater risk aversion. Panel A presents the certainty equivalence (CE) without short sale constraint. The results of two robustness validation methods are provided. Method I bootstraps the CE under alternative hypothesis and provides the bootstrapped standard deviation of CE. Method II bootstraps the CE under null hypothesis, i.e., no diversification benefits. For Method II, P-values are provided. Panel B presents the CE with short-sale constraints. Bootstrapped CE of Method I is provided. Method II is not feasible with short-sale constraint.

γ	1.1	2	3	5	7	10	15	20
Panel A: No Short-Sale Constraints								
Certainty Equivalence								
(%)	17.35	9.36	6.12	3.55	2.46	1.68	1.11	0.87
Validation Method I								
S.D. (%)	1.28	0.56	0.4	0.29	0.21	0.15	0.1	0.09
Validation Method II								
P-Value	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.00
Panel B: with Short-Sale Constraints								
Certainty Equivalence								
(%)	10.21	6.36	4.19	2.5	1.72	1.27	0.74	0.52
Validation Method I								
S.D. (%)	0.67	0.54	0.38	0.21	0.16	0.11	0.08	0.06

Table 4: Other Spanning Tests of Expanding from G7 Stocks to EM3 Regional Stocks (1988-2002)

This table presents the results of mean-variance spanning tests for adding EM3 regional stock markets to the portfolio of G7 stock markets. Panel A shows both the asymptotic and small sample P-values of regression based mean spanning test (Kan and Zhou, 2001). Panel B shows the P-value of the stochastic discount factor based GMM test (Bekaert and Urias, 1996).

Panel A: Regression Based Test Kan and Zhou (2001)			
	Wald Test	Likelihood Ratio Test	Lagrangian Multiplier Test
Asymptotic P-Value	0.309	0.317	0.324
Samll Sample P-Value	0.155	0.330	0.349
Panel B: GMM Test Bekaert and Urias (1996) Wald Test P-Value	0.996		

Table 5: Portfolio Weights of G7 Stocks Plus EM3 Stocks (1988-2002)

This table shows the optimal portfolio weights of G7 (U.S., U.K., France, Germany, Japan, Canada, Italy) stocks plus EM3 (Far East, East Europe and Middle East, and Latin America) stocks for investors with different risk tolerances. γ is the relative risk aversion coefficient. Larger γ stands for greater risk aversion. Panel A shows the portfolio weights without short-sale constraints and Panel B shows the portfolio weights with short-sale constraints.

γ	1.1	2	3	5	7	10	15	20
Panel A: without: Short-Sale Constraints								
G7 Stocks								
U.S.	2.572	1.691	1.333	1.046	0.923	0.830	0.759	0.723
U.K.	-0.319	-0.053	0.056	0.142	0.179	0.207	0.229	0.240
France	1.623	0.893	0.595	0.358	0.256	0.179	0.120	0.090
Germany	-0.055	-0.038	-0.032	-0.027	-0.024	-0.023	-0.021	-0.021
Japan	-2.099	-1.124	-0.728	-0.410	-0.274	-0.172	-0.092	-0.053
Canada	-2.389	-1.300	-0.857	-0.502	-0.349	-0.235	-0.147	-0.102
Italy	-0.046	0.016	0.041	0.061	0.070	0.076	0.081	0.084
EM3 Stocks								
Far East	0.158	0.049	0.004	-0.032	-0.047	-0.058	-0.067	-0.072
EUME	-0.390	-0.199	-0.121	-0.059	-0.032	-0.012	0.004	0.012
Latin	1.943	1.066	0.708	0.422	0.299	0.208	0.136	0.100
Panel B: with Short-Sale Constraints								
G7 Stocks								
U.S.	0.000	0.132	0.240	0.488	0.512	0.606	0.537	0.600
U.K.	0.000	0.000	0.000	0.000	0.099	0.103	0.230	0.207
France	0.000	0.038	0.124	0.177	0.147	0.097	0.088	0.049
Germany	0.000	0.000	0.059	0.001	0.000	0.018	0.001	0.000
Japan	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Canada	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Italy	0.000	0.000	0.000	0.000	0.000	0.016	0.031	0.066
EM3 Stocks								
Far East	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
EUME	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
Latin	1.000	0.830	0.578	0.335	0.243	0.159	0.114	0.077

Table 6: Certainty Equivalence of Expanding from G7 stocks to G4 Bonds (1970-2002)

This table evaluates the diversification benefits of adding G4 long-term government bonds (U.S., U.K., Germany, Japan) to G7 stock markets for investors with different risk tolerance. γ is the relative risk aversion coefficient. Larger γ stands for greater risk aversion. Panel A presents the certainty equivalence (CE) without short sale constraint. The results of two robustness validation methods are provided. Method I bootstraps the CE under alternative hypothesis and provides the bootstrapped standard deviation of CE. Method II bootstraps the CE under null hypothesis, i.e., no diversification benefits. For Method II, P-values are provided. Panel B presents the CE with short-sale constraint. Bootstrapped CE of Method I is provided. Method II is not feasible with short-sale constraints.

γ	1.1	2	3	5	7	10	15	20
Panel A: No Short-Sale Constraints								
Certainty Equivalence								
(%)	1.35	0.16	0.07	0.75	1.77	3.47	6.47	9.55
Validation Method I								
S.D. (%)	0.14	0.06	0.02	0.05	0.07	0.1	0.16	0.17
Validation Method II								
P-Value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Panel B: with Short-Sale Constraints								
Certainty Equivalence								
(%)	0.00	0.02	0.04	0.67	1.81	3.48	6.47	9.52
Validation Method I								
S.D. (%)	0.01	0.01	0.03	0.09	0.09	0.09	0.14	0.12

Table 7: Other Tests of G7 Equity Market Plus G4 Bonds (1970-2002)

This table presents the results of mean-variance spanning tests for adding G4 long-term government bond markets to the portfolio of G7 stock markets. Panel A shows both the asymptotic and small sample P-values of regression based mean spanning test (Kan and Zhou, 2001). Panel B shows the P-value of the stochastic discount factor based GMM test (Bekaert and Urias, 1996).

Panel A: Regression Based Test Kan and Zhou (2001)			
	Wald Test	Likelihood Ratio Test	Lagrangian Multiplier Test
Asymptotic P-Value	0.000	0.000	0.000
Samll Sample P-Value	0.000	0.000	0.000
Panel B: GMM Test Bekaert and Urias (1996)			
Wald Test P-Value	0.000		

Table 8: Portfolio Weights of G7 Stocks G4 Bonds (1970-2002)

This table shows the optimal portfolio weights of G7 (U.S., U.K., France, Germany, Japan, Canada, Italy) stocks plus G4 (U.S., U.K., Japan, Germany) bonds for investors with different risk tolerances. γ is the relative risk aversion coefficient. Larger γ stands for greater risk aversion. Panel A shows the portfolio weights without short-sale constraints and Panel B shows the portfolio weights with short-sale constraints.

γ	1.1	2	3	5	7	10	15	20
Panel A: without Short-Sale Constraints								
G7 Stocks								
U.S.	0.779	0.533	0.432	0.352	0.318	0.292	0.272	0.262
U.K.	0.713	0.358	0.213	0.097	0.047	0.010	-0.019	-0.034
France	0.553	0.285	0.176	0.089	0.051	0.023	0.001	-0.010
Germany	0.400	0.208	0.130	0.067	0.040	0.020	0.004	-0.003
Japan	0.679	0.362	0.233	0.130	0.086	0.053	0.027	0.014
Canada	-0.667	-0.359	-0.233	-0.132	-0.089	-0.057	-0.032	-0.019
Italy	-0.309	-0.147	-0.081	-0.028	-0.006	0.011	0.025	0.031
G4 Bonds								
U.S.	-0.558	-0.136	0.036	0.174	0.233	0.277	0.311	0.329
U.K.	-0.047	0.027	0.057	0.081	0.091	0.099	0.105	0.108
Japan	0.119	0.140	0.148	0.155	0.158	0.160	0.162	0.163
Germany	-0.661	-0.270	-0.111	0.016	0.071	0.112	0.144	0.160
Panel B: with Short-Sale Constraints								
G7 Stocks								
U.S.	0.000	0.159	0.189	0.172	0.242	0.238	0.228	0.220
U.K.	0.353	0.250	0.174	0.058	0.042	0.014	0.000	0.000
France	0.272	0.210	0.147	0.071	0.041	0.026	0.000	0.000
Germany	0.057	0.114	0.098	0.072	0.040	0.014	0.009	0.000
Japan	0.318	0.267	0.210	0.114	0.081	0.041	0.015	0.013
Canada	0.000	0.000	0.000	0.054	0.000	0.000	0.000	0.000
Italy	0.000	0.000	0.000	0.000	0.000	0.005	0.026	0.022
G4 Bonds								
U.S.	0.000	0.000	0.027	0.128	0.214	0.266	0.308	0.332
U.K.	0.000	0.000	0.058	0.107	0.095	0.102	0.095	0.086
Japan	0.000	0.000	0.080	0.123	0.140	0.157	0.166	0.167
Germany	0.000	0.000	0.016	0.102	0.105	0.136	0.153	0.160

Figure 1: Adding EM3 Regional Stocks to G7 Stocks without Portfolio Constraints

This figure shows the change of efficiency frontier and investors' optimal portfolios when the investment opportunity set expands from G7 (U.S., U.K., France, Germany, Japan, Canada, and Italy) stock markets to include EM3 (Far East, East Europe and Middle East, and Latin America) regional stock markets. No portfolio constraints are imposed. The dashed line is the mean-standard deviation frontier of G7 stock markets. The solid line is the mean-standard deviation frontier of G7 plus EM3 stock markets. The diamond plots are the G7 stock markets and the circle plots are the EM3 regional stock markets. The dot plots along the frontiers are investors' optimal portfolios in G7, and G7 plus EM3 stocks respectively.

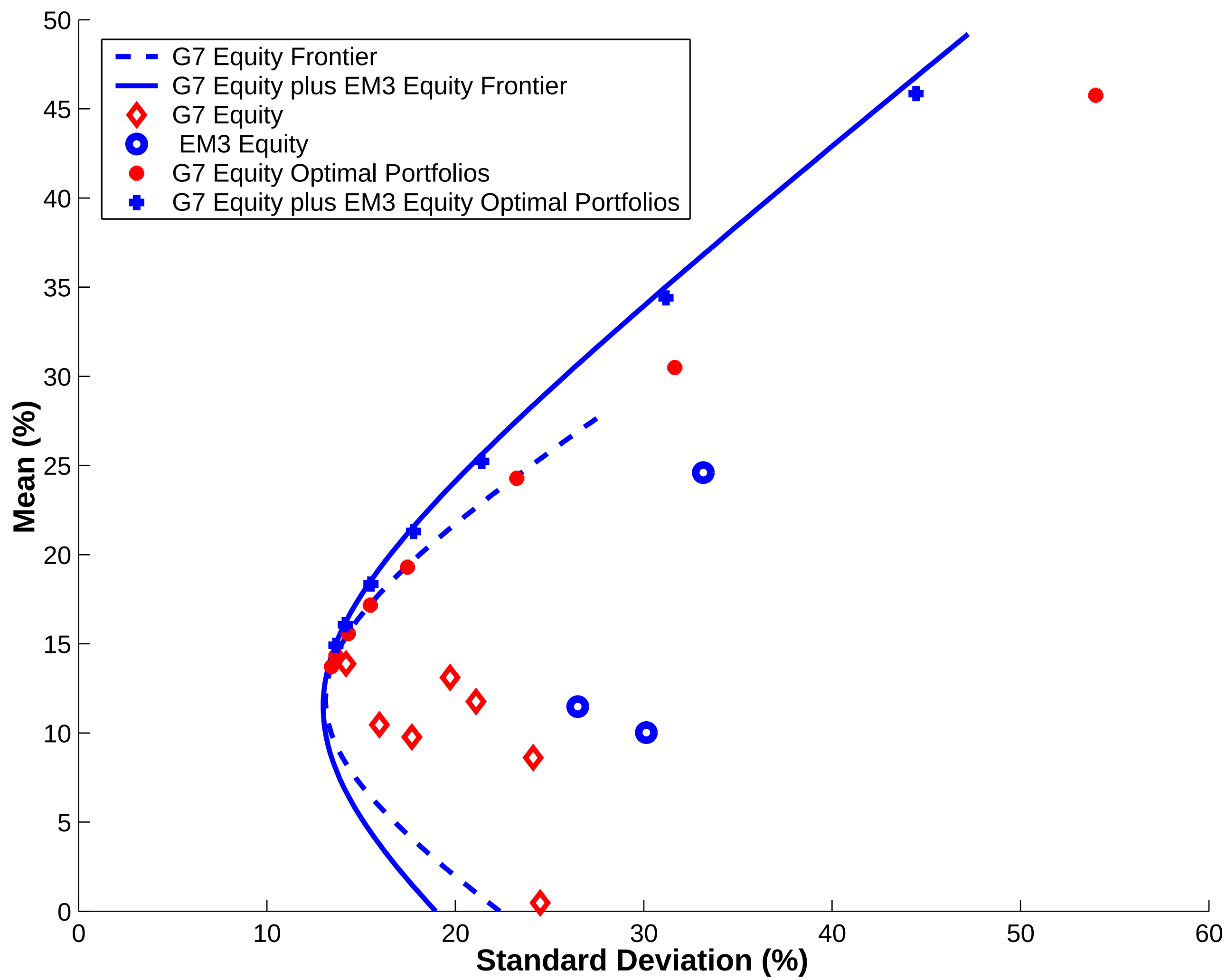

Figure 2: Adding EM3 Regional Stocks to G7 Stocks with Short-Sale Constraints

This figure shows the change of efficiency frontier and investors' optimal portfolios when the investment opportunity set expands from G7 (U.S., U.K., France, Germany, Japan, Canada, and Italy) stock markets to include EM3 (Far East, East Europe and Middle East, and Latin America) regional stock markets. Short-Sale constraints are imposed. The dashed line is the mean-standard deviation frontier of G7 stock markets. The solid line is the mean-standard deviation frontier of G7 plus EM3 stock markets. The diamond plots are the G7 stock markets and the circle plots are the EM3 regional stock markets. The dot plots along the frontiers are investors' optimal portfolios in G7, and G7 plus EM3 stocks respectively.

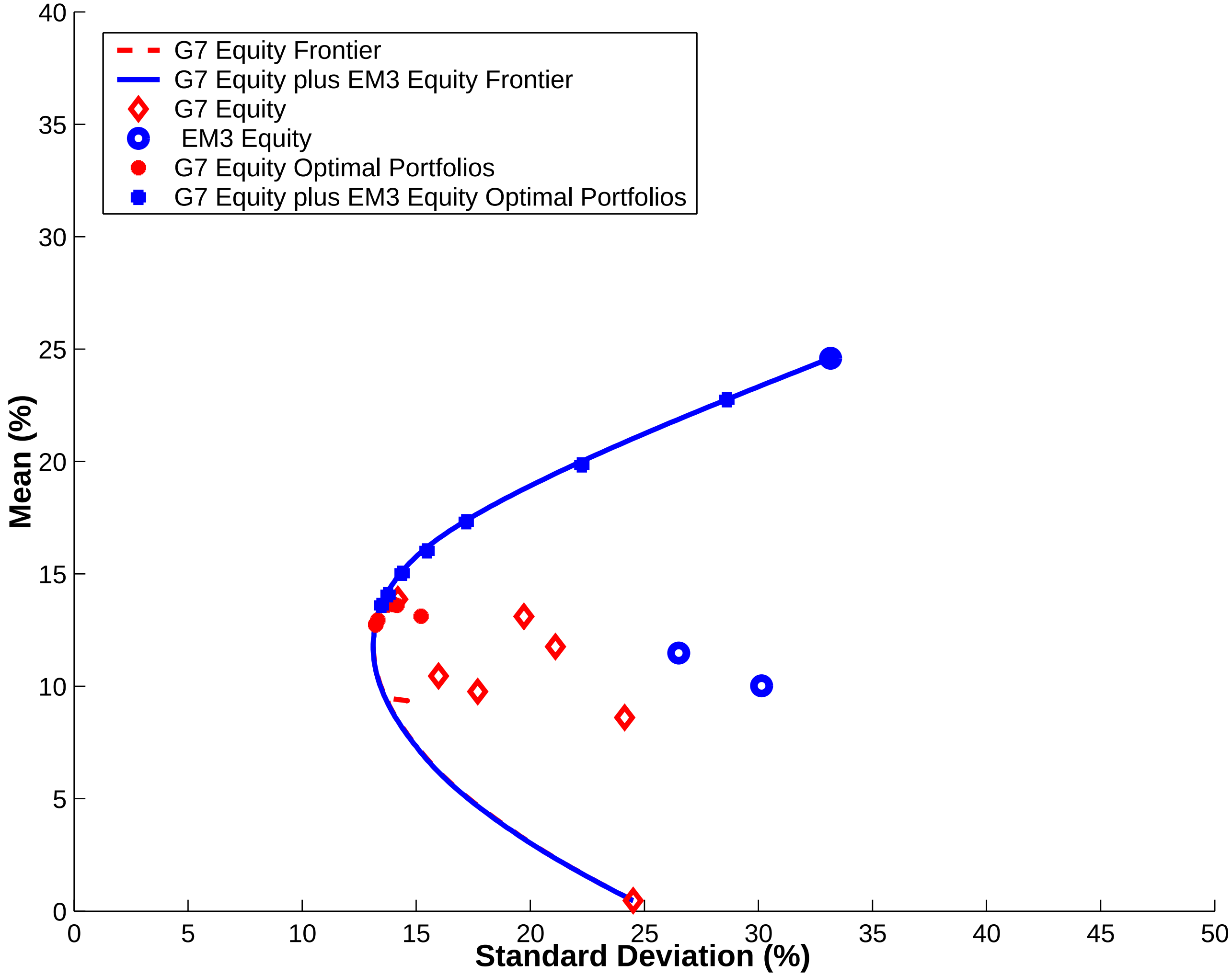

Figure 3: Certainty Equivalence of Adding EM3 Regional Stocks to G7 Stocks

This figure shows the CE (certainty equivalence) for expanding from G7 (U.S., U.K., France, Germany, Japan, Canada, and Italy) stocks to include EM3 (Far East, East Europe and Middle East, and Latin America) regional stocks for investors with different risk tolerances. The solid line shows the CE without short-sale constraints, and the dashed line shows the CE with short-sale constraints.

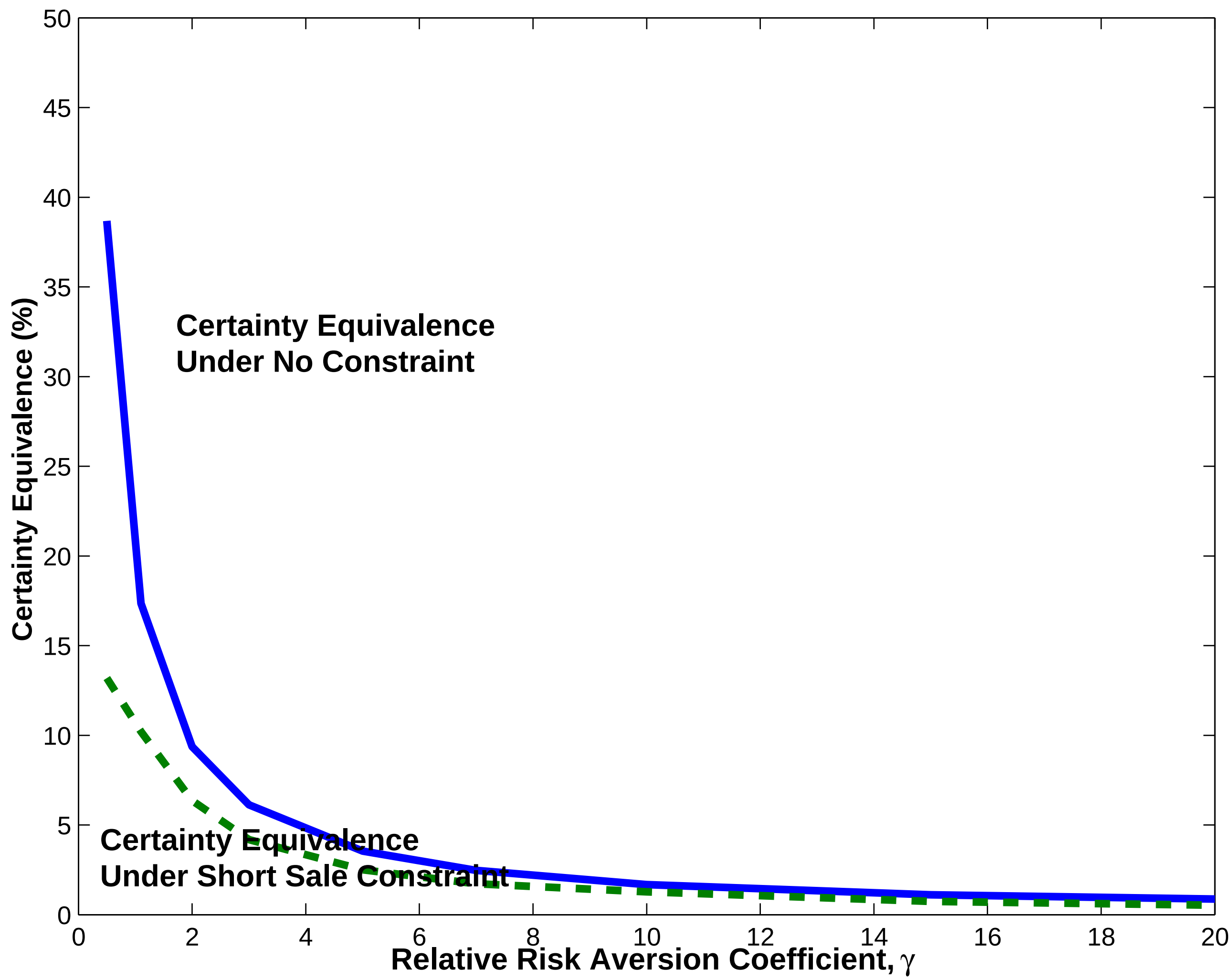

Figure 4: Adding G4 Bonds to G7 Stocks without Portfolio Constraints

This figure shows the change of efficiency frontier and investors' optimal portfolios when the investment opportunity set expands from G7 (U.S., U.K., France, Germany, Japan, Canada, and Italy) stock markets to include G4 bond (U.S., U.K., Germany, and Japan) markets. No portfolio constraints are imposed. The dashed line is the mean-standard deviation frontier of G7 stock markets. The solid line is the mean-standard deviation frontier of G7 stocks plus G4 bonds. The diamond plots are the G7 stock markets and the circle plots are the G4 bond markets. The dot plots along the frontiers are investors' optimal portfolios in G7 stocks, and G7 stocks plus G4 bonds respectively.

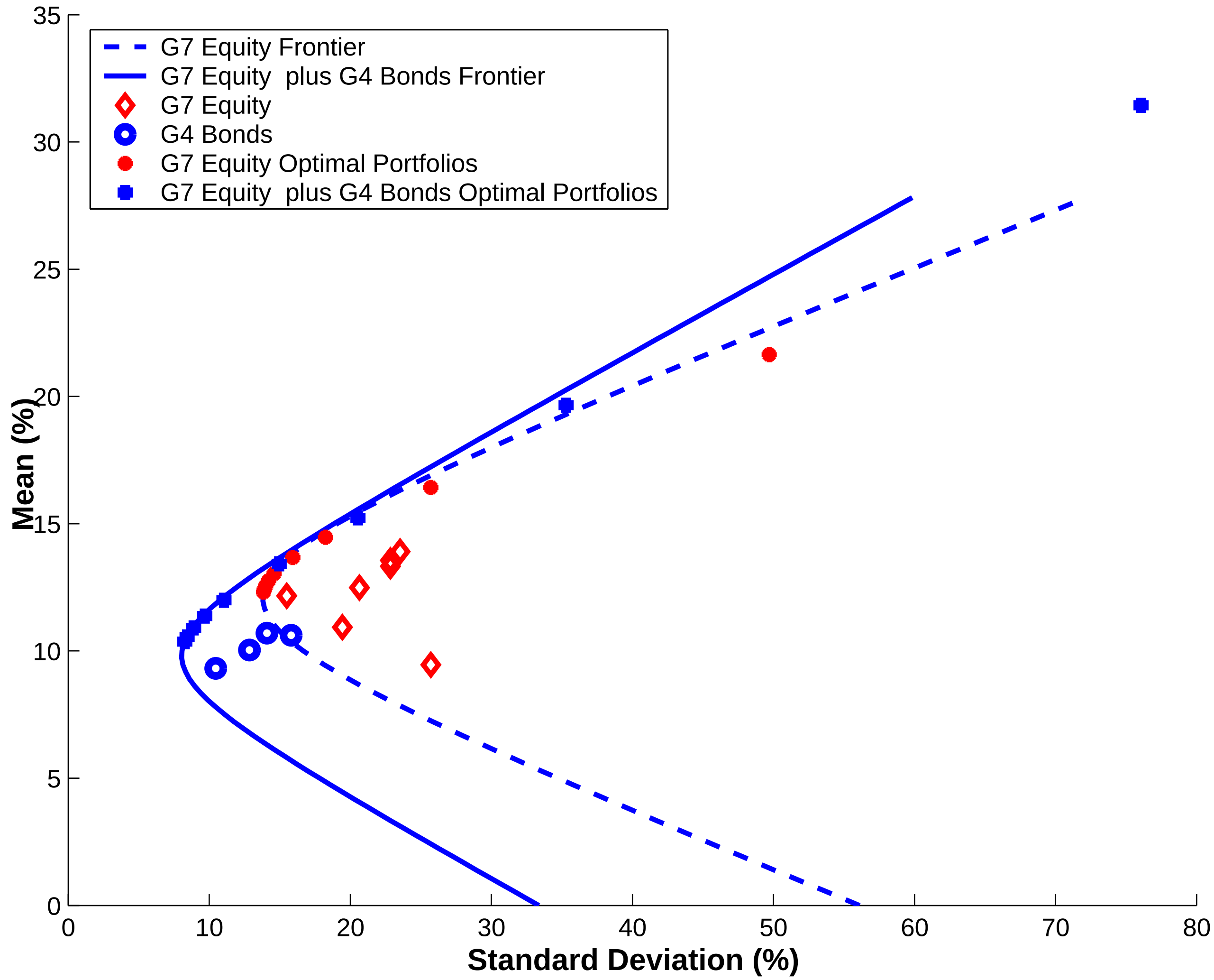

Figure 5: Adding G4 Bonds to G7 Stocks with Short-Sale Constraints

This figure shows the change of efficiency frontier and investors' optimal portfolios when the investment opportunity set expands from G7 (U.S., U.K., France, Germany, Japan, Canada, and Italy) stock markets to include G4 bond (U.S., U.K., Germany, and Japan) markets. Short-sale constraints are imposed. The dashed line is the mean-standard deviation frontier of G7 stock markets. The solid line is the mean-standard deviation frontier of G7 stocks plus G4 bonds. The diamond plots are the G7 stock markets and the circle plots are the G4 bond markets. The dot plots along the frontiers are investors' optimal portfolios in G7 stocks, and G7 stocks plus G4 bonds respectively.

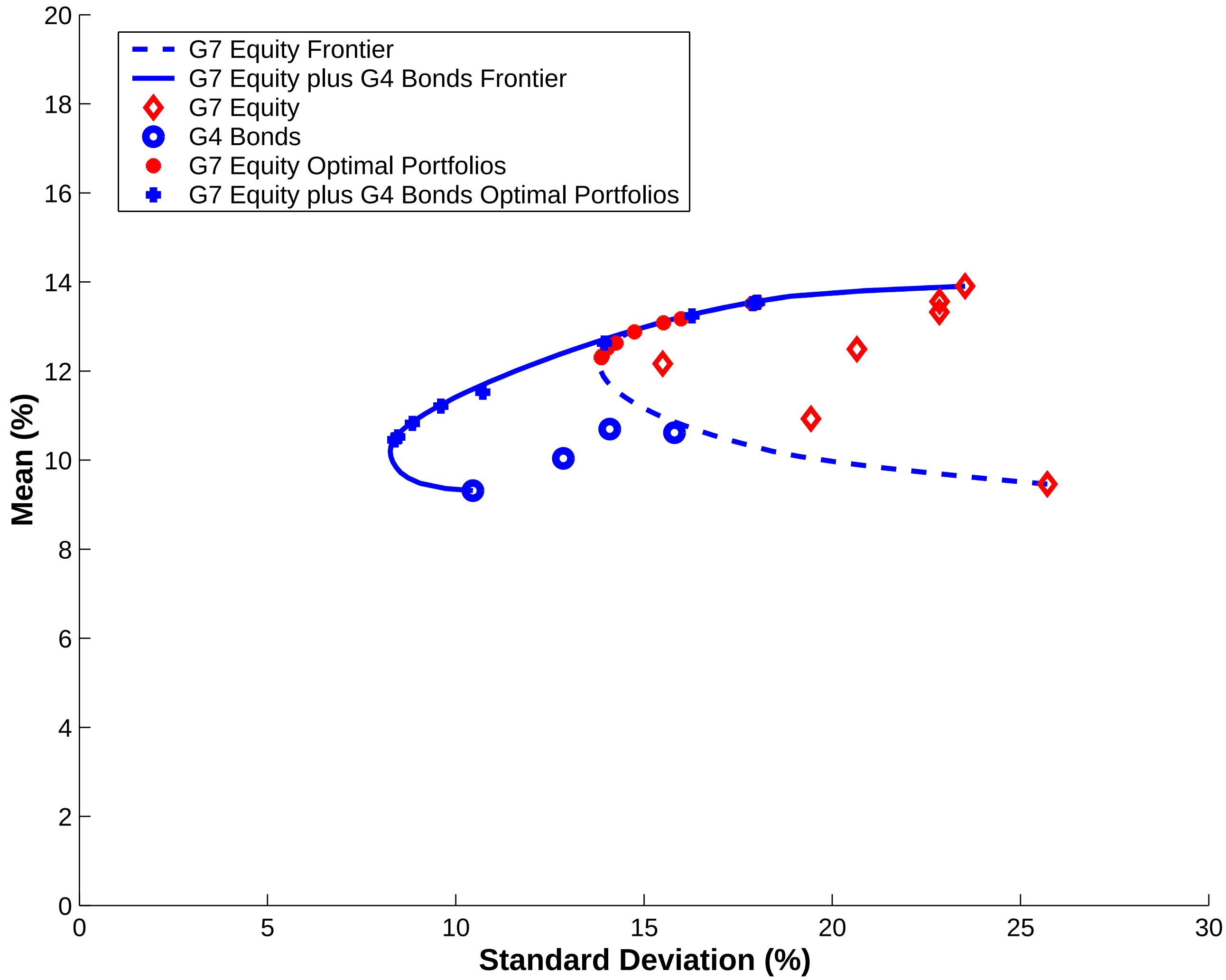

Figure 6: Certainty Equivalence of Adding G4 bonds to G7 Stocks

This figure shows the CE (certainty equivalence) for expanding from G7 (U.S., U.K., France, Germany, Japan, Canada, and Italy) stocks to include G4 bonds (U.S., U.K., Germany, and Japan) for investors with different risk tolerance. The solid line shows the CE without short-sale constraints, and the dashed line shows the CE with short-sale constraints.