Equilibrium Asset Prices and Investor Behavior in the Presence of Money Illusion*

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Abstract

This article analyzes the implications of money illusion for investor behavior and asset prices in a securities market economy with inflationary fluctuations. We provide a belief-based formulation of money illusion which accounts for the systematic mistakes in evaluating real and nominal quantities. The impact of money illusion on security prices and their dynamics is demonstrated to be considerable even though its welfare cost on investors is small in typical environments. A money-illusioned investor's real consumption is shown to generally depend on the price level, and specifically to decrease in the price level. A general-equilibrium analysis in the presence of money illusion generates implications that are consistent with several empirical regularities. In particular, the real bond yields and dividend price ratios are positively related to expected inflation, the real short rate is negatively correlated with realized inflation, and money illusion may induce predictability and excess volatility in stock returns. The basic analysis is generalized to incorporate heterogeneous investors with differing degrees of illusion.

Journal of Economic Literature Classification Numbers: C60, D50, D90, G12.
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1. Introduction

Money illusion refers to "a tendency to think in terms of nominal rather than real monetary values" (Shafir, Diamond and Tversky (1997)). Recognition of this seemingly simple confusion dates back to Fisher (1928) who argued that "Almost every one is subject to the 'Money Illusion' in respect to his own country's currency." Though the difference between nominal and real quantities is obvious, money illusion remains pervasive. As Akerlof (2002) notes, seven decades after Fisher, "... rules of thumb involving 'money illusion' are not only commonplace but also sensible—neither foolhardy nor implausible." Despite its prevalence, money illusion has largely been ignored by economists after the rational expectations revolution in 1970's.[1] Yet, there has recently been growing, convincing support for money illusion. Shafir, Diamond and Tversky (1997) provide a psychological foundation for money illusion based on their analysis of survey data. These authors argue that although people are generally aware of the difference between real and nominal values, they still often think of transactions predominantly in nominal terms since, relative to real quantities, the nominal quantities are much more salient. Fehr and Tyran (2001) demonstrate the role of money illusion in nominal price rigidity by conducting a series of rigorous experiments. Interestingly, they also show that money illusion arises only when payoffs are presented in nominal terms to the participants, that is, when nominal terms are more salient. The bias towards more *accessible* information arises naturally, as argued in cognitive psychology (Kahneman (2003)). Moreover, as reasoned by Akerlof (2002), this bias can also be justified in many situations when losses from the bias are small.

In this paper, we analyze money illusion in financial markets and do indeed demonstrate that the welfare loss of money illusion to an investor is small for typical environments, while its impact on equilibrium can be considerable. Money illusion can then be interpreted as a form of bounded rationality (Simon (1968)) in financial markets: investors may partially overlook inflation in their decision-making since the cost of this negligence is small. This result parallels the main idea in the New Keynesian macro economics literature, which argues that small frictions and departures from full rationality may generate large fluctuations in the real economy (Akerlof and Yellen (1985), Mankiw (1985)). In addition, our analysis also shows that some of the consumption and asset prices behavior can be better understood in the context of money illusion.

The presence of money illusion in financial markets has long been noted. Modigliani and Cohn (1979) hypothesize that money-illusioned investors essentially discount future real

[1] Early literature on money illusion emphasized its impact on labor supply and unemployment (Keynes (1936)).

payoffs at nominal rather than real rates, causing the undervaluation of stocks in 1970's when inflation was excessively high. In the psychology literature, Svedsater, Gamble, and Garling (2007) highlight the role of money illusion in financial decision-making via a number of experiments, and show that investors may be influenced by the nominal representation of stock prices when evaluating financial information. The existing analysis in the finance literature is primarily limited to empirically understanding dividend-price ratios based on this partial-equilibrium stock valuation framework (Modigliani and Cohn (1979), Ritter and Warr (2002), Campbell and Vuolteenaho (2004)). At present, we lack rigorous understanding of the financial economic implications of money illusion.

Our goal is fill this gap and undertake an analysis of money illusion within a familiar asset pricing framework. Our strategy is to deviate only in one dimension, the incorporation of money illusion, while retaining all other standard paradigms in finance (no-arbitrage, market clearing). In particular, we study the implications of money illusion for investor behavior and asset prices in a pure exchange, security market economy that is exposed to inflationary fluctuations. To demonstrate the role of money illusion as clearly as possible, we employ the common constant-relative-risk-aversion (CRRA) preferences as the benchmark case with no money illusion. To the best of our knowledge, ours is the first attempt to directly embed money illusion into an investor's decision-making and valuation framework.

Towards that, we develop a belief-based formulation of money illusion which accounts for the systematic mistakes in evaluating real and nominal payoffs. In our analysis, we consider the general case of partial illusion. In a mildly inflationary environment, an investor may partially overlook and may not bother to fully take into account of the impact of inflation on the purchasing power of currency in his intertemporal choice. That is, he partially overlooks inflation in his decision-making, since ignoring inflation simplifies his decision-making process and only incurs a small welfare loss. To capture this idea, we employ a generalized version of the Modigliani-Cohn hypothesis, in which the illusioned investor discounts future real payoffs by a combination of nominal and real rates, with a degree of money illusion parameter controlling the bias towards nominal rates. We decompose this perception error in discounting into a change of measure, representing the money-illusioned investor's beliefs, and an ensuing average discounting error, capturing the aspect that a money-illusioned investor tends to over-discount payoffs in an inflationary environment. This tractable formulation nests the benchmark case of a normal investor with no illusion, where the investor discounts real payoffs by real rates, as well as the Modigliani-Cohn case of complete illusion, where the investor discounts real payoffs by nominal rates. The money-illusioned investor's dynamic consumption-investment problem then takes place under his money-illusioned probability measure and the ensuing average discounting error, together which fully represent his perception bias.

Our main findings are as follows. First, at a partial equilibrium level, a money-illusioned investor's real consumption is decreasing in the price level (holding all else fixed). To see the intuition, recall that a normal investor's optimal consumption plan is such that his marginal utility is proportional to the cost of consumption in a given state. Consequently, the investor consumes less in states for which consumption is relatively expensive, and more otherwise. When the price level is increased, a normal investor understands that there is no change in real prices, and so does not alter his consumption. A money-illusioned investor, however, is confused between nominal and real quantities, and so perceives the consumption to be more expensive, thereby consuming less. This prediction is consistent with the experimental evidence in Raghubir and Srivastava (2002), who show that an individual decreases his expenditure when faced with higher nominal price, and vice versa for lower nominal price.

Second, since the effects of money illusion at an individual investor level are similar across investors, it is natural to expect those effects to show up at the aggregate level. To investigate the impact of possibly widespread behavior of money-illusioned investors in financial markets, we move to a general equilibrium setting. Our focus is on the implications for asset prices and their dynamics. Equilibrium in the presence of money illusion emerges rich in implications and is able to generate various predictions, pertaining to inflation and asset price behavior. In particular, the real long term bond yields and the dividend price ratio are positively related to expected inflation under money illusion. The reason is that, as discussed above, money illusion induces an investor to consume less when the price level increases. Thus, higher expected inflation tends to lower future demand for consumption, and hence makes future consumption"less valuable" in equilibrium. Since the stock and long term bonds are claims to future consumption, higher expected inflation leads to lower stock and bond prices, or equivalently to higher dividend price ratio and long bond yields.

The implication of a positive relation between expected inflation and dividend-price ratio is consistent with the Modigliani and Cohn (1979) hypothesis. Further supportive empirical evidence for this implication is recently documented by Ritter and Warr (2002), Sharpe (2002), Campbell and Vuolteenaho (2004), Cohen, Polk and Vuolteenaho (2005), Chordia and Shivakumar (2005), Basu, Markov and Shivakumar (2005). More recently, in the context of housing prices, Brunnermeier and Julliard (2008) also provide evidence supporting this argument. Piazzesi and Schneider (2008) combine the Modigliani-Cohn argument with disagreement about inflation, and argue that both high and low inflation can lead to house price frenzies. Also in the context of housing markets, Genesove and Meyer (2001) document that home sellers appear to be averse to realizing nominal losses.

While the Modigliani and Cohn hypothesis is intuitive, by assumption it is silent on the impact of money illusion on bond yields. This issue is addressed in our equilibrium model in which interest rates are endogenously determined, and long bond yields are positively related to long term expected inflation, via a mechanism similar to the one for dividend-price ratio. Our bond yield implication is also supported by the empirical findings in both U.K. data (Barr and Campbell (1997)) and U.S. data (Sharpe (2002)). More importantly, our analysis provides a micro foundation for money illusion by showing that partially overlooking inflation has a large impact on security prices but only a small utility cost. This result parallels that in Fehr and Tyran (2001), who show that a small amount of individual-level money illusion can cause considerable price rigidity at the aggregate level, with the impact of money-illusioned agents being amplified by rational agents' strategic responses. Both their analysis and ours underscore the importance of money illusion by demonstrating that a slight degree of illusion can induce a large impact on equilibrium. We also demonstrate how money illusion may induce stock return predictability and excess volatility. The dividend-price ratio depends on expected inflation under money illusion, and hence time variation in expected inflation naturally generates time variation in dividend-price ratio, and so induces stock return predictability and higher volatility.

Finally, the baseline economic setting is extended to feature multiple investors with different degrees of money illusion, in order to obtain further insights as to the effects of money illusion, as well as to demonstrate the robustness of our main results. Now, the equilibrium is additionally driven by the stochastic distribution of consumption across investors, leading to richer dynamics. Here, the degree of money illusion in the economy is endogenously determined by the price level and the initial wealth distribution across investors. An important implication is that the real short interest rate and realized inflation are negatively correlated for empirically reasonable parameters. The reason is that when the realized inflation is high (high price level), the money-illusioned investor's consumption decreases, hence decreasing his consumption share in the economy. This brings the equilibrium closer to that in a normal investor economy for which the interest rate is lower for empirically reasonable parameters. This negative correlation result is consistent with empirical evidence (see Ang, Bekaert and Wei (2008) and the references therein).

The remainder of the article is organized as follows. In Section 2, we outline the economy in the presence of money illusion and analyze the behavior of a money-illusioned investor at a partial equilibrium level. In Section 3, we investigate the impact of money illusion on equilibrium asset prices and their dynamics. Section 4 generalizes the analysis to incorporate heterogeneity across investors. Section 5 concludes and the Appendix provides all proofs.

2. Economy with Money Illusion

We consider a continuous-time, pure-exchange security market economy with an infinite horizon. There is a single consumption good which serves as the numeraire. The economy is exposed to inflationary fluctuations and is populated by investors with money illusion.

2.1. Economic Set-Up

The uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ on which is defined a two-dimensional Brownian motion $\omega = (\omega_1, \omega_2)^\top$. Absent any behavioral biases, rational investors' beliefs are represented by the probability measure $\mathcal{P}$. All stochastic processes are assumed adapted to the information filtration $\{\mathcal{F}_t\}$. In what follows, given our focus, we assume all processes and expectations introduced are well-defined, without explicitly stating the required regularity conditions.

The aggregate consumption supply D is assumed to follow a geometric Brownian motion process

$$dD(t) = D(t)\left[\mu_D dt + \sigma_D^\top d\omega(t)\right],$$

with μ_D, σ_D constants, and $D(0) > 0$. The price level of the good p, representing the consumer price index (CPI) in the economy, is similarly modeled as

$$dp(t) = p(t)\left[\mu_p dt + \sigma_p^\top d\omega(t)\right],$$

where the expected inflation μ_p and inflation volatility vector σ_p are constants, and $p(0) = 1$ without loss of generality. The lognormality assumptions on D and p are for simplicity, and most of our main results and insights remain valid for more general processes (See Remark 2 of Section 3), for which the analysis can readily be extended. A more general inflation process, incorporating mean reversion, is considered in Section 3.4.

Financial investment opportunities are represented by three securities: a real riskless bond, a nominal riskless bond, and a stock. The two bonds are assumed to be in zero net supply, and the stock, which is a claim to aggregate consumption D, is assumed to be in a positive net supply of one share. The real riskless bond is (locally) riskless in real terms with an (instantaneous) real interest rate of r. The nominal riskless bond is (locally) riskless in nominal terms with an (instantaneous) nominal interest rate of R, while being risky in real terms. The prices, in real terms, of the nominal bond and the stock, B and S, respectively, are posited to have dynamics

$$\begin{aligned} dB(t) &= B(t)\left[\left(R(t) - \mu_p + \|\sigma_p\|^2\right) dt - \sigma_p^\top d\omega(t)\right], \\ dS(t) + D(t)dt &= S(t)[\mu_S(t)\, dt + \sigma_S(t)^\top\, d\omega(t)]. \end{aligned}$$

The real interest r, the stock mean return μ_S and volatility vector σ_S are to be determined endogenously in equilibrium. The nominal price parameters (R, μ_p, σ_p) are taken as given in our current analysis, but are endogenously determined in a subsequent analysis in concurrent work as discussed in the Conclusion.[2]

An investor in the economy is endowed at time 0 with one share of the stock, providing him with an initial wealth of $W(0) = S(0)$. He then chooses a nonnegative consumption process c and a portfolio process π, where $\pi(t) \equiv (\pi_B(t), \pi_S(t))^\top$ denotes the vector of fractions of wealth invested in the nominal bond and the stock at time t, respectively. The investor's real financial wealth process W follows

$$dW(t) = W(t)r(t)dt - c(t)dt + W(t)\pi(t)^\top \left[(\mu(t) - r(t)\mathbf{1})\, dt + \sigma(t) d\omega(t) \right]. \tag{1}$$

An investor derives utility from intertemporal consumption, given by the standard CRRA utility function

$$u(c(t), t) = e^{-\rho t} \frac{c(t)^{1-\gamma}}{1-\gamma}, \qquad \gamma > 0,$$

with ρ representing the time discount factor, γ the relative risk aversion coefficient, and $\gamma = 1$ the logarithmic utility function.

2.2. Modeling Money Illusion

As discussed in the Introduction, money illusion can be attributed to intuitive decision making based on more accessible information since real-life economic situations are dominantly expressed in nominal terms (Shafir, Diamond and Tversky (1997), Fehr and Tyran (2001, 2007), Kahneman (2003)). Moreover, Modigliani and Cohn (1979) postulate that "investors capitalize equity earnings at a rate that parallels the nominal interest rate, rather than the economically correct real rate." That is, according to this view, an investor with money illusion effectively discounts future real payoffs at nominal rather than real rates. We formalize this view below by modeling money illusion directly as a bias in beliefs, in two steps. We first identify the stochastic process capturing the discounting error (as hypothesized by Modigliani and Cohn) by comparing a money-illusioned investor's valuation of the stock with that of a rational investor's. Then, from this stochastic process, we recover the implied bias in perception.

Specifically, if an investor is rational, he discounts real payoffs by his real discount rate. In this set-up, his real discount rate over the time interval $[t, s]$ is given by his marginal

[2] The presence of a nominal bond and a real bond is not necessary in our analysis. Given the completeness of markets, the presence of any other two nonredundant financial securities would do. However, we have introduced these two bonds here for expositional convenience.

rate of substitution, $u'(s)/u'(t)$, where $u'(t) \equiv \partial u(c(t),t)/\partial c(t)$ denotes his marginal utility of consumption at time t. Therefore, a rational investor's valuation of the stock is given by

$$S(t) = E_t \left[\int_t^\infty \frac{u'(s)}{u'(t)} D(s) ds \right], \tag{2}$$

where the expectation $E_t[\cdot]$ is under rational beliefs represented by the probability measure $\mathcal{P}$. Now we turn to a money-illusioned investor. Under the Modigliani-Cohn hypothesis, a money-illusioned investor mistakenly uses his nominal discount rate to discount real payoffs. Since an investor's nominal discount rate over the time interval $[t, s]$ is $(u'(s)/p(s))/(u'(t)/p(t))$, a money-illusioned investor's stock valuation is given by

$$S(t) = E_t \left[\int_t^\infty \frac{u'(s)/p(s)}{u'(t)/p(t)} D(s) ds \right]. \tag{3}$$

We note that expression (3) captures the extreme case of complete money illusion, where the investor is completely confused between nominal and real discount rates.

In our analysis, we consider the more general case of *partial* illusion, where the investor partially overlooks the difference between the real and nominal discount rates. Here, the investor uses a combination of nominal and real rates to discount real payoffs, and so his stock valuation is

$$\begin{aligned} S(t) &= E_t \left[\int_t^\infty \left(\frac{u'(s)/p(s)}{u'(t)/p(t)} \right)^\delta \left(\frac{u'(s)}{u'(t)} \right)^{1-\delta} D(s) ds \right], \\ &= E_t \left[\int_t^\infty \left(\frac{p(t)}{p(s)} \right)^\delta \frac{u'(s)}{u'(t)} D(s) ds \right], \qquad 0 \le \delta \le 1, \end{aligned} \tag{4}$$

where the parameter δ represents the degree of money illusion. This formulation turns out to not only be tractable, but to also have the convenient property of nesting the benchmark case of a normal investor with no illusion as a special case (for $\delta = 0$), in addition to the Modigliani-Cohn case of complete illusion (for $\delta = 1$).[3] By comparing the rational investor's valuation (equation (2)) with that of the money-illusioned investor's (equation (4)), we find that the money-illusioned investor's error is fully captured by the process $(p(t)/p(s))^\delta$.

Since we wish to formulate the perception error in (4) as a bias in beliefs under our generalized Modigliani-Cohn hypothesis, we are interested in identifying money-illusioned beliefs, represented by a probability measure $\mathcal{P}^*$, consistent with equation (4). It is clear from (4) that the process $(p(t)/p(s))^\delta$ quantifies the error in discounting and hence would link the rational beliefs $\mathcal{P}$ with the money-illusioned beliefs $\mathcal{P}^*$. However, $(p(t)/p(s))^\delta$ generally would

[3]One can imagine that investors may more properly account for inflation (i.e., δ decreases) after a period of high inflation. However, as an initial step towards understanding money illusion, we treat δ as an exogenous constant for simplicity and leave the dynamics of δ for future research.

not exactly coincide with a change of measure, unless restricted to be a martingale. Hence, we may consider normalizing it by its average so as to obtain the desired change of measure. In particular, we decompose the discounting error $(p(t)/p(s))^\delta$ into a positive martingale process and a finite variation process, and after some algebra, reach the following up-to-a-constant unique decomposition:

$$\left(\frac{p(t)}{p(s)}\right)^\delta = e^{-b(s-t)}\frac{\eta(s)}{\eta(t)}, \qquad s > t, \tag{5}$$

where

$$\begin{aligned} b &\equiv \delta\mu_p - \frac{1}{2}\delta(1+\delta)\|\sigma_p\|^2, \\ \eta(t) &\equiv e^{-\frac{1}{2}\delta^2\|\sigma_p\|^2 t - \delta\sigma_p^\top \omega(t)}. \end{aligned}$$

The first component $e^{-b(s-t)}$ quantifies the average discounting error since $E_t[(p(t)/p(s))^\delta] = e^{-b(s-t)}$. This component is less than one in an inflationary environment ($\mu_p > \frac{1}{2}(1+\delta)\|\sigma_p\|^2$), and hence captures the aspect that a money-illusioned investor tends to over-discount payoffs in this environment. Similarly, this component is greater than one in a deflationary environment ($\mu_p < \frac{1}{2}(1+\delta)\|\sigma_p\|^2$). The second, martingale component $\eta(s)/\eta(t)$ recovers the density process (Radon-Nikodym derivative) of the money-illusioned probability measure $\mathcal{P}^*$ with respect to the original measure $\mathcal{P}$:

$$\left(\frac{d\mathcal{P}^*}{d\mathcal{P}}\right)_s = \frac{\eta(s)}{\eta(t)}, \qquad s > t.$$

That is, decomposition (5) reveals that money illusion manifests itself through two channels: The first captures the tendency of over-discounting in an inflationary environment, and the second reflects the implied change of measure. Similarly, Jouini and Napp (2007), in the context of aggregating heterogeneous beliefs into a consensus belief, also decompose a pertinent process (their so-called characteristic process) into a discounting component and a change of measure.

The consequence of the above belief-based formulation is that the money-illusioned investor is effectively endowed with the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P}^*)$ and an ensuing average discounting error $e^{-b(s-t)}$. The probability measure $\mathcal{P}^*$ and the parameter b account for the investor's bias in perception that causes systematic mistakes in evaluating real and nominal quantities, including discounting real payoffs by nominal rates.[4] For example, under our generalized Modigliani-Cohn hypothesis, the money-illusioned investor's valuation of the stock under his perception satisfies

$$S(t) = E_t^*\left[\int_t^\infty e^{-b(s-t)}\frac{u'(s)}{u'(t)}D(s)ds\right] = E_t\left[\int_t^\infty \left(\frac{p(t)}{p(s)}\right)^\delta \frac{u'(s)}{u'(t)}D(s)ds\right],$$

[4]Moreover, under the money-illusioned investor's beliefs $\mathcal{P}^*$ is defined a Brownian motion $\omega^*(t)$, which by Girsanov's theorem, is given by $d\omega^*(t) = d\omega(t) + \delta\sigma_p dt$.

where the illusioned investor's expectation $E_t^*[\cdot]$ is taken under $\mathcal{P}^*$. That is, the probability measure $\mathcal{P}^*$ and the ensuing average discounting error captured by b fully represent the money-illusioned investor's behavior of effectively discounting real payoffs by a combination of nominal and real rates.

Our formulation implies that, as in Modigliani and Cohn (1979), money-illusioned investors overlook the fact that higher expected inflation leads to higher future nominal payoffs, and hence underestimate the expected dividend growth rate and the expected stock return in an inflationary environment. However, the Modigliani-Cohn hypothesis does not address the impact of money illusion on nominal bonds, while our formulation does. In ours, the money-illusioned investors know the nominal payoffs from nominal bonds but are confused about their real payoffs, and hence affect the bond markets. Consequently, our model allows us to study the impact of money illusion on both the stock and bond markets in a unified framework, complementing the Modigliani-Cohn analysis.

2.3. Investor's Behavior Under Money Illusion

The money-illusioned investor's dynamic problem under his perception, namely his money-illusioned probability measure $\mathcal{P}^*$ and the ensuing average discounting error driven by b, is

$$\max_{c,\pi} E^*\left[\int_0^\infty e^{-bt}u(c(t),t)dt\right] = E\left[\int_0^\infty \eta(t)e^{-bt}u(c(t),t)dt\right], \tag{6}$$

subject to (1). That is, the money-illusioned investor's objective function under rational beliefs inherits the change of probability measure η and an additional discounting parameter b, reflecting his tendency to over-discount in an inflationary environment. Hereafter, for expositional convenience, we will present our analysis under the original probability measure $\mathcal{P}$.

To solve for the optimal consumption-portfolio policies, we employ standard martingale methods (Cox and Huang (1989), Karatzas, Lehoczky and Shreve (1987)). In particular, we first construct the state price density process from the available securities in financial markets. The posited dynamic market completeness implies the existence of a unique real state price density process ξ, given by

$$d\xi(t) = -\xi(t)\left[r(t)dt + \kappa(t)^\top d\omega(t)\right],$$

where the market price of risk (or Sharpe ratio) process κ is given by $\kappa(t) = \sigma(t)^{-1}\left[\mu(t) - r(t)\mathbf{1}\right]$, with vector of drifts $\mu \equiv \left(R - \mu_p + \|\sigma_p\|^2, \mu_S\right)^\top$, the volatility matrix $\sigma \equiv \left(-\sigma_p, \sigma_S\right)^\top$ and $\mathbf{1} \equiv (1,1)^\top$. The quantity $\xi(t,\omega)$ is interpreted as the Arrow-Debreu state price of one unit of the good to be delivered at (t,ω), $\omega \in \Omega$. We then can conveniently characterize the optimal solution, as reported in Proposition 1, in terms of this state price density and price level p. For ease of comparison, we note that a normal investor with no illusion ($\delta = 0$), N, chooses the optimal consumption to be (e.g., Cox and Huang (1989))

$$c^N(t) = \left(y^N e^{\rho t}\xi(t)\right)^{-1/\gamma}, \tag{7}$$

where the Lagrange multiplier associated with the static budget constraint y^N is given by

$$\left(y^N\right)^{-1/\gamma} = W(0)\left[\mathrm{E}\int_0^\infty e^{-\rho t/\gamma}\xi(t)^{(\gamma-1)/\gamma}dt\right]^{-1}. \tag{8}$$

Proposition 1. *A money-illusioned investor's real consumption choice is given by*

$$c(t) = \left(ye^{\rho t}\xi(t)\right)^{-1/\gamma} p(t)^{-\delta/\gamma}, \tag{9}$$

and the Lagrange multiplier y is

$$y^{-1/\gamma} = W(0)\left[\mathrm{E}\int_0^\infty e^{-\rho t/\gamma}\xi(t)^{(\gamma-1)/\gamma}p(t)^{-\delta/\gamma}dt\right]^{-1}. \tag{10}$$

Consequently:

(i) Real consumption choice c is decreasing in the price level p (holding all else fixed);

(ii) the Lagrange multiplier y is decreasing in expected inflation μ_p;

(iii) under the condition $\mathrm{E}\left[\int_0^\infty e^{-\rho t/\gamma}\xi(t)^{(\gamma-1)/\gamma}p(t)^{-\delta/\gamma}\ln p(t)dt\right] > 0$, *the Lagrange multiplier y is decreasing in δ.*

Proposition 1 reveals that the money-illusioned investor's real consumption is generally dependent on the price level, and in particular, decreases in the price level.[5] To see this result, first consider a normal investor who chooses his optimal consumption plan so that his marginal utility is proportional to the cost of consumption (real state price density) in a given state. Consequently, the investor consumes more in states for which consumption is relatively cheap, and less for which consumption is relatively expensive. Now suppose that the price level in a state is increased. This only increases the nominal price of consumption in that state but has no impact on the real price. Recognizing this rationally, the normal investor then does not alter his consumption and is not affected by the price level change. A money-illusioned investor, however, is confused between nominal and real quantities and perceives the consumption in that state to be more expensive. As a result, the money-illusioned investor allocates less consumption to that state, leading to implication (i) in Proposition 1. Supportive evidence

[5] In the early literature (e.g., Leontief (1936), Patinkin (1965, p22)), money illusion is defined as a violation of the homogeneity property, the demand and supply functions being homogenous of degree zero in all nominal prices. This definition is consistent with (9).

for this implication is documented, via a series of experiments, by Raghubir and Srivastava (2002). These authors demonstrate that individuals purchase less of a product in real terms when the price of the product is denominated in a "cheaper" currency, that is, when the nominal price level is higher, and vice versa when the nominal price level is lower.

Properties (ii)–(iii) reveal that there is an additional, indirect effect of money illusion, via the Lagrange multiplier y associated with the investor's static budget constraint. This "wealth effect" captures how wealthy the investor is initially, and is present even if there is currently no inflation ($p(t) = 1$). According to property (ii), higher expected inflation reduces this multiplier, and hence inducing the investor to consume more (c is decreasing in y from (9)) even if the price level may not have changed from the initial date. Intuitively, a higher expected inflation implies a higher nominal wealth growth rate, making a money-illusioned investor feel richer and hence increase his current consumption, which reduces his future consumption growth. Similarly, property (iii) reveals that in an inflationary environment with higher expected inflation (when the condition in (iii) holds), a higher degree of money illusion (higher δ) leads the investor to consume more now through the indirect y effect, even if there is no concurrent inflation.

3. Equilibrium in the Presence of Money Illusion

In this Section, we investigate the equilibrium effects of money illusion, and focus on the case of there being a representative money-illusioned investor, where the parameter δ may be interpreted as the degree of money illusion in the economy. Section 4 relaxes this single investor assumption to heterogeneous investors with varying degrees of illusion. Following the standard definition of competitive equilibrium, we define equilibrium in the economy with money illusion as follows.[6]

Definition 1. An *equilibrium* is a price system (r, S) and consumption-portfolio policies (c, π) such that: (i) the investor chooses his optimal consumption-portfolio strategies given his money-illusioned perception bias, and (ii) good and security markets clear, i.e.,

$$c(t) = D(t), \;\; \pi_B(t) = 0, \;\; \pi_S(t) = 1, \;\; W(t) = S(t). \tag{11}$$

[6]There is no explicit economic role of money (such as via a cash-in-advance constraint or a money-in-the-utility-function formulation). Hence, in this Section and subsequent Section 4, the nominal price parameters (R, μ_p, σ_p) are exogenously specified, and need only to obey the extended Fisher equation, $R(t) = r(t) + \mu_p - \|\sigma_p\|^2 - \sigma_p^\top \kappa$, a no-arbitrage restriction implied by the market price of risk process.

3.1. Interest Rate and Market Price of Risk

In the benchmark economy with no money illusion ($\delta = 0$), the price of real consumption, ξ^N, and its dynamics are not dependent on the price level, and are given by

$$\xi^N(t) = e^{-\rho t} D(t)^{-\gamma}, \tag{12}$$

$$r^N = \rho + \gamma\mu_D - \frac{1}{2}\gamma(\gamma+1)\|\sigma_D\|^2, \tag{13}$$

$$\kappa^N = \gamma\sigma_D. \tag{14}$$

Proposition 2 solves for the equilibrium state price density and its dynamics in the presence of money illusion.

Proposition 2. *In the economy with money illusion, under the probability measure $\mathcal{P}$, the state price density, real interest rate and market price of risk are given by*

$$\xi(t) = e^{-\rho t} D(t)^{-\gamma} p(t)^{-\delta}, \tag{15}$$

$$r = \rho + \gamma\mu_D - \frac{1}{2}\gamma(\gamma+1)\|\sigma_D\|^2 + \delta\left[\mu_p - \frac{1+\delta}{2}\|\sigma_p\|^2 - \gamma\sigma_D^\top\sigma_p\right], \tag{16}$$

$$\kappa = \gamma\sigma_D + \delta\sigma_p. \tag{17}$$

Consequently:

(i) The state price density ξ is decreasing in the price level p;

(ii) the short real interest rate is increasing in expected inflation.

In the presence of money illusion ($\delta > 0$), the price of real consumption ξ is directly driven by the price level, and so its dynamics (r, κ) are affected by expected inflation and volatility. As stated in property (i), real state prices are decreasing in the price level. This result is a consequence of the consumption behavior of a money-illusioned investor in that his real consumption decreases when the price level increases (Proposition 1(i)). The lower demand due to money illusion makes the consumption good "less valuable," thereby decreasing the state prices. Moreover, property (ii) reveals that the real interest rate and expected inflation are positively related. The intuition is that when expected inflation increases, the future price levels tend to be higher, and hence the demand for real consumption tends to be lower in the future (Proposition 1(i)). The real bond is then less appealing to the investor, and so the interest rate has to rise to counteract this.[7]

[7] The empirical evidence for the relation between the real short rate and expected inflation is mixed. Barr and Campbell (1997) find that the levels of real interest rates and expected inflation have positive correlation, but the changes in real interest rates and expected inflation have negative correlation. Clearly, the rich dynamics of the real short rate cannot fully be captured by our simple model. However, as we demonstrate in Sections 3.4 and 4, our baseline setting can easily be extended to capture richer dynamics.

As for the market price of risk in the presence of money illusion, it is now also driven by the fluctuations in price level σ_p, in addition to the aggregate consumption volatility σ_D. As a consequence, the variability in state prices, $\|\kappa\|$, is increased by the presence of money illusion, for low correlation between price level and aggregate consumption ($\sigma_D^\top \sigma_p$), as empirical evidence suggests.

3.2. Stock Price and Dividend Price Ratio and Bond Yields

We now turn to the behavior of the stock price and long term bond yields in the presence of money illusion. Letting $B(t,T)$ to denote the time-t price of a zero coupon bond with payoff one unit of consumption upon its maturity T, we define its yield $r(t,T)$ as

$$r(t,T) \equiv -\frac{1}{T-t} \ln B(t,T).$$

We also define the long term expected inflation from time t to T, $\mu_p(t,T)$, as[8]

$$\mu_p(t,T) \equiv \frac{1}{T-t} \mathrm{E}_t \left(\ln \frac{p(T)}{p(t)} \right) + \frac{1}{2} \|\sigma_p\|^2. \tag{18}$$

We note that under the geometric Brownian motion assumption on p, $\mu_p(t,T) = \mu_p$ for all T. Note also that the instantaneous real interest rate r and expected inflation μ_p are the limits of $r(t,T)$ and $\mu_p(t,T)$ when T goes to t. Proposition 3 reports the equilibrium stock price and bond yields.

Proposition 3. *In the economy with money illusion, the stock price is given by*

$$S(t) = \frac{D(t)}{\rho + (\gamma - 1)\left(\mu_D - \frac{1}{2}\gamma\|\sigma_D\|^2\right) + \phi}, \tag{19}$$

where

$$\phi \equiv \delta \left(\mu_p - \frac{1+\delta}{2} \|\sigma_p\|^2 - (\gamma - 1)\sigma_D^\top \sigma_p \right), \tag{20}$$

and the real term structure is flat, that is, $r(t,T) = r$ for all T.

Moreover:

(i) The dividend-price ratio D/S is increasing in expected inflation;

(ii) the long term bond yields $r(t,T)$ are increasing in long term expected inflation $\mu_p(t,T)$.

The effect of money illusion on stock valuation is to make the stock price level directly depend on expected inflation and its volatility, in contrast to the benchmark economy ($\delta = 0$)

[8] The variance term $\frac{1}{2}\|\sigma_p\|^2$ is a standard Jensen's Inequality adjustment for taking expectation of a logarithm function.

in which there is no such effect ($\phi = 0$). Moreover, as stated in property (i), the dividend-price ratio is increasing in expected inflation. The reason for this is that higher expected inflation implies higher future price levels, and hence future consumption becomes "less valuable" (Proposition 2(i)). Since the stock is a claim to future aggregate consumption, a higher expected inflation then leads to a lower stock price or equivalently, to a higher dividend-price ratio. We note that this positive relation between expected inflation and dividend-price ratio is consistent with the empirical evidence documented in various studies including Modigliani and Cohn (1979), Ritter and Warr (2002), Sharpe (2002), and Campbell and Vuolteenaho (2004).

In fact, these studies also attribute the positive relation between expected inflation and dividend-price ratio to money illusion. For example, Modigliani and Cohn (1979) postulate that the low stock valuation (higher dividend-price ratio) in 1970's, when the expected inflation was high, is caused by money illusion. More specifically, money-illusioned investors discount real payoffs by nominal rates, and so tend to over-discount in an inflationary environment. Consequently, a higher expected inflation leads to a lower stock value, i.e., a higher dividend-price ratio.

This hypothesis is intuitive, yet by assumption, leaves the discount rate indeterminate and hence is silent on the impact of money illusion on bond yields. This issue is addressed in our equilibrium model where the interest rates are endogenously determined. Property (ii) shows that long term bond yields are also positively related to long term expected inflation. Similar to (i), higher expected inflation makes future real consumption "less valuable," and hence the long term bonds, increasing the long term bond yields. This implication is also consistent with the empirical findings in both U.K. data (Barr and Campbell (1997)) and U.S. data (Sharpe (2002)).

3.3. Economic Significance of Money Illusion

We here perform a simple calibration of our baseline model to quantitatively assess the economic significance of money illusion on an investor's welfare and the equilibrium security prices and their dynamics. Towards this, we assume throughout our baseline economic environment with a representative investor exhibiting a degree of money illusion δ, and hence the equilibrium security prices and their dynamics are as given by Propositions 2–3 in our calibrations.

We quantify the welfare loss due to money illusion as follows. Consider a normal investor without money illusion who chooses the consumption plan $c^N(t)$ given by (7) in Section 2.3, and so his indirect utility is

$$U^N(W(0)) = E\left[\int_0^\infty e^{-\rho t}\frac{c^N(t)^{1-\gamma}}{1-\gamma}dt\right], \tag{21}$$

for given initial wealth $W(0)$. Suppose now that the investor has to follow the alternative money-illusioned consumption plan $c(t)$ given by (9), and so his indirect utility becomes

$$U(W(0)) = E\left[\int_0^\infty e^{-\rho t}\frac{c(t)^{1-\gamma}}{1-\gamma}dt\right]. \tag{22}$$

We define the welfare loss measure, $k(\delta)$, for a given degree of money illusion δ, as the investor's loss quantified in units of initial wealth and satisfying

$$U^N(W(0)(1-k(\delta))) = U(W(0)). \tag{23}$$

The left hand side of (23) is the normal investor's indirect utility with initial wealth reduced by a fraction of $k(\delta)$, while the right hand side is his indirect utility if forced to follow the optimal consumption plan of an investor with a degree of illusion δ. That is, for a normal investor, taking the consumption plan of an investor with a degree of money illusion δ is equivalent to giving up a proportion $k(\delta)$ of his wealth.

The rationale behind (23) is as follows. An investor uses a standard CRRA specification to guide his consumption and investment decisions. However, due to overlook or various constraints such as the cost of obtaining detailed information on inflation, the investor just follows a simpler decision rule, which often ignores inflation. Proposition 4 explicitly solves for this loss measure $k(\delta)$ as a function of the primitive parameters in the model.

Proposition 4. *The welfare loss $k(\delta)$ defined in (23) is given by*

$$k(\delta) = 1 - \frac{(\rho+\eta_2)^{1/(\gamma-1)}(\rho+\eta_3)}{(\eta_1+\rho/\gamma)^{\gamma/(\gamma-1)}}, \tag{24}$$

where

$$\begin{aligned}
\eta_1 &\equiv \frac{\gamma-1}{\gamma}r + \frac{\gamma-1}{\gamma^2}\frac{\|\kappa\|^2}{2}, \\
\eta_2 &\equiv (\gamma-1)\mu_D - \frac{1}{2}\gamma(\gamma-1)\|\sigma_D\|^2, \\
\eta_3 &\equiv \eta_2 + \delta\left(\mu_p - \frac{1}{2}(1+\delta)\|\sigma_p\|^2 - (\gamma-1)\sigma_D^\top\sigma_p\right),
\end{aligned}$$

and r and κ are as in Proposition 2.

For our calibration, the dividend parameters, μ_D and $\|\sigma_D\|$, are chosen to match the consumption data, based on the U.S. quarterly data from 1891–1998 in Campbell (2003):

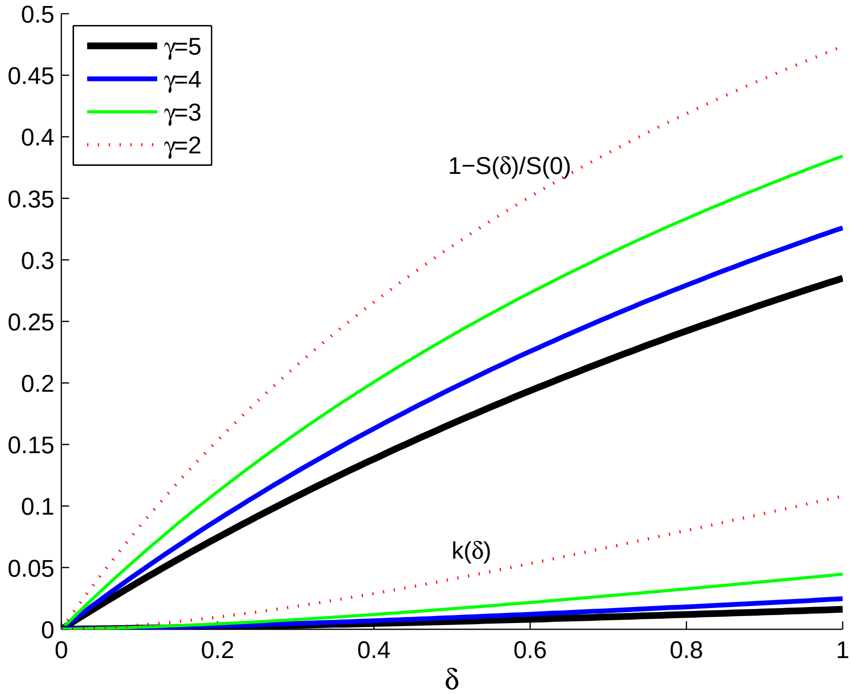

Figure 1: Welfare Loss and Stock Price Effect of Money Illusion. This figure plots an investor's welfare loss measure $k(\delta)$, given by equation (24), and $1-S(\delta)/S(0)$, where $S(\delta)$ and $S(0)$ are the stock prices when the degree of money illusion is δ and 0, respectively, for varying levels of investor's relative risk aversion γ. The relevant parameter values are set as: $\mu_D = 0.01789$, $\|\sigma_D\| = 0.03218$, (Campbell (2003)), $\mu_p = 0.034$, $\|\sigma_p\| = 0.025$, $\sigma_p^T \sigma_D = 0.00025$ (U.S. Department of Commerce), and $\rho = 0.02$.

$\mu_D = 0.01789$, $\|\sigma_D\| = 0.03218$. Based on the quarterly price index data from the U.S. Department of Commerce, Bureau of Economic Analysis during the post-war 1951–2003, we set $\mu_p = 0.034$, $\|\sigma_p\| = 0.025$, and $\sigma_p^T \sigma_D = 0.00025$. Substituting the above parameters into the expression of $k(\delta)$ in (24) leads to a function of δ, γ, and ρ. We then plot $k(\delta)$, $\delta \in [0, 1]$, for various values of $\gamma \in [2, 5]$ and $\rho \in [0.01, 0.05]$. It turns out that for all plots, $k(\delta)$ in general is relatively small. Figure 1 plots $k(\delta)$ for the intermediate case of $\rho = 0.02$ and is representative of our conclusions. For instance, for $\gamma = 3$ and $\delta = 0.2$, $k(\delta)$ is merely 0.004. That is, for a normal investor, following the consumption plan of an investor with a degree of money illusion 0.2 is equivalent to giving up 0.4% of his initial wealth. The results for other parameter values for γ and ρ are similar: the welfare loss $k(\delta)$ is generally less than 1% when $\delta = 0.2$ and only becomes non-negligible when the degree of money illusion δ is large. Similar results are obtained from the calibrations for the cases in which the investor has a finite horizon. These results support the bounded rationality viewpoint that instead of going through the trouble of fully incorporating inflation all the time, an investor may partially overlook the impact of inflation. Our welfare loss findings are also consistent with the analysis of Cochrane (1989), who demonstrates that simple rules of thumb for consumption ("near rational" alternatives) only incur very small utility costs. Similarly, in our setting with a mildly inflationary environment, a money-illusioned investor's consumption choice does not deviate much from the optimal one, and hence only leads to a small welfare loss.

We next consider the quantitative effects of money illusion on security prices and their dynamics. These are simply obtained from the explicit expressions in Propositions 2–3 for varying levels of δ given the calibrated model parameters. Figure 1 also plots the effect on the stock price level, and is representative of all other plots. We see that in contrast to the small welfare loss, money illusion has a significant impact on the equilibrium stock price. For example, for $\gamma = 3$, $\rho = 0.02$ and $\delta = 0.2$, the stock price in the presence of money illusion is 10.67% lower than that in the benchmark case with no illusion. Similar large effects on other variables, such as the interest rate and the market price of risk, obtain. To understand why money illusion has such a large impact on asset prices in typical economic environments, we note that the historical inflation rate is 3.4%. Hence, a money-illusioned investor with $\delta = 0.2$ generally over-discounts by 0.68% (= $0.2 \times 3.4\%$). This is substantial given that in the absence of money illusion, the real interest rate is on par with the real consumption growth rate (equation (18)), which historically is only around 1.8%.

Finally, it is worth noting that the average inflation during the 1970s was twice that of the entire post-war period. To assess our model's quantitative implication for this high inflation period on the stock market valuation, we repeat our calibration based on the 1970s inflation data $\mu_p = 0.068, \|\sigma_p\| = 0.026$ (U.S. Department of Commerce). By comparing the post-war and 1970s calibrations, our model implies that for the case of $\gamma = 3, \delta = 0.2$, the stock market valuation during the 1970s should be 9.9% lower than that during the post-war period. This implication is fairly close to the actual data: According to the estimates by Robert Shiller, the stock market valuation during the 1970s was 11.4% lower than the average during the post-war period.[9]

3.4. Time Varying Expected Inflation

In this Section, we generalize the current economic setting to incorporate mean reversion in expected inflation, and hence obtain richer dynamics for the equilibrium price processes. In particular, we assume the price level to follow

$$dp(t) = p(t) \left[\mu_p(t)dt + \sigma_p^\top d\omega(t) \right], \tag{25}$$

[9] The stock market valuation is measured by the ratio of price and 10 year moving average of earnings of the S&P 500 Index. The data is available at http://www.econ.yale.edu/ shiller/data.htm.

with the instantaneous expected inflation μ_p following a mean reverting process

$$d\mu_p(t) = \beta\left(\overline{\mu}_p - \mu_p(t)\right)dt + \sigma_{\mu_p}^\top d\omega(t), \tag{26}$$

where $\overline{\mu}_p$ and σ_{μ_p} are constants. As shown in the Appendix, the long term expected inflation $\mu(t,T)$ is a linear combination of the instantaneous expected inflation $\mu_p(t)$ and the long run expected inflation $\overline{\mu}_p$, given by

$$\mu(t,T) = \frac{1-e^{-\beta(T-t)}}{\beta(T-t)}\mu_p(t) + \left(1 - \frac{1-e^{-\beta(T-t)}}{\beta(T-t)}\right)\overline{\mu}_p. \tag{27}$$

The previous analysis can straightforwardly be extended to this setting, as demonstrated in the Appendix. We hereby only report the expressions for the stock price and real long term bond yields for brevity.

Proposition 5. *In the economy with money illusion and mean reverting expected inflation, the equilibrium stock price and the long term bond yields are given by*

$$S(t) = D(t)f(\mu_p(t)), \tag{28}$$

$$r(t,T) = g(T-t) + \delta\mu_p(t,T), \tag{29}$$

where $f(\cdot)$ and $g(\cdot)$ are deterministic functions with $f(\cdot) > 0$, $f'(\cdot) < 0$, and are provided in the Appendix.

Consequently:

(i) The dividend-price ratio does not predict the future dividend growth but predicts the future stock returns;

(ii) the volatility vector for the stock return is $\sigma_S(t) = \sigma_D + \frac{f'(\mu_p(t))}{f(\mu_p(t))}\sigma_{\mu_p}$;

(iii) the dividend-price ratio is increasing in expected inflation $\mu_p(t)$;

(iv) the long term bond yields $r(t,T)$ are increasing in long term expected inflation $\mu_p(t,T)$.

Under this richer setting, money illusion induces the price dividend ratio to be time varying, even though the dividends follow a geometric random walk (since D/S is monotonic in μ_p and μ_p is mean reverting). This implies that, consistent with empirical findings (e.g., Campbell and Shiller (2004)), the dividend-price ratio does not predict the future dividend growth but predicts future stock returns. Moreover, due to money illusion, the variation in expected inflation induces variation in the dividend-price ratio, and hence in stock returns. In particular, the stock return volatility is increased under empirically plausible parameters. Specifically, since empirical evidence suggests $\sigma_{\mu_p}^\top\sigma_D < 0$ (Hess and Lee (1999)), the σ_S expression in property (ii) implies that $\|\sigma_S\| > \|\sigma_D\|$. Since the original studies on "excess volatility" (Shiller (1981), LeRoy and Porter (1981)) and "predictability" (Fama and French (1988)), extensive efforts have been devoted to these issues, though a consensus view is still not reached.[10] The results in properties (i)–(ii) suggest that money illusion may further add an ingredient to this ongoing debate.

Properties (iii)–(iv) on the dependence of the dividend-price ratio and long term bond yields on expected inflation confirm the earlier results of Proposition 3, but now in a richer economic setting. The intuition, again, is that higher expected inflation implies higher price levels in the future, making future real consumption "less valuable," and hence leading to higher dividend-price ratio and bond yields.

Remark 1 (A preference-based formulation of money illusion.) We have so far employed a belief-based formulation to analyze the implications of money illusion. We here present an alternative tractable formulation, based on a preference specifications, which incorporates the potential confusion between real and nominal quantities into an investor's decision-making and valuation framework. In particular, we posit that an investor's objective function is represented over real, as well as nominal consumption, with a degree of money illusion θ controlling the bias towards nominal evaluation:

$$u(c, p) = \frac{1}{1-\gamma}\left(c^{1-\theta}(pc)^{\theta}\right)^{1-\gamma}, \quad \gamma > 0, \ \ 0 \leq \theta \leq 1.$$

This formulation has essentially the same foundation as traditional preferences[11] and captures in reduced-form that an investor may partially evaluate his objectives based on nominal, instead of real terms, and nests the benchmark case of a normal investor with no illusion ($\theta = 0$), as well as the case of complete illusion ($\theta = 1$). As in our belief-based formulation, the investor is guided by CRRA preferences with the risk aversion coefficient γ.

As shown in Basak and Yan (2007), it turns out that this preference-based formulation for the case of more-risk-averse than logarithmic ($\gamma > 1$), which is the more empirically plausible case, leads to the same results as in our belief-based formulation, with the exception of Section 5. Technically, the analysis in the current paper goes through with δ essentially replaced by $\theta(\gamma - 1)$, though the economic intuition for a number of implications is somewhat different since the mechanism for money illusion is via preferences rather than beliefs. More recently, Miao and Xie (2007) adopt our preference-based formulation of money illusion to explore its implications on monetary policy and long-run growth.

Remark 2 (More general consumption supply and price level processes.) For most of our analysis, we employ the geometric Brownian motion specification, with constant drift

[10] See Campbell (2003) and Barberis and Thaler (2003) for recent surveys.

[11] See Loffler (2001) for an analysis on the axiomatic foundation for a demand function that violates homogeneity and so, as he discusses, exhibits money illusion.

and volatility, for both the consumption supply D and the price level process p. However, much of our analysis and conclusions are valid for more general processes for which the drifts and volatilities of D and p are general adapted processes. Indeed, Propositions 1–2 (equations (9)–(10), (15)–(17)) and their implications (properties (i)–(ii)) pertaining to consumption and state price density behavior clearly hold for general D and p Itô processes. With the price level generally modeled as

$$dp(t) = p(t)[\mu_p(t)dt + \sigma_p(t)^\top d\omega(t)],$$

where μ_p, σ_p are possibly stochastic processes, the illusioned investor's objective function is as in (6), with b replaced by its stochastic counterpart $b(t) \equiv \frac{\delta}{t}\int_0^t \left(\mu_p(\tau) - \frac{1}{2}(1+\delta)\left\|\sigma_p(\tau)\right\|^2\right) d\tau$ and the illusioned probability measure $\mathcal{P}^*$ given by $(d\mathcal{P}^*/d\mathcal{P})_s = e^{-\frac{1}{2}\int_t^s \|\delta\sigma_p(\tau)\|^2 d\tau - \int_t^s \delta\sigma_p(\tau)^\top d\omega(\tau)}$. Nevertheless, we did rely on the geometric Brownian motion specification, at places, to obtain closed-form expressions for the prices for long-lived securities, such as the stock price and long term bond prices in Proposition 3 (Section 3.2). However, as properties (iii)–(iv) of Proposition 5 confirm, our main findings pertaining to long-lived assets hold more generally under richer settings than geometric Brownian motion.

4. Investor Heterogeneity and Money Illusion

We now generalize our equilibrium analysis of Section 3 containing a single representative illusioned investor to a setting with multiple investors having different degrees of money illusion. This will allow us to investigate the effects of investor heterogeneity in the presence of money illusion on the ensuing equilibrium. Apart from confirming the insights we get from the homogeneous investor model, this modification also generates some new insights arising from the interaction between money-illusioned and normal investors.

In particular, we assume that the economy is populated by two types of investors, identical normal investors with no money illusion ($\delta = 0$) and investors with complete money illusion ($\delta = 1$). The partial equilibrium economic setting is as in Section 2 and both of these types of investors' optimal policies are presented in Section 2.3 (equations (7)–(8) for the normal; equations (9)–(10) for the illusioned). The definition of equilibrium is then a two-investor version of Definition 1 in Section 3. Proposition 6 characterizes the equilibrium in this heterogeneous investor economy.

Proposition 6. *In the economy with a normal investor and a completely money-illusioned, the equilibrium state price density, interest rate, market price of risk, and stock price are given* *by*

$$\xi(t) = e^{-\rho t} D(t)^{-\gamma} \left(1 - \hat{\delta}(t)\right)^{-\gamma}, \tag{30}$$

$$\begin{aligned} r(t) &= \rho + \gamma\mu_D - \frac{1}{2}\gamma(\gamma+1)\|\sigma_D\|^2 \\ &\quad + \hat{\delta}(t)\left[\mu_p + \left(\frac{(\gamma-1)\,(1-\hat{\delta}(t))}{2\gamma} - 1\right)\|\sigma_p\|^2 - \gamma\sigma_D^\top\sigma_p\right], \end{aligned} \tag{31}$$

$$\kappa(t) = \gamma\sigma_D + \hat{\delta}(t)\sigma_p, \tag{32}$$

$$S(t) = D(t)\mathrm{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\left(\frac{D(s)}{D(t)}\right)^{-(\gamma-1)}\left(\frac{1-\hat{\delta}(s)}{1-\hat{\delta}(t)}\right)^{-\gamma} ds\right], \tag{33}$$

where the consumption share, $\hat{\delta}$, is the ratio of the illusioned investor's consumption to the aggregate consumption, representing the degree of money illusion, and is given by

$$\hat{\delta}(t) = \frac{1}{1 + y^{1/\gamma} p(t)^{1/\gamma}}, \tag{34}$$

and the initial wealth weight y satisfies the illusioned investor's static budget constraint (10) with (30) and (34) substituted in.

Consequently:

(i) The state price density is decreasing in the price level;

(ii) the real short rate and realized inflation are negatively correlated, $\mathrm{cov}_t(dr(t), dp(t)) < 0$, *under the condition* $\mu_p > \left(1 + \frac{\gamma-1}{\gamma}(\hat{\delta}(t) - \frac{1}{2})\right)\|\sigma_p\|^2 + \gamma\sigma_D^\top\sigma_p$.

As compared with the single illusioned-investor economy, the equilibrium is now additionally driven by the stochastic distribution of consumption across investors ($\hat{\delta}$) and the relative initial wealth distribution (y). The driving process $\hat{\delta}$ is the consumption share (c/D) of the illusioned investor and plays the role of the exogenously specified constant parameter δ in the single investor economy. In particular, $\hat{\delta}$ captures the degree of money illusion in the economy, which is now stochastic and is endogenously determined by the price level and the initial wealth distribution. Indeed, when the relative weight of the illusioned investor $1/y$ approaches 0, the degree of illusion $\hat{\delta}$ gradually vanishes, and all the equilibrium quantities collapse to their no-illusion counterparts.

The effect of money illusion in this heterogeneous investor economy is to make the state prices negatively depend on the price level. This is as in the single investor case, and hence much of our earlier discussion remains valid in this heterogeneous investor case. More interestingly though, the equilibrium interest rate and market price of risk are stochastic, in contrast to being constants in the single investor case, and depend on the price level (via $\hat{\delta}$). An important implication is that the real short rate and realized inflation are negatively correlated under the condition in property (ii), which holds for empirically reasonable parameters.[12] The intuition is that when the realized inflation (or price level) is higher, the money-illusioned investor's consumption decreases (Proposition 1(i)), decreasing his consumption share $\hat{\delta}$. This in turn brings the equilibrium closer to that in a normal investor economy for which the interest rate is lower since in an inflationary environment (the condition in property (ii)), money illusion makes future consumption "less valuable" (Proposition 2(i)), and hence increases the interest rate. This leads to a negative correlation between the real short rate and realized inflation, which is consistent with empirical findings in various previous studies (see Ang, Bekaert and Wei (2008) and the references therein). The rich dynamics induced by the interaction between the money-illusioned and normal investors have the potential to address other issues such as the long-run inflation hedging properties of stocks and bonds. We leave this to future research.

Our analysis also offers some insights on the long-run properties of the economy. As shown in equation (34), the long-run properties of the illusioned investor's consumption share depends on the long-run properties of the price level. In an inflationary environment with $\mu_p > \frac{1}{2}\|\sigma_p\|^2$, the price level converges to infinity in the long run, implying that the illusioned investor's consumption share $\hat{\delta}$ converges to zero. That is, the illusioned investor disappears in the long run. Otherwise, if $\mu_p < \frac{1}{2}\|\sigma_p\|^2$, the price level converges to zero in the long run, implying that the illusioned investor dominates the market in the long run. The intuition is that in an inflationary environment, the money-illusioned investor overdiscounts future payoffs and so saves too little for the future. As a result, the illusioned investor's consumption share decreases over time and disappears in the long run, and vice versa dominates in a deflationary environment. Under reasonable parameter values (e.g., those in Figure 1) $\mu_p > \frac{1}{2}\|\sigma_p\|^2$ holds, implying that the money-illusioned investor's wealth share will disappear in the long run. However, this process is extremely slow. For the case of $\gamma = 3$, for example, it takes on average around 100 years for the illusioned investor to lose half of his consumption share. This is consistent with the findings of Yan (2008), who analyzes a model with heterogeneous beliefs and shows that it takes hundreds of years for an investor with a significantly wrong belief to lose half of his wealth share.

[12] The condition in property (ii) requires the expected inflation to be high enough relative to the consumption and inflation volatilities, and typically holds in the data for empirically plausible relative risk aversions. Indeed, according to the U.S. quarterly data on price index and consumption during 1951–2003 (source: U.S. Department of Commerce, Bureau of Economic Analysis), $\mu_p = 0.034$, $\|\sigma_p\| = 0.025$, $\sigma_D^\top \sigma_p = 0.00025$, the condition in property (ii) holds for $\gamma \in [1, 148]$ and $\hat{\delta} \in [0, 1]$.

5. Conclusion

We provide a tractable continuous-time pure exchange framework to understand the implications of the presence of money illusion in financial markets. We develop a belief-based formulation of money illusion and demonstrate an illusioned investor's behavior to be generally driven by the price level, in addition to real economic conditions in the economy. We provide a full characterization of security prices and their dynamics in the ensuing equilibrium, and demonstrate that although money illusion only induces a small welfare loss, it has a significant impact on equilibrium prices, and that some of the empirical regularities can be better understood in the context of money illusion. The basic setting readily extends to additionally incorporate richer dynamics for pertinent processes and investor heterogeneity in the degree of money illusion. In concurrent work we also generalize the current economic setting, where the price level process is taken as given, to a dynamic monetary economy, and carry out the analogous equilibrium analysis while simultaneously endogenously determining the inflation in the presence of money illusion. We build on the nominal pricing models of Bakshi and Chen (1996) and Basak and Gallmeyer (1999), and consider an illusioned investor who holds money for the purpose of transaction services via a money-in-the-utility-function formulation. As in current analysis, a money-illusioned investor's real consumption and investment decisions are affected by the price level. Hence, due to money illusion, the monetary authority can affect the real side of the economy (e.g., real interest rates and market price of risk) through money supply, a feature not possible under no illusion in our monetary model.

Our formulation of money illusion can be extended to further analyze the effects of money illusion under different economic settings. One such generalization is to study the impact of money illusion on macroeconomic variables (e.g., consumption and economy growth) in a production economy. As shown in our monetary economy extension, due to money illusion, an investor's real consumption and investment choices are affected by money supply in equilibrium. This violation of money neutrality implies that money supply would potentially have a large impact on the output and that money illusion could play an important role in the fluctuations of the aggregate economy and the monetary policy. Another generalization is to an international setting, along the lines of Fisher's (1928) original observation, that people tend to think in terms of their own currency. It would then be of interest to investigate the implications of this "currency illusion" on exchange rates and international portfolio choice, in addition to the quantities analyzed in the current paper. Finally, as an initial attempt to model money illusion, our formulation only incorporates an investor's bias towards nominal quantities in his decision-making. Survey and experimental studies, however, reveal additional considerations as to the nature of money illusion (Shiller (1997), Fehr and Tyran (2001, 2007)). For instance, in their experiments, Fehr and Tyran (2007) find that strategic uncertainty about other investors' behavior may play an important role in sustaining the impact of money illusion. Extending our formulation to incorporate an investor's consideration of other investors' reaction to changes in the price level may be a fruitful direction for future research.

Appendix: Proofs

Proof of Proposition 1. Using martingale techniques (Cox and Huang, 1989; Karatzas et al., 1987), the investor's dynamic budget constraint (1) can be written as

$$\mathrm{E}\left[\int_0^\infty \xi(t)c(t)dt\right] \leq W(0). \tag{35}$$

Then, the necessary and sufficient conditions for optimality of the consumption stream are

$$e^{-\rho t}c(t)^{-\gamma}p(t)^{-\delta} = y\xi(t),$$

leading to expression (9), with y solving (35) holding with equality and (9) substituted in. Properties (i)–(iii) follow from equations (9)–(10) straightforwardly. *Q.E.D.*

Proof of Proposition 2. From the consumption choice (9) and the market clearing condition (11), we obtain

$$\left(e^{\rho t}\xi(t)\right)^{-1/\gamma} p(t)^{-\delta/\gamma} = D(t).$$

Solving for $\xi(t)$ from the above expression yields (15). Equations (16)–(17) can be obtained by applying Itô's lemma to both sides of (15) and matching the drift and volatility terms. Properties (i)–(ii) are direct implications from equations (15)–(16). *Q.E.D.*

Proof of Proposition 3. The no-arbitrage stock price is given by

$$S(t) = \frac{1}{\xi(t)}\mathrm{E}_t\left[\int_t^\infty \xi(s)D(s)ds\right]. \tag{36}$$

Substituting (15) into the above expression and manipulating, we obtain

$$S(t) = D(t)\int_t^\infty e^{-\rho(s-t)}\mathrm{E}_t\left[\left(\frac{p(s)}{p(t)}\right)^{-\delta}\left(\frac{D(s)}{D(t)}\right)^{-(\gamma-1)}\right] ds. \tag{37}$$

From the geometric Brownian motion assumption on D and p, we have

$$D(s) = D(t)e^{\left(\mu_D - \frac{1}{2}\|\sigma_D\|^2\right)(s-t)+\sigma_D^\top(\omega(s)-\omega(t))}, \tag{38}$$

$$p(s) = p(t)e^{\left(\mu_p - \frac{1}{2}\|\sigma_p\|^2\right)(s-t)+\sigma_p^\top(\omega(s)-\omega(t))}. \tag{39}$$

Substitute (38) and (39) into the conditional expectation in (37), we obtain

$$\mathrm{E}_t\left[\left(\frac{p(s)}{p(t)}\right)^{-\delta}\left(\frac{D(s)}{D(t)}\right)^{-(\gamma-1)}\right] = e^{-\left((\gamma-1)\left(\mu_D-\frac{1}{2}\|\sigma_D\|^2\right)+\delta\left(\mu_p-\frac{1}{2}\|\sigma_p\|^2\right)\right)(s-t)}\mathrm{E}_t\left[e^{X(s)}\right], \tag{40}$$

where $X(s) = -\left((\gamma-1)\sigma_D+\delta\sigma_p\right)^\top(\omega(s)-\omega(t))$ is normally distributed with mean 0 and variance $\|(\gamma-1)\sigma_D+\delta\sigma_p\|^2(s-t)$. We then have $\mathrm{E}_t[e^{X(s)}] = e^{\frac{1}{2}\|(\gamma-1)\sigma_D+\delta\sigma_p\|^2(s-t)}$. Substituting this expression and (40) into (37), after some manipulation, we obtain (19). Since the short rate is constant in this economy, the term structure is flat, that is, $r(t,T) = r$ for all T. Properties (i)–(ii) are immediate. *Q.E.D.*

Proof of Proposition 4. Equations (7), (9) and (15) for ξ imply that $c^N(t)$ and $c(t)$ are geometric Brownian motions. Thus, substituting (7)–(10) into (21) and (22), we can explicitly evaluate the expectations and integrals in the expressions of $U^N(W(0))$ and $U(W(0))$. Substituting these into $U^N(W(0)(1-k(\delta))) = U(W(0))$, after some algebra, we can solve for $k(\delta)$, as stated in (24). *Q.E.D.*

Proof of Proposition 5. In this richer setting, the objective function of the illusioned investor is as in (6), with now the additional discounting parameter b being replaced by its stochastic counterpart $b(t) \equiv \frac{\delta}{t}\int_0^t \mu_p(\tau)d\tau - \frac{1}{2}\delta(1+\delta)\left\|\sigma_p\right\|^2$. We first prove (27). Assumptions in (25) and (26) imply that

$$p(s) = p(t)e^{\int_t^s\left(\mu_p(\tau)-\frac{1}{2}\|\sigma_p\|^2\right)d\tau+\int_t^s\sigma_p^\top d\omega(\tau)}, \tag{41}$$

$$\mu_p(\tau) = \overline{\mu}_p + \left(\mu_p(t) - \overline{\mu}_p\right)e^{-\beta(\tau-t)} + \int_t^\tau e^{-\beta(\tau-\ell)}\sigma_{\mu_p}^\top d\omega(\ell). \tag{42}$$

From (41), we obtain

$$\mathrm{E}_t\left[\ln\frac{p(T)}{p(t)}\right] = \mathrm{E}_t\left[\int_t^T\left(\mu_p(\tau) - \frac{1}{2}\left\|\sigma_p\right\|^2\right)d\tau\right].$$

Substituting the above expression and (42) into (18), after some algebra, we obtain (27).

To compute the stock price, we first need to compute the conditional expectation in (37). Substituting (38) and (41) into the conditional expectation in (37) we obtain

$$\mathrm{E}_t\left[\left(\frac{p(s)}{p(t)}\right)^{-\delta}\left(\frac{D(s)}{D(t)}\right)^{-(\gamma-1)}\right] = e^{-\left((\gamma-1)\left(\mu_D-\frac{1}{2}\|\sigma_D\|^2\right)-\frac{1}{2}\delta\|\sigma_p\|^2\right)(s-t)}\mathrm{E}_t\left[e^{Y(s)}\right], \tag{43}$$

where

$$Y(s) = -\delta\int_t^s \mu_p(\tau)\,d\tau - \int_t^s\left((\gamma-1)\,\sigma_D^\top + \delta\sigma_p^\top\right)d\omega(\tau). \tag{44}$$

Substituting (42) into (44), we obtain

$$\begin{aligned} Y(s) = & -\delta\left(\overline{\mu}_p(s-t) + \left(\mu_p(t) - \overline{\mu}_p\right)\frac{1-e^{-\beta(s-t)}}{\beta}\right) \\ & -\delta\int_t^s\int_t^\tau e^{-\beta(\tau-\ell)}\sigma_{\mu_p}^\top d\omega(\ell)\,d\tau - \int_t^s\left((\gamma-1)\,\sigma_D^\top + \delta\sigma_p^\top\right)d\omega(\tau). \end{aligned} \tag{45}$$

From the interchangeability of Lebesgue and Itô integrals (Karatzas and Shreve (1988, p. 209)), we have

$$\begin{aligned} \int_t^s\int_t^\tau e^{-\beta(\tau-\ell)}\sigma_{\mu_p}^\top d\omega(\ell)\,d\tau &= \int_t^s\int_\ell^s e^{-\beta(\tau-\ell)}d\tau\sigma_{\mu_p}^\top d\omega(\ell) \\ &= \int_t^s\frac{1-e^{-\beta(s-\ell)}}{\beta}\sigma_{\mu_p}^\top d\omega(\ell). \end{aligned}$$

Substituting the above expression into (45), we obtain $Y(s)$ that is normally distributed, and so $E_t[e^{Y(s)}]$ can explicitly be evaluated. Substituting the explicit expression for $E_t[e^{Y(s)}]$ and (43) into (37), after some manipulation, we obtain (28) with

$$f(\mu_p(t)) = \int_0^\infty e^{h(\mu_p(t),s-t)} ds,$$

where $h(\mu_p(t), s-t)$ is given by

$$h(\mu_p(t), s-t) = -\left[\rho + (\gamma - 1)\left(\mu_D - \frac{1}{2}\|\sigma_D\|^2\right) - \frac{1}{2}\delta\|\sigma_p\|^2\right](s-t) + \mathrm{E}_t Y(s) + \frac{1}{2}\mathrm{var}_t Y(s).$$

Clearly, $f(\cdot)$ is positive. Differentiating $f(\cdot)$, we obtain that $f'(\cdot) < 0$. Similar computations for the long bond yields lead to (29) with

$$\begin{aligned} g(x) \;=\; & \gamma\mu_D - \frac{1}{2}\gamma\|\sigma_D\|^2 - \frac{1}{2}\delta\|\sigma_p\|^2 - \frac{1}{2}\left\|\gamma\sigma_D - \delta(\sigma_p + \frac{1}{\beta}\sigma_{\mu_p})\right\|^2 \\ & + \frac{1 - e^{-\beta x}}{\beta^2 x}\sigma_{\mu_p}^\top\left(\delta\gamma\sigma_D + \delta^2(\sigma_p + \frac{1}{\beta}\sigma_{\mu_p})\right) - \frac{1 - e^{-2\beta x}}{4\beta^3 x}\delta^2\left\|\sigma_{\mu_p}\right\|^2. \end{aligned}$$

Properties (i)–(iv) are immediate from equations (28)–(29). *Q.E.D.*

Proof of Proposition 6. The necessary and sufficient conditions for optimality for the two investors are

$$\begin{aligned} e^{-\rho t}c(t)^{-\gamma}p(t)^{-1} &= y\xi(t), \\ e^{-\rho t}c^N(t)^{-\gamma} &= y^N\xi(t), \end{aligned}$$

where c and c^N are the consumption processes of the illusioned and the normal investors, y and y^N Lagrange multipliers associated with the budget constraints of the illusioned and the normal investors. We set $y^N = 1$ without loss of generality since equilibrium is determined only up to a multiplicative constant.

Solving for the optimal consumption from the above conditions, we obtain

$$c(t) = \left(e^{\rho t}y\xi(t)\right)^{-1/\gamma} p(t)^{-1/\gamma}, \tag{46}$$

$$c^N(t) = \left(e^{\rho t}\xi(t)\right)^{-1/\gamma}. \tag{47}$$

Substituting the above expressions into the market clearing condition $c(t) + c^N(t) = D(t)$, after some algebra, we obtain (30). Applying Itô's Lemma to (30), we obtain (31) and (32). From (46) and (47), we can easily verify that $c(t)/D(t) = \hat{\delta}(t)$. Substituting (30) into the no-arbitrage stock pricing equation (36), we obtain (33).

Property (i) follows from (30), and equation (31) implies that $dr(t)/dp(t) < 0$ if the condition in property (ii) holds, which leads to property (ii). *Q.E.D.*

References

Akerlof, G., 2002, "Behavioral Macroeconomics and Macroeconomic Behavior," *American Economic Review*, 92, 411–433.

Akerlof, G., and J. Yellen, 1985, "Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria?" *American Economic Review*, 75, 708–720.

Ang, A., G. Bekaert, and M. Wei, 2008, "The Term Structure of Real Rates and Expected Inflation," *Journal of Finance*, 63, 797–849.

Bakshi, G., and Z. Chen, 1996, "Inflation, Asset Prices, and the Term Structure of Interest Rates in Monetary Economies," *Review of Financial Studies*, 9, 241–275.

Barberis, N., and R. Thaler, 2003, "A Survey of Behavioral Finance," in *Handbook of the Economics of Finance*, G. Constantinides, M. Harris, and R. Stulz (ed.), North-Holland, Amsterdam.

Barr, D., and J. Campbell, 1997, "Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices," *Journal of Monetary Economics*, 39, 361–383.

Basak, S., and M. Gallmeyer, 1999, "Currency Prices, the Nominal Exchange Rate, and Security Prices in a Two-country Dynamic Monetary Equilibrium," *Mathematical Finance*, 9, 1–30.

Basak, S., and H. Yan, 2007, "Equilibrium Asset Prices and Investor Behavior in the Presence of Money Illusion: A Preference-Based Formulation," SSRN working paper #623561.

Basu, S., S. Markov and L. Shivakumar 2005, "Do Financial Analysts Suffer from Inflation Illusion," working paper, Emory University and London Business School.

Brunnermeier, M., and C. Julliard, 2008, "Money Illusion and Housing Frenzies," *Review of Financial Studies*, 21, 135–180.

Campbell, J., 2003, "Consumption-Based Asset Pricing," in *Handbook of the Economics of Finance* G. Constantinides, M. Harris, and R. Stulz (ed.), North-Holland, Amsterdam.

Campbell, J., and R. Shiller, 2005, "Valuation Ratios and the Long-Run Stock Market Outlook: An Update," in *Advances in Behavioral Finance*, Volume II, R. Thaler (ed.), Princeton University Press, Princeton.

Campbell, J., and T. Vuolteenaho, 2004, "Inflation Illusion and Stock Prices," *American Economic Review Papers and Proceedings*, 94, 19–23.

Chordia, T., and L. Shivakumar, 2005, "Inflation Illusion and Post-earnings-announcement Drift," *Journal of Accounting Research*, 43, 521–556.

Cochrane, J., 1989, "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives," *American Economic Review*, 79, 319–337.

Cohen, R., C. Polk, and T. Vuolteenaho, 2005, "Money Illusion in the Stock Market: The Modigliani-Cohn Hypothesis," *Quarterly Journal of Economics*, 120, 639–668.

Cox, J., and C. Huang, 1989, "Optimal Consumption and Portfolio Policies when Asset Prices Follow a Diffusion Process," *Journal of Economic Theory*, 49, 33–83.

Fama, E., and K. French, 1988, "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics*, 22, 3–25.

Fehr, E., and J. Tyran, 2001, "Does Money Illusion Matter?" *American Economic Review*, 91, 1239–1262.

Fehr, E., and J. Tyran, 2007, "Money Illusion and Coordination Failure," *Games and Economic Behavior*, 58, 246–268.

Fisher, I., 1928, *The Money Illusion*, Longmans, Toronto.

Genesove, D. and C. Mayer, 2001, "Loss Aversion and Seller Behavior: Evidence from the Housing Market," *Quarterly Journal of Economics*, 116, 1233–1260.

Hess, P., and B. Lee, 1999, "Stock Returns and Inflation with Supply and Demand Disturbances," *Review of Financial Studies*, 12, 1203–1218.

Jouini, E. and C. Napp, 2007, Consensus Consumer and Intertemporal Asset Pricing with Heterogeneous Beliefs, *Review of Economic Studies*, 74, 1149–1174.

Kahneman, D., 2003, "Maps of Bounded Rationality: Psychology for Behavioral Economics," *American Economic Review*, 93, 1449–1475.

Karatzas, I., J. Lehoczky, and S. Shreve, 1987, "Optimal Portfolio and Consumption Decisions for a 'Small Investor' on a Finite Horizon," *SIAM Journal of Control and Optimization*, 25, 1157–1186.

Karatzas, I., and S. Shreve, 1988, *Brownian Motion and Stochastic Calculus*, Springer-Verlag, New York.

Keynes, J., 1936, *The General Theory of Employment, Interest and Money*, Macmillan, New York.

Leontief, W. 1936, "The Fundamental Assumptions of Mr. Keynes' Monetary Theory of Unemployment," *Quarterly Journal of Economics*, 5, 192–97.

LeRoy, S., and R. Porter, 1981, "The Present-value Relation: Tests Based on Implied Variance Bounds," *Econometrica*, 49, 97–113.

Loffler, A., 2001, "A μ-σ-Risk Aversion Paradox and Wealth Dependent Uility," *Journal of Risk and Uncertainty*, 23, 57–73.

Mankiw G., 1985, "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly," *Quarterly Journal of Economics*, 100, 529–537.

Miao, J. and D. Xie, 2007, "Monetary Policy and Economic Growth under Money Illusion", working paper, Boston University and HKUST.

Modigliani, F., and R. Cohn, 1979, "Inflation, Rational Valuation, and the Market," *Financial Analysts Journal*, 35, 24–44.

Patinkin, D., 1965, *Money, Interest, and Prices* (2nd ed.), Harper and Row, New York.

Piazzesi, M., and M. Schneider, 2008, Inflation Illusion, Credit, and Asset Pricing, in *Asset Prices and Monetary Policy*, J. Campbell (ed.), University Of Chicago Press, Chicago.

Raghubir, P., and J. Srivastava, 2002, "Effect of Face Value on Product Valuation in Foreign Currencies," *Journal of Consumer Research*, 29, 335–348.

Ritter, J., and R. Warr, 2002, "The Decline of Inflation and the Bull Market of 1982-1999," *Journal of Financial and Quantitative Analysis*, 37, 29–61.

Shafir, E., P. Diamond, and A. Tversky, 1997, "Money Illusion," *Quarterly Journal of Economics*, 112, 341–374.

Sharpe, S., 2002, "Reexamining Stock Valuation and Inflation: The Implications of Analysts' Earnings Forecasts," *Review of Economics and Statistics*, 84, 632–648.

Shiller, R., 1981, "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review*, 71, 421–436.

Shiller, R., 1997, "Public Resistance to Indexation: A Puzzle," *Brookings Papers on Economic Activity*, 1, 159–228.

Simon, H., "Rationality as Process and as Product of Thought," *American Economic Review*, 68, 1–16.

Svedsater, H., A. Gamble, and T. Garling, 2007, "Money Illusion in Financial Decision Making: The Influence of Nominal Representation of Share Price," *Journal of Socio-Economics*, 36, 698–712.

Yan, Hongjun, 2008, "Natural Selection in Financial Markets: Does It Work?" *Management Science* 54, 1935–1950.