



# Cowles Foundation

for Research in Economics  
at Yale University

Cowles Foundation Discussion Paper No. 1298

April 2001  
Revised June 2003

## ESTIMATES OF THE EFFECTIVENESS OF MONETARY POLICY

Ray C. Fair

This paper can be downloaded without charge from the  
Social Science Research Network Electronic Paper Collection:  
<http://ssrn.com/abstract=274017>

An index to the working papers in the  
Cowles Foundation Discussion Paper Series is located at:  
<http://cowles.econ.yale.edu/P/au/DINDEX.htm>

# Estimates of the Effectiveness of Monetary Policy

Ray C. Fair\*

revised June 2003

## Abstract

This paper examines various interest rate rules, as well as policies derived by solving optimal control problems, for their ability to dampen economic fluctuations caused by random shocks. A tax rate rule is also considered. A multicountry econometric model is used for the experiments. The results differ sharply from those obtained using recent models in which the coefficient on inflation in the nominal interest rate rule must be greater than one in order for the economy to be stable.

## 1 Introduction

Many of the recent studies examining monetary policy effects have used macroeconomic models with a particular feature. In these models an increase in inflation with the nominal interest rate held constant is expansionary, and the economy is not stable unless the coefficient on inflation in the nominal interest rate rule is greater than one.<sup>1</sup> Although these models, which will be called “modern-view”

---

\*Cowles Foundation and International Center for Finance, Yale University, New Haven, CT 06520-8281. Voice: 203-432-3715; Fax: 203-432-6167; e-mail: ray.fair@yale.edu; website: <http://fairmodel.econ.yale.edu>.

<sup>1</sup>These models are nicely summarized in Clarida, Galí, and Gertler (1999), Taylor (2000), and Romer (2000). They are used in Clarida, Galí, and Gertler (2000) and in virtually all the papers in Taylor (1999) to examine monetary policy rules.

models, have been widely used, it is not clear that they have adequately captured the effects of inflation shocks. The results in Fair (2002), which are based on a structural macroeconomic model (discussed below), suggest that an increase in inflation is contractionary even when the nominal interest rate is held constant. Essentially the same results are reached in Giordani (2002) from analyzing VAR models. The results from these two quite different approaches thus cast doubt on a key property of modern-view models. They suggest that the coefficient on inflation in the interest rate rule does not have to be greater than one for the economy to be stable, which, as will be seen, has important consequences for monetary policy.

This paper uses the multicountry econometric (MC) model in Fair (1994) to examine monetary policy effects. The MC model has been extensively tested, including tests for rational expectations, and it appears to be a good approximation of the economy. These tests are in Fair (1994) and on the website mentioned in the introductory footnote. The MC model is briefly outlined in the appendix. The appendix includes an explanation of the property of the model that a positive inflation shock with the nominal interest rate held constant is contractionary, which, as mentioned above, is a key difference from modern-view models. In the MC model a positive inflation shock lowers real wage income (because wages lag prices) and real wealth (because nominal wealth lags prices), both of which have a negative effect on real consumer expenditures. In addition, the estimation results suggest that households respond to nominal interest rates rather than real interest rates. There is thus no estimated positive household response to lower real interest rates when there is a positive inflation shock with nominal interest rates held constant.

Section 2 examines the stabilization features of four interest rate rules for the United States. The first is the estimated rule in the MC model, which has an estimated long run coefficient on inflation of 1.0. The other three rules are modifications of the estimated rule, with imposed long run coefficients on inflation of 0.0, 1.5, and 2.5 respectively. It will be seen that as the inflation coefficient increases there is a reduction in price variability at a cost of an increase in interest rate variability. Even the rule with a zero inflation coefficient is stabilizing, which is contrary to what would be obtained using modern-view models, since in these models the economy is not stable if the inflation coefficient is less than one.

Section 3 then computes optimal rules for particular loss functions. These solutions require a combination of stochastic simulation and solving deterministic optimal control problems, and this is the first time that such solutions have been obtained for a large scale model. It will be seen that the optimal control results are similar to those obtained using the estimated rule mentioned above for a loss function with a much higher weight on inflation than on output.

Another feature of the results in Sections 2 and 3 is that considerable variance of the endogenous variables is left using even the best interest rate rule. Section 4 then adds a fiscal policy rule—a tax rate rule—to see how much help it can be to monetary policy in trying to stabilize the economy. The results show that the tax rate rule provides some help. This is also the first time that such a rule has been analyzed using a large scale model.

## **2 Stabilization Effectiveness of Four Nominal Interest Rate Rules**

### **The Estimated U.S. Rule and Three Modifications**

There is an estimated nominal interest rate rule for each of the major countries in the MC model. The estimated rule for the United States is one of the rules used in this paper, and the following is a brief discussion of the rule. Estimated U.S. interest rate rules have a long history. The first one is in Dewald and Johnson (1963), who regressed the Treasury bill rate on a constant, the Treasury bill rate lagged once, real GNP, the unemployment rate, the balance-of-payments deficit, and the consumer price index. The next example can be found in Christian (1968), followed by many others. In 1978, I added an estimated interest rate rule to my U.S. model—Fair (1978)—and an updated version of this rule is the one used in this paper.

The main modification that has been made to the 1978 rule is the addition of a dummy variable term to account for the change in Fed operating procedure during the period 1979:4–1982:3 (to be called the “early Volcker” period).<sup>2</sup> The stated policy of the Fed during this period was that it was focusing more on monetary aggregates than it had done in the past. The estimated interest rate rule already had the lagged growth of the money supply as an explanatory variable, and the change in policy was modeled by adding the lagged growth of the money supply multiplied by a dummy variable as another explanatory variable. The dummy variable is 1

---

<sup>2</sup>Paul Volcker was chair of the Fed between 1979:3 and 1987:2, but the period in question is only 1979:4–1982:3.

for the 1979:4–1982:3 period and 0 otherwise.

The exact specification of the rule that is used in this paper is:

$$\begin{aligned} r = & \alpha_1 + \alpha_2 \dot{p} + \alpha_3 u + \alpha_4 \Delta u + \alpha_5 \dot{m}_{-1} + \alpha_6 D1 \times \dot{m}_{-1} \\ & + \alpha_7 r_{-1} + \alpha_8 \Delta r_{-1} + \alpha_9 \Delta r_{-2} + \epsilon \end{aligned} \quad (1)$$

where  $r$  is the three month Treasury bill rate,  $\dot{p}$  is the quarterly rate of inflation at an annual rate,  $u$  is the unemployment rate,  $\dot{m}$  is the quarterly rate of growth of the money supply at an annual rate, and  $D1$  equals 1 for 1979:4–1982:3 and 0 otherwise. The results of estimating equation (1) for the 1954:1–2002:3 period are presented in Table 1.<sup>3</sup>

The endogenous variables on the right hand side of equation (1) are inflation and the unemployment rate, and two stage least squares was used to estimate the equation. In the first stage regressions inflation and the unemployment rate are regressed on a set of predetermined variables (the main predetermined variables in the U.S. subset of the MC model). The predicted values from these regressions are then used in the second stage. One can look on the these regressions as those used by the Fed to predict inflation and the unemployment rate, and so it need not be assumed that the Fed has perfect foresight.

---

<sup>3</sup>Note that the three month Treasury bill rate is used for the interest rate. Although in practice the Fed controls the federal funds rate, the quarterly average of the federal funds rate and the quarterly average of the three month Treasury bill rate are so highly correlated that it makes little difference which rate is used in estimated interest rate rules using quarterly data. The money supply data are taken from the flow of funds accounts.

**Table 1**  
**Estimated U.S. Interest Rate Rule**  
 Dependent Variable is  $r$

	Coef.	t-stat.
constant	.748	4.90
$\dot{p}$	.080	4.50
$u$	-.113	-3.64
$\Delta u$	-.756	-5.65
$\dot{m}_{-1}$	.011	1.88
$D1 \times \dot{m}_{-1}$	.217	9.52
$r_{-1}$	.909	46.16
$\Delta r_{-1}$	.225	3.97
$\Delta r_{-2}$	-.327	-6.36
SE	.476	
$R^2$	.970	
DW	1.83	
Wald (p-value)	15.32 (.053)	

Estimation period: 1954:1–2002:3 (195 obs.)

Estimation technique: two stage least squares

$r$  = three month Treasury bill rate

$\dot{p}$  = inflation rate

$u$  = unemployment rate

$\dot{m}$  = growth rate of the money supply

$D1 = 1$  for 1979:4–1982:3; 0 otherwise

Wald test: see text

Equation (1) is a “leaning against the wind” equation.  $r$  is estimated to depend positively on the inflation rate and the lagged growth of the money supply and negatively on the unemployment rate and the change in the unemployment rate. Adjustment and smoothing effects are captured by the lagged values of  $r$ . The coefficient on lagged money supply growth is .228 (= .217 + .011) for the early Volcker period compared to only .011 before and after, which is consistent with the Fed’s stated policy of focusing more on monetary aggregates during this period. This way of accounting for the Fed policy shift does not, of course, capture the richness of the change in behavior, but at least it seems to capture some of the

change.

The Wald test in Table 1 is of the hypothesis that the coefficients in the rule are the same before the early Volcker period as after (but not including the early Volcker period). The Wald statistic is distributed as  $\chi^2$  with (in this case) 8 degrees of freedom. The p-value is .053, and so the hypothesis of stability is not rejected at the 5 percent level. Other tests of the equation, including other stability tests, are reported in Fair (2001) for the sample period ending in 1999:3.

Assuming that in the long run  $\dot{m}$  equals  $\dot{p}$ , the long run coefficient on inflation in Table 1 is 1.0  $[(.080+.011)/(1 - .909)]$ . As noted in the Introduction, the other three rules have imposed long run coefficients of 0.0, 1.5, and 2.5 respectively. This was done for each rule by changing the coefficient for  $\dot{p}$  in the estimated rule in Table 1. The respective coefficients are -.011, .1255, and .2165. None of the other coefficients in the estimated equation were changed for the three rules. This process is similar to that followed for the studies in Taylor (1999), where the five main rules tried had inflation coefficients varying from 1.2 to 3.0. No inflation coefficient less than 1.0 was tried in these studies because the models, which are modern-view models, are not stable in this case.

The MC model is a standard structural macroeconometric model in the Cowles Commission tradition, where the structural equations are estimated by a consistent technique (2SLS). When a coefficient in the interest rate rule is changed, this changes the reduced form of the model. If the model were linear, analytic expressions for the reduced form equations could be derived, and one could see directly how the reduced form coefficients were changed. Since the model is nonlinear, no analytic expressions are available, and the model must be solved by an iterative



technique (in the present case, the Gauss-Seidel technique). It is still the case, of course, that the reduced form is changed when the inflation coefficient in the rule is changed, and the solution accounts for this.

## **The Stochastic Simulation Procedure**

The four interest rate rules are examined using stochastic simulation. The focus in this paper, as in much of the literature, is on variances, not means. The aim of monetary policy is taken to smooth the effects of shocks. In order to examine the ability of monetary policy to do this, one needs an estimate of the likely shocks that monetary policy would need to smooth, and this can be done by means of stochastic simulation. Given an econometric model, shocks can be generated by drawing errors.

There are 362 stochastic equations in the MC model, 191 quarterly and 171 annual. There is an estimated error term for each of these equations for each period. Although the equations do not all have the same estimation period, the period 1976–1998 is common to almost all equations.<sup>4</sup> There are thus available 23 vectors of annual error terms and 92 vectors of quarterly error terms. These vectors are taken as estimates of the economic shocks, and they are drawn in the manner discussed below. Since these vectors are vectors of the historical shocks, they pick up the historical correlations of the error terms. If, for example, shocks in two consumption equations are highly positively correlated, the error terms in the two equations will tend to be high together or low together.

---

<sup>4</sup>For the few equations whose estimation periods began later than 1976, zero errors were used for the missing observations.

The period used for the stabilization experiments is 1994:1–1998:4, five years or 20 quarters. Since the concern here is with stabilization around base paths and not with positions of the base paths themselves, it does not matter much which path is chosen for the base path. The choice here is simply to take as the base path the historical path. The base path is generated by adding the historical errors to the equations and taking them to be exogenous. Thus, for all the stochastic simulations in this paper the historical errors are added to the model and the draws are around these errors.

Each trial for the stochastic simulation is a dynamic deterministic simulation for 1994:1–1998:4 using a particular draw of the error terms. For each of the five years for a given trial an integer is drawn between 1 and 23 with probability  $1/23$  for each integer. This draw determines which of the 23 vectors of annual error terms is used for that year. The four vectors of quarterly error terms used are the four that correspond to that year. Each trial is thus based on drawing five integers. The solution of the model for this trial is an estimate of what the world economy would have been like had the particular drawn error terms actually occurred. (Remember that the drawn error terms are on top of the historical error terms for 1994:1–1998:4, which are always used.) The number of trials taken is 100, so 100 world economic outcomes for 1994:1–1998:4 are available for analysis.<sup>5</sup>

---

<sup>5</sup>Another way of drawing error terms would be from an estimated distribution. Let  $\hat{V}$  be an estimate of the  $362 \times 362$  covariance matrix  $V$  of the error terms. One could, for example, assume that the error terms are multivariate normal and draw errors from the  $N(\hat{\mu}_t, \hat{V})$  distribution, where  $\hat{\mu}_t$  is the vector of the historical errors for  $t$ . Because of the quarterly-annual difference,  $\hat{V}$  would have to be taken to be block diagonal, one quarterly block and one annual block. Even for this matrix, however, there are not enough observations to estimate all the nonzero elements, and so many other zero restrictions would have to be imposed. The advantage of drawing the historical error vectors is that no distributional assumption has to be made and no zero restrictions have to be

The historical errors are added to the interest rate rule, but no errors are drawn for it. Adding the historical errors means that when the model inclusive of the rule is solved with no errors for any equation drawn, a perfect tracking solution results.<sup>6</sup> Not drawing errors for the rule means that the Fed does not behave randomly but simply follows the rule.

Let  $y_t^j$  be the predicted value of endogenous variable  $y$  for quarter  $t$  on trial  $j$ , and let  $y_t^*$  be the base (actual) value. How best to summarize the  $100 \times 20$  values of  $y_t^j$ ? One possibility for a variability measure is to compute the variability of  $y_t^j$  around  $y_t^*$  for each  $t$ :  $(1/J) \sum_{j=1}^J (y_t^j - y_t^*)^2$ , where  $J$  is the total number of trials.<sup>7</sup> The problem with this measure, however, is that there are 20 values per variable, which makes summary difficult. A more useful measure is the following.

Let  $L^j$  be:

$$L^j = \frac{1}{T} \sum_{i=1}^T (y_i^j - y_i^*)^2 \quad (2)$$

where  $T$  is the length of the simulation period (20). Then the measure is

$$L = \frac{1}{J} \sum_{j=1}^J L^j \quad (3)$$

$L$  is a measure of the deviation of the variable from its base values over the whole period.<sup>8</sup>

---

imposed.

<sup>6</sup>Each of the four rules used has a different set of historical errors associated with it because the predicted values from the rules differ due to the different inflation coefficients.

<sup>7</sup>If  $y_t^*$  were the estimated mean of  $y_t$ , this measure would be the estimated variance of  $y_t$ . Given the  $J$  values of  $y_t^j$ , the estimated mean of  $y_t$  is  $(1/J) \sum_{j=1}^J y_t^j$ , and for a nonlinear model it is not the case that this mean equals  $y_t^*$  even as  $J$  goes to infinity. As an empirical matter, however, the difference in these two values is quite small for almost all macroeconomic models, and so it is approximately the case that the above measure of variability is the estimated variance.

<sup>8</sup> $L$  is, of course, not an estimated variance. Aside from the fact that for a nonlinear model the

## The Results

The results for this section are presented in the first five rows in Table 2. The first row (“No rule”) treats  $r$  as exogenous. This means that the value of  $r$  in a given quarter is the historic value for all the trials:  $r$  does not respond to the shocks. Values of  $L$  are presented for real GDP ( $Y$ ), the level of the private nonfarm deflator ( $P$ ), the percentage change in  $P$  ( $\dot{P}$ ), and  $r$ . The following discussion will focus on  $Y$ ,  $P$ , and  $r$ . The results for  $\dot{P}$  are generally similar to those for  $P$ , although the differences in  $L$  across rules are larger for  $P$  than for  $\dot{P}$ . All the experiments for the MC model use the same error draws. This considerably lessens stochastic simulation error across experiments.

The results in Table 2 are easy to summarize. Consider row 1 versus row 3 first.  $L$  for  $Y$  falls from 2.75 for the no rule case to 2.31 for the estimated rule, and  $L$  for  $P$  falls from 3.07 to 2.40. Both output and price variability are thus lowered considerably by the estimated rule. Now consider rows 2 through 5. As the long run inflation coefficient increases from 0.0 to 2.5, the variability of  $P$  falls, the variability of  $r$  rises, and the variability of  $Y$  is little affected. The cost of lowering  $P$  variability is thus an increase in  $r$  variability, not an increase in  $Y$  variability. Which rule one thinks is best depends on the weights one attaches to  $P$  and  $r$  variability,

---

mean of  $y_t$  is not  $y_t^*$ ,  $L^j$  is an average across a number of quarters or years, and variances are not in general constant across time.  $L$  is just a summary measure of variability.

**Table 2**  
**Variability Estimates: Values of  $L$**

<b>MC Model</b>				
	$Y$	$P$	$\dot{P}$	$r$
1. No rule ( $r$ exogenous)	2.75	3.07	2.00	0.00
2. Modified rule (0.0)	2.32	2.72	1.91	0.42
3. Estimated rule(1.0)	2.31	2.40	1.85	0.58
4. Modified rule (1.5)	2.32	2.27	1.82	0.73
5. Modified rule (2.5)	2.34	2.03	1.78	1.15
6. 3. with tax rule	2.01	2.28	1.82	0.52
<b>US Model</b>				
7. No rule ( $r$ exogenous)	3.42	3.12	2.04	0.00
8. Estimated rule	2.94	2.60	1.94	0.55
9. Optimal ( $\lambda_1 = 0.5, \lambda_2 = 0.5$ )	2.54	3.17	2.05	0.96
10. Optimal ( $\lambda_1 = 0.5, \lambda_2 = 1.5$ )	2.67	2.83	1.97	0.78
11. Optimal ( $\lambda_1 = 0.5, \lambda_2 = 2.5$ )	2.79	2.59	1.91	0.75

$Y$  = real GDP

$P$  = private nonfarm deflator

$\dot{P}$  = percentage change in the private nonfarm deflator

$r$  = three month Treasury bill rate

Simulation period = 1994:1–1998:4

Number of trials = 100

Estimated rule (1.0) = estimated rule, where long run  
inflation coefficient = 1.0

Modified rule (0.0) = estimated rule with long run  
inflation coefficient = 0.0

Modified rule (1.5) = estimated rule with long run  
inflation coefficient = 1.5

Modified rule (2.5) = estimated rule with long run  
inflation coefficient = 2.5

How do these results compare to those in the literature? Probably the largest difference concerns row 2, where the variability in row 2 is less than the variability in row 1. This shows that even the rule with a long run inflation coefficient of zero lowers variability. As discussed above, in modern-view models the rule in row 2 would be destabilizing. Clarida, Galí, and Gertler (2000) have a clear discussion of this. They conclude that the rule used by the Fed in the pre-1979 period probably

had an inflation coefficient less than one (p. 177), and they leave as an open question why the Fed followed a rule that was “clearly inferior” (p. 178) during this period. The results in this paper suggest that such a rule is not necessarily bad.

Results regarding the tradeoff between output variability and price variability as coefficients in a rule change appear to be quite dependent on the model used. This is evident in Tables 2 and 3 in Taylor (1999a, p. 43), and McCallum and Nelson (1999) point out that increasing the inflation or output coefficient in their rule leads to a tradeoff in one of their models but a reduction in *both* output and price variability in another. In this paper the tradeoff is between price variability and interest rate variability as the inflation coefficient is increased. There is little tradeoff between output and price variability. Because the tradeoffs are so model specific, one must have confidence in the model used to have confidence in the tradeoff results. As discussed above, the MC model has been extensively tested and appears to be a good approximation to the economy, and so the results in Table 2 may be conveying useful information.

### **3 Optimal Control**

#### **The Procedure**

Much of the literature on examining rules has not been concerned with deriving rules by solving optimal control problems,<sup>9</sup> but optimal control techniques are obvious ones to use in this context. The following procedure has been applied

---

<sup>9</sup>Exceptions are Feldstein and Stock (1993), Fair and Howrey (1996), and Rudebusch (1999).

to the U.S. subset of the MC model. This subset, which will be called the “US model,” is discussed in the appendix.

The US model is completely quarterly, and quarterly historical errors for the 1976:1–1998:4 period (92 quarters) were used for the draws. Each vector of quarterly errors had a probability of 1/92 of being drawn. Not counting the estimated interest rate rule, there are 29 estimated equations in the US model plus the export (*EX*) and price of imports (*PIM*) equations discussed in the appendix.

The optimal control methodology requires that a loss function be postulated for the Fed. In the loss function used here the Fed is assumed to care about output about inflation deviations and about interest rate fluctuations. In particular, the loss for quarter  $t$  is assumed to be:

$$H_t = \lambda_1 100[(Y - Y^*)/Y^*]^2 + \lambda_2 100(\dot{P} - \dot{P}^*)^2 + \alpha(\Delta r_t - \Delta r_t^*)^2 + 1.0/(r_t - 0.999) + 1.0/(16.001 - r_t) \quad (4)$$

where  $Y$  is real GDP,  $\dot{P}$  is the percentage change in the private nonfarm deflator, and \* denotes a base value.  $\lambda_1$  is the weight on output deviations, and  $\lambda_2$  is the weight on inflation deviations. The last two terms in (4) insure that the optimal values of  $r$  will be between 1.0 and 16.0. The value of  $\alpha$  was chosen by experimentation to yield an optimal solution with a value of  $L$  for  $r$  about the same as the value that results when the estimated rule is used. The value chosen was 8.0.

Assume that the control period of interest is 1 through  $T$ , where in this paper 1 is 1994:1 and  $T$  is 1998:4. Although this is the control period of interest, in order not to have to assume that life ends in  $T$ , the control problem should be thought of as one of minimizing the expected value of  $\sum_{t=1}^{T+n} H_t$ , where  $n$  is chosen to be large enough to avoid unusual end-of-horizon effects near  $T$ . The overall control

problem should thus be thought of as choosing values of  $r$  that minimize the expected value of  $\sum_{t=1}^{T+n} H_t$  subject to the model used.

If the model used is linear and the loss function quadratic, it is possible to derive analytically optimal feedback equations for the control variables.<sup>10</sup> In general, however, optimal feedback equations cannot be derived for nonlinear models or for loss functions with nonlinear constraints on the instruments, and a numerical procedure must be used. The following procedure was used for the results in this paper. It is based on a sequence of solutions of deterministic control problems, one sequence per trial.

Recall what a trial for the stochastic simulation is. A trial is a set of draws of 20 vectors of error terms, one vector per quarter. Given this set, the model is solved dynamically for the 20 quarters using an interest rate rule (or no rule). This entire procedure is then repeated 100 times (the chosen number of trials), at which time the summary statistics are computed. As will now be discussed, each trial for the optimal control procedure requires that 20 deterministic control problems be solved, and so with 100 trials, 2,000 solutions are required.

For purposes of solving the control problems, the Fed is assumed to know the model (its structure and coefficient estimates) and the exogenous variables, both past and future. The Fed is assumed *not* to know the future values of any endogenous variable or any error draw when solving the control problems.<sup>11</sup> The Fed is assumed to know the error draws for the first quarter for each solution. This

---

<sup>10</sup>See, for example, Chow (1981).

<sup>11</sup>The main exogenous variables in the US model are fiscal policy variables. The other exogenous variables are either unimportant or easy to forecast. Remember that since the base is the perfect tracking solution, the historical errors are always added to the model.



is consistent with the use of the above rules, where the error draws for the quarter are used when solving the model with the rule.

The procedure for solving the overall control problem is as follows.

1. Draw a vector of errors for quarter 1, and add these errors to the equations. Take the errors for quarters 2 through  $k$  to be their historical values (no draws), where  $k$  is defined shortly. Choose values of  $r$  for quarters 1 through  $k$  that minimize  $\sum_{t=1}^k H_t$  subject to the model as just described. This is just a deterministic optimal control problem, which can be solved, for example, by the method in Fair (1974).<sup>12</sup> Let  $r_1^*$  denote the optimal value of  $r$  for quarter 1 that results from this solution. The value of  $k$  should be chosen to be large enough so that making it larger has a negligible effect on  $r_1^*$ . (This value can be chosen ahead of time by experimentation.)  $r_1^*$  is a value that the Fed could have computed at the beginning of quarter 1 (assuming the model and exogenous variables were known) having knowledge of the error draws for quarter 1, but not for future quarters.
2. Record the solution values from the model for quarter 1 using  $r_1^*$  and the error draws. These solution values are what the model estimates would have occurred in quarter 1 had the Fed chosen  $r_1^*$  and had the error terms been as drawn.
3. Repeat steps 1 and 2 for the control problem beginning in quarter 2, then for the control problem beginning in quarter 3, and so on through the control problem beginning in quarter  $T$ . For an arbitrary beginning quarter  $s$ , use the solution values of all endogenous variables for quarters  $s - 1$  and back, as well as the values of  $r_{s-1}^*$  and back.
4. Steps 1 through 3 constitute one trial, i.e., one set of  $T$  drawn vectors of errors. Do these steps again for another set of  $T$  drawn vectors. Keep doing this until the specified number of trials has been completed.

---

<sup>12</sup>This method sets up the problem as an unconstrained nonlinear optimization problem and uses an optimization algorithm like DFP to find the optimum. Almost all the computer time for the overall procedure in this section is spent solving these optimization problems. The total computer time taken to solve the 2,000 optimization problems was about 3 hours on a computer with a 1.7 GHz Pentium chip.

The solution values of the endogenous variables carried along for a given trial from quarter to quarter in the above procedure are estimates of what the economy would have been like had the Fed chosen  $r_1^*, \dots, r_T^*$  and the error terms been as drawn.<sup>13</sup>

By “optimal rule” in this paper is meant the entire procedure just discussed. There is obviously no analytic rule computed, just a numerical value of  $r^*$  for each period.

## The Results

The results are presented in rows 7–11 in Table 2. The experiments in these rows use the same error draws to lessen stochastic simulation error across experiments, although these error draws are different from those used for the experiments in rows 1–6. Rows 7 and 8 are equivalent to rows 1 and 3: no rule and estimated rule, respectively. The same pattern holds for both the MC model and the US model results, namely that the estimated rule substantially lowers the variability of both  $Y$  and  $P$ .

Row 9 presents the results for the optimal solution with equal weights (i.e.,  $\lambda_1 = 0.5$  and  $\lambda_2 = 0.5$ ) on output and inflation in the loss function. Comparing rows 7 and 9, the optimal control procedure lowered the variability of  $Y$  substantially and

---

<sup>13</sup>The optimal control procedure just outlined differs somewhat for the procedure used in Fair and Howrey (1996, pp. 178-179). In Fair and Howrey (1996) the Fed is assumed not to know the exogenous variable values, but instead to use estimated autoregressive equations to predict these values for the current and future quarters. Also, the Fed is assumed not to know the error draws for the current quarter when solving its problem. In addition, stochastic simulation is not done. Instead, the error terms are set to zero (instead of to their historical values), the target values are taken to be the historical means (instead of the actual values), and the (one) trial uses for the error draws for a given quarter the actual errors for that quarter.

had little effect on the variability of  $P$ . This is quite different than the estimated rule (row 8). The estimated rule lowered the variability of both  $Y$  and  $P$ , but the fall in the variability of  $Y$  was much less than it was for the optimal control procedure.

For rows 10 and 11 the weight on inflation in the loss function is increased. This, not surprisingly, increases the variability of  $Y$  and lowers the variability of  $P$ . Row 11, which has a weight of 2.5 on inflation, gives similar results to those in row 8, which uses the estimated rule. In this sense the estimated rule is consistent with the Fed minimizing the loss function (4) with weights  $\lambda_1 = 0.5$  and  $\lambda_2 = 2.5$ .

Again, how do these results compare to those in the literature? A common result in the Taylor (1999) volume is that simple rules perform nearly as well as optimal rules or more complicated rules. See Taylor (1999a, p. 10), Rotemberg and Woodford (1999, p. 109), Rudebusch and Svensson (1999, p. 238), and Levin, Wieland, and Williams (1999, p. 294). The results in rows 8 and 11 are consistent with this theme, where the estimated rule performs nearly as well as the optimal control procedure. The optimal control procedure in this case is one in which the Fed puts a considerably higher weight on inflation than on output in the loss function.

## 4 Adding a Tax Rate Rule

Turning back to the MC model, it is clear in Table 2 that considerable overall variability is left in rows 2–5. In this section a tax rate rule is analyzed to see how much help it can be to monetary policy in stabilizing the economy. The idea is that a particular tax rate or set of rates would be automatically adjusted each quarter as a function of the state of the economy. Congress would vote on the parameters of the tax rate rule as it was voting on the general budget plan, and the tax rate or set of rates would then become an added automatic stabilizer.

Consider, for example, the federal gasoline tax rate. If the short run demand for gasoline is fairly price inelastic, a change in the after-tax price at the pump will have only a small effect on the number of gallons purchased. In this case a change in the gasoline tax rate is like a change in after-tax income. Another possibility would be a national sales tax if such a tax existed. If the sales tax were broad enough, a change in the sales tax rate would also be like a change in after-tax income.

For the results in this paper a constructed federal indirect business tax (IBT) rate based on data from the national income and product accounts is used for the tax rate rule. In practice a specific tax rate or rates, such as the gasoline tax rate, would have to be used, and this would be decided by the political process. The constructed tax rate for quarter  $t$ , denoted  $\tau_t$ , is the ratio of overall federal indirect business taxes to total consumption expenditures. In the regular version of the model  $\tau_t$  is taken to be exogenous.

The following equation is used for the tax rate rule:

$$\begin{aligned} \tau_t = \tau_t^* + 0.125[.5((Y_{t-1} - Y_{t-1}^*)/Y_{t-1}^*) + .5((Y_{t-2} - Y_{t-2}^*)/Y_{t-2}^*)] \\ + 0.125 * [.5(\dot{P}_{t-1} - \dot{P}_{t-1}^*) + .5(\dot{P}_{t-2} - \dot{P}_{t-2}^*)] \end{aligned} \quad (5)$$

where, as before,  $Y$  denotes real GDP and  $\dot{P}$  denotes the percentage change in a private nonfarm price deflator. It is not realistic to have tax rates respond contemporaneously to the economy, and so lags have been used in (5). Lags of both one and two quarters have been used to smooth tax rate changes somewhat. The rule says that the tax rate exceeds its base value as output and the inflation rate exceed their base values. Note that unlike the basic interest rate rule used in this paper, equation (5) is not based on an estimated rule. It would not make sense to try to estimate such a rule since it is clear that the government has never followed a tax rule policy.

Results using this rule along with the estimated interest rate rule are reported in row 6 in Table 2. The use of the rule lowers  $L$  for  $Y$  from 2.31 when only the estimated interest rate rule is used to 2.01 when both rules are used. The respective numbers for  $P$  are 2.40 and 2.28. The tax rate rule is thus of some help in lowering output and price variability, with a little more effect on output variability than on price variability. The variability of  $r$  falls slightly when the tax rate rule is added, since there is less for monetary policy to do when fiscal policy is helping.

## 5 Conclusion

The main conclusions about monetary policy from the results in Table 2 are the following:

1. The estimated rule explaining Fed behavior (in Table 1) substantially reduces output and price variability (row 3 versus row 1).
2. Variability is reduced even when the long run coefficient on inflation in the interest rate rule is set to zero (row 2 versus row 1). This is contrary to what would be the case in modern-view models, where such a rule would be destabilizing.
3. Increasing the long run coefficient on inflation in the interest rate rule lowers price variability, but it comes at a cost of increased interest rate variability (for example, row 5 versus row 3).
4. A tax rate rule is a noticeable help to monetary policy in its stabilization effort (row 6 versus row 3).
5. The optimal control procedure with  $\lambda_1 = 0.5$  and  $\lambda_2 = 2.5$ , which means a higher weight on inflation than on output in the loss function, gives results that are similar to the use of the estimated rule (row 11 versus row 8). The fact that the estimated rule does about as well as the optimal control procedure is consistent with many results in the literature, where simple rules tend to do fairly well.
6. Even when both the estimated interest rate rule and the tax rate rule are used, the values of  $L$  in Table 2 are not close to zero (row 6). Monetary policy even with the help of a fiscal policy rule does not come close to eliminating the effects of typical historical shocks.

# Appendix

## The MC Model

An updated version of the MC model in Fair (1994) has been used for the present work. This version is presented on the website mentioned in the introductory footnote. There are 38 countries in the MC model for which stochastic equations are estimated.<sup>14</sup> There are 31 stochastic equations for the United States and up to 15 each for the other countries. The total number of stochastic equations is 362, and the total number of estimated coefficients is 1649. In addition, there are 1111 estimated trade share equations. The total number of endogenous and exogenous variables, not counting the trade shares, is about 5000. Trade share data were collected for 59 countries, and so the trade share matrix is  $59 \times 59$ .<sup>15</sup>

The estimation periods begin in 1954 for the United States and as soon after 1960 as data permit for the other countries. They end between 1998 and 2002. The estimation technique is two stage least squares except when there are too few observations to make the technique practical, where ordinary least squares is used. The estimation accounts for possible serial correlation of the error terms. The variables used for the first stage regressors for a country are the main predetermined variables in the model for the country. A list of these variables is available from

---

<sup>14</sup>The 38 countries are the United States, Canada, Japan, Austria, France, Germany, Italy, the Netherlands, Switzerland, the United Kingdom, Finland, Australia, South Africa, Korea, Belgium, Denmark, Norway, Sweden, Greece, Ireland, Portugal, Spain, New Zealand, Saudi Arabia, Venezuela, Colombia, Jordan, Syria, India, Malaysia, Pakistan, the Philippines, Thailand, China, Argentina, Chile, Mexico, and Peru.

<sup>15</sup>The 21 other countries that fill out the trade share matrix are Brazil, Turkey, Poland, Russia, Ukraine, Egypt, Israel, Kenya, Bangladesh, Hong Kong, Singapore, Vietnam, Nigeria, Algeria, Indonesia, Iran, Iraq, Kuwait, Libya, the United Arab Emirates, and an all other category.

the website.<sup>16</sup>

There is a mixture of quarterly and annual data in the MC model. Quarterly equations are estimated for 14 countries (the first 14 in footnote 19), and annual equations are estimated for the remaining 24. However, all the trade share equations are quarterly. There are quarterly data on all the variables that feed into the trade share equations, namely the exchange rate, the local currency price of exports, and the total value of imports per country. When the model is solved, the predicted annual values of these variables for the annual countries are converted to predicted quarterly values using a simple distribution assumption. The quarterly predicted values from the trade share equations are converted to annual values by summation or averaging when this is needed.

Two properties of the model will be outlined here. The first is the effect of inflation shocks on the economy with the nominal interest rate held constant, and the second is the effect of nominal interest rates on the economy.

### **Inflation Shock with Nominal Interest Rate Constant**

A positive inflation shock with the nominal interest rate held constant is contractionary in the United States in the MC model. The three main reasons are:

1. The wage rate appears in the price equation and the price level appears in the wage rate equation, and these two equations have the feature that a positive price shock leads to an initial fall in the real wage rate. In other words, wage rates lag prices. Real wage income thus falls because of the fall in the

---

<sup>16</sup>Some of the equations in the model are changed beginning in 1999 to incorporate the EMU. Beginning in 1999, the exchange rate equations of the individual EMU countries are replaced with one exchange rate equation, and the individual interest rate rules are replaced with one rule. These changes are not relevant for this paper because the simulation period ends in 1998.



real wage rate. This has a negative effect on real household expenditures because real income is an explanatory variable in the household expenditure equations.

2. Real wealth is an explanatory variable in the consumer expenditure equations. Much of this wealth is held in the form of corporate stocks. The response of nominal stock prices in the model to an increase in the price level is fairly modest, and it is the case that a positive price shock leads to a fall in real wealth. In other words, nominal wealth lags prices. The fall in real wealth then has a negative effect on real consumer expenditures.
3. Empirical results from estimating the household expenditure equations suggest that households respond to nominal interest rates and not real interest rates, and so nominal interest rates are used as explanatory variables in these equations. There is thus no estimated positive household response to lower real interest rates when there is a positive inflation shock with nominal interest rates held constant.

This property of the model is discussed in more detail in Fair (2002). In this paper an experiment was performed in which there was a positive shock to the price equation with the nominal interest rate held constant, and this led to a decrease in real GDP—Fair (2002, Table 3).

## **Nominal Interest Rate Changes**

The U.S. short term nominal interest rate in the model ( $r$ ), which is the three-month Treasury bill rate, affects the economy in a number of ways:

1. Long term interest rates depend on current and lagged values of  $r$ .
2. An interest rate appears as an explanatory variable in each of the household expenditure equations (three consumption and one housing investment) and the plant and equipment investment equation, all with negative coefficient estimates.

3. An interest rate has a negative effect on capital gains ( $CG$ ) in the  $CG$  equation, and changes in  $CG$  change household wealth. Household wealth is an explanatory variable in the consumption and housing investment equations with positive coefficient estimates.
4. Interest payments of firms and the government—and thus interest income of households—change when interest rates change, and household interest income appears in the household expenditure equations through a disposable income variable, which has a positive effect in these equations.
5. A change in  $r$  leads to a change in the value of the dollar vis-a-vis the other major currencies through exchange rate equations—an increase in  $r$  leads to an appreciation of the dollar and a decrease leads to a depreciation. A change in the value of the dollar leads to a change in U.S. import prices, which then results in a change U.S. domestic prices through an import price variable in the domestic price equation. The change in the value of the dollar also leads to a change in the demand for U.S. exports through the trade share equations, and it leads to a change in U.S. import demand through an import price variable in the U.S. import equation.
6.  $r$  appears as an explanatory variable in some of the other countries' interest rate rules, and so foreign interest rates in part follow U.S. rates.

The net effects of, say, a decrease in  $r$  on U.S. output and the price level are positive. Output increases because there is an increase in the demand for U.S. domestically produced goods, and the price level increases because of the increase in demand and the depreciation of the dollar.

## **The US Model**

The optimal control procedure is too costly in terms of computer time to be able to be used for the MC model, and for this work the U.S. subset of the model was used. The “US model” used for the optimal control results is exactly the same

as the model for the United States in the MC model except for the treatment of U.S. exports ( $EX$ ) and the U.S. price of imports ( $PIM$ ). These two variables change when  $r$  changes—primarily because the value of the dollar changes—and the effects of  $r$  on  $EX$  and  $PIM$  were approximated in the following way.

First,  $\log EX_t - \alpha_1 r_t$  was regressed on a constant,  $t$ ,  $\log EX_{t-1}$ ,  $\log EX_{t-2}$ ,  $\log EX_{t-3}$ , and  $\log EX_{t-4}$ , and  $\log PIM_t - \alpha_2 r_t$  was regressed on a constant,  $t$ ,  $\log PIM_{t-1}$ ,  $\log PIM_{t-2}$ ,  $\log PIM_{t-3}$ , and  $\log PIM_{t-4}$ . Second, these two equations were added to the US model for particular values of  $\alpha_1$  and  $\alpha_2$ , and an experiment was run in which the estimated interest rate rule of the Fed was dropped and  $r$  was decreased by one percentage point. This was done many times for different values of  $\alpha_1$  and  $\alpha_2$ . The final values of  $\alpha_1$  and  $\alpha_2$  chosen were ones whose experimental results most closely matched the results for the same experiment using the complete MC model. The final values chosen were -.0004 and -.0007 respectively. Third, the experiment in the third row of Table 2 was run for the US model with the  $EX$  and  $PIM$  equations added and with the estimated errors from these equations being used in the drawing of the errors. When an error for the  $EX$  equation was drawn, it was multiplied by  $\beta_1$ , and when an error for the  $PIM$  equation was drawn, it was multiplied by  $\beta_2$ . The experiment was run many times for different values of  $\beta_1$  and  $\beta_2$ , and the final values chosen were ones that led to results similar to those in the third row of Table 2. The values were  $\beta_1 = .4$  and  $\beta_2 = .75$ . The results using these values are in row 8 of Table 2. The chosen values of  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ , and  $\beta_2$  were then used for the experiments in rows 7–11.

## References

- [1] Chow, G. C., 1981, *Econometric Analysis by Control Methods*, New York: John Wiley & Sons.
- [2] Christian, James W., 1968, "A Further Analysis of the Objectives of American Monetary Policy," *Journal of Finance*, 23, 465-477.
- [3] Clarida, Richard, Jordi Galí, and Mark Gertler, 1999, "The Science of Monetary Policy: A New Keynesian Perspective," *The Journal of Economic Literature*, 37, 1661-1707.
- [4] \_\_\_\_\_, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *The Quarterly Journal of Economics*, 115, 147-180.
- [5] Dewald, William G., and Harry G. Johnson, 1963, "An Objective Analysis of the Objectives of American Monetary Policy, 1952-61," in *Banking and Monetary Studies*, Deane Carson, ed., Homewood, Illinois: Richard D. Irwin, 171-189.
- [6] Fair, R. C., 1974, "On the Solution of Optimal Control Problems as Maximization Problems," *Annals of Economic and Social Measurement*, 3, 135-154.
- [7] \_\_\_\_\_, 1978, "The Sensitivity of Fiscal Policy Effects to Assumptions about the Behavior of the Federal Reserve," *Econometrica*, 46, 1165-1179.
- [8] \_\_\_\_\_, 1994, *Testing Macroeconometric Models*, Cambridge, MA: Harvard University Press.
- [9] \_\_\_\_\_, 2001, "Actual Federal Reserve Policy Behavior and Interest Rate Rules," *FRBNY Economic Policy Review*, 7, 61-72.
- [10] \_\_\_\_\_, 2002 "On Modeling the Effects of Inflation Shocks," *Contributions to Macroeconomics: Vol. 2: No. 1, Article 3*. <http://www.bepress.com/bejm/contributions/vol2/iss1/art3>
- [11] \_\_\_\_\_, and E. Philip Howrey, 1996, "Evaluating Alternative Monetary Policy Rules," *Journal of Monetary Economics*, 38, 173-193.

- [12] Feldstein, Martin, and James H. Stock, 1993, "The Use of a Monetary Aggregate to Target Nominal GDP," NBER Working Paper No. 4304.
- [13] Giordani, Paolo, 2002, "On Modeling the Effects of Inflation Shocks: Comments and Some Further Evidence," forthcoming bepress.
- [14] Levin, Andrew, Volker Wieland, and John C. Williams, 1999, "Robustness of Simple Monetary Policy Rules under Model Uncertainty," in Taylor (1999), 263-299.
- [15] McCallum, Bennett T., and Edward Nelson, 1999, "Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model," in Taylor (1999), 15-45.
- [16] Romer, David, 2000, "Keynesian Macroeconomics without the LM Curve," *The Journal of Economic Perspectives*, 14, 149-169.
- [17] Rotemberg, Julio J., and Michael Woodford, 1999, "Interest Rate Rules in an Estimated Sticky Price Model," in Taylor (1999), 57-119.
- [18] Rudebusch, Glenn D., 1999, "Is the Fed Too Timid? Monetary Policy in an Uncertain World," March.
- [19] \_\_\_\_\_, and Lars E. O. Svensson, 1999, "Policy Rules for Inflation Targeting," in Taylor (1999), 203-246.
- [20] Taylor, John B., ed., 1999, *Monetary Policy Rules*, Chicago: The University of Chicago Press.
- [21] \_\_\_\_\_, 1999a, "Introduction," in Taylor (1999), 1-14.
- [22] \_\_\_\_\_, 2000, "Teaching Modern Macroeconomics at the Principles Level," *The American Economic Review*, 90, 90-94.