Are Chads Democrats?
An Analysis of the Florida Presidential Recount

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Abstract

This paper presents the results from a statistical analysis of the first Florida recount. The findings indicate that it is highly unlikely that the relative increase in Gore's vote total can be explained by mechanical reading errors. Rather it appears partisan biases influenced the outcome. Estimates indicate that on average if a ballot's status changed from no vote to a vote, the chance that it went to Gore was about 15% higher than one would expect given his fraction of that county's vote. Overall then, controlling for each candidate's vote in a county and the type of ballot used, this paper estimates that Gore picked up 903 "too many" votes in the recount relative to what would have been expected by chance machine read errors. If humans influenced the results how did they do it? During the recount the ballots were put through the tabulating machines several times. However, machine readings tend to vary from run to run and this means humans, partisan humans, have to decide which of several tallies to report. Potentially, this discretion allowed the preferences of those conducting the recount to impact the reported totals.

1 Introduction

As everybody by now knows Florida has conducted a machine recount of the votes cast in the

presidential election. Since that recount reduced Bush's lead some are now asking for yet

another recount, this time by hand. Naturally, people tend to think that each recount will produce

a more accurate estimate of the vote total. However, this may or may not be the case. Recounts

offer the possibility of greater precision, but also the possibility that human leanings will alter the

outcome. One place to look for evidence on this is the machine recount. Thus, this analysis asks

if the votes picked up by Gore during the recount were statistically likely to have arisen from

natural fluctuations or is there evidence that human biases may have impacted the result.[1] The

evidence strongly points to the conclusion that human intervention was in fact responsible for

Gore's performance in the recount.

In order to understand how those conducting the recount may have influenced the

outcome it is necessary to understand both how the Florida ballots operate and how the recount

was conducted. Within Florida there are three ballot types in use. Two counties use old

fashioned hand tabulated paper ballots. Another thirty-eight use optical scanner ballots which

require the voter to mark up a card which a machine then reads. Finally, twenty-seven counties

use so called punch card ballots. These ballots ask the voter to poke out little paper dots, called

chads, from the cards to register their preferences. Neither the optical scanner or punch card

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[1]I am not claiming either here or anywhere else in this note that anybody involved with
the recount did anything to deliberately influence the vote total one way or the other. However,
the people that conducted the recount are human and as such suffer from the same weaknesses
we all do. It is therefore possible that they could have taken reasonable actions that were
unconsciously directed to favor their preferred candidate.

ballots are infallible. Two readings of the same ballot through the same machine can apparently

produce different answers. Because the vote totals change after each count, the possibility arises

that the political leanings of any human vote counter will impact the outcome *whether or not* the

recount is by hand or by machine. In the machine recount, press reports indicate that the cards

were passed through the tabulating equipment multiple times to "ensure an accurate recount."[2]

However, with the count frequently changing after each pass when do you stop? For example,

suppose the person running the cards through supports Nader. After the first pass Nader's vote

goes up. After the second pass his vote goes down. Which total do you report, or do you run

them through again? If you run them through again do you stop when two consecutive passes

produce the same result, when a second pass produces as many votes for Nader as the first pass,

or do you use some other rule?

Absent a mechanical stopping and reporting rule it seems impossible to believe the

political passions of the counters did not impact the result. Bush supporters probably argued for

stopping rules that helped Bush, while Gore supporters favored rules that helped Gore.[3] These

battles among the human counters no doubt took place county by county in every office where a

recount was conducted. The question then comes down to whether the Bush or Gore supporters

were the more fervent? If you know the answer to that you probably know something about the

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[2]An Associated Press article by David Royse (2000) claims each ballot was run through three times. However, Lucy Morgan (November 15, 2000) interviewed a number of elections supervisors and found that in reality the number of times a ballot was reread depended upon the people conducting the recount. Neither article states what was done when discrepancies occurred across counts.

[3]I am not saying those counting the ballots necessarily did this deliberately. It is just human nature to adapt the rules towards the side you favor.

change in votes for a particular county.

The evidence in this paper suggesting that the recount numbers are unlikely to have resulted from machine read errors comes in three forms. First, the data is split into counties that did and did not use punch card ballots. These counties are then analyzed separately throughout the analysis. The primary finding is that it makes very little difference. Gore does unexpectedly well in both data subsets. Second, the paper asks what fraction of the ballots would have to have been read differently in the recount to explain the number of votes Gore gained relative to Bush. The answer is approximately 10%, and that number appears to be implausibly high.[4]

The third source of evidence presented here paper comes from a statistical model representing the recount process. The model allows for machine read errors in which votes in the first recount that went to a candidate revert to ballots without a recorded vote, and for the converse in which a ballot previously read as not voting then goes to a particular candidate. Parameter estimates indicate a bias in the latter type of error in Gore's favor across all ballots of about 15%. Further, Gore also retained a surprisingly large number of votes relative to Bush in those counties that do not use punch card ballots. In other words, if a ballot was read as a Gore vote in the first count it was more likely than a Bush vote to be reread that way. Accounting for all of these effects, it appears that Gore gained approximately 903 more votes from the recount than one would have expected by the chance fluctuations associated with machine read errors.[5]

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[4]Rusin (2000) estimates that about 1% of the ballots were read differently during the recount, while the vote counting model estimated in this paper places it at .36%.

[5]This number derives from the statistical model estimated under the null hypothesis. Under the null each candidate is equally likely to see a vote that was read as for him reread as no vote. Similarly, each ballot that changes reading from no vote to a vote is allocated to a candidate in proportion to the votes he received in the initial count.

From these three sources of evidence it is difficult to see how Gore's relative gain in the recount could have derived from machine errors, or quirks associated with particular ballot types rather than the influence of those counting the ballots.

This paper is organized as follows. Section 2 discusses issues with the data. Subsection 2.1 explores the chad issue and explains it potential impact on the recount results. Next, Subsection 2.2 discusses the data generated from the recount in Pinellas and Palm Beach Counties. Section 3 contains the paper's primary statistical findings. This section is further divided into several subsections. Section 3.1 provides estimates of the probability that Gore's relative gain in votes from the recount can be explained via natural fluctuations from mechanical read errors. Section 3.2 discusses and estimates an empirical model of the recount process, which yields estimates of the source of Gore's strong performance in the recount. Section 3.3 explores the possible alternative hypothesis that the first count was biased by human actions and that the second was the more accurate. Finally, Section 4 concludes.

2 Data Issues

2.1 The Chad Issue, Or Lack Thereof

When people poke their punch card ballots they do not always fully dislodge the chad indicating their preference. Instead the chad hangs on by one or more corners, an object apparently known as a hanging chad. This raises any number of problems for those attempting to count the votes whether they be machines or humans. If a chad blocks its hole the counting machine assumes the chad was not punched and therefore does not register a vote for whatever or whoever that hole represents.

The chad problem arises from the fact that as ballots are handled some hanging chads will fall off, while others close. Still others that were never poked by the voter are doubtlessly broken off by the repeated fanning of the cards to ensure that they do not stick together. All together this means that every time the votes are counted they produce a different answer, sometimes up and sometimes down. Because it is so easy to see how punch card ballots can produce different readings during each count the press has been quick to label them as unreliable.[6] There has also been the further assertion that, in contrast, the optical scanner ballots produce far more consistent readings from tally to tally. For example Morgan (November 14, 2000) claims that ". . . the 38 counties using optical scanner ballots, a system in which a machine reads stiff paper ballots marked with black ink, had far fewer changes in the total number of votes cast."[7] This is basically misleading. The counties using punch card ballots had 2277 changes in the total number of votes cast, and those using optical scanner ballots 1948. Furthermore, if you allow for the total number of ballots cast by type, the number of changes per ballot is smaller for the punch card ballots (one change per 1614 votes) than the optical scanner ballots (one change per 1094 votes). Overall then, the Florida recount does not lend any evidence to the idea that error rates across ballot types are likely to explain very much of the data.

2.2 Pinellas and Palm Beach

Both Pinellas and Palm Beach counties introduce unique problems to the analysis because they both produced large lopsided gains for Gore. In Pinellas there are conflicting reports regarding the source of this Gore bonanza. A Reuters news article claims that 400 of the Gore votes came

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[6]See, for example, Jackson (2000).

[7]Morgan (November 14, 2000).

from a batch of "some 400 ballots" found after the initial count in Pinellas County.[8] A

conflicting report in *Time Magazine* by Lacayo (2000) claims that, "In Pinellas County, when

election officials removed the chaff from the ballots before they were submitted for recount by

the machines, Gore picked up an additional 417 votes." Which explanation holds has a

substantial impact upon how one should interpret results with and without Pinellas County's

data. If the ballots were simply found, then they are not really due to changes in machine

readings and should be excluded from the analysis. However, if they arose from human

intervention then they should be included. In this case there is simply eye witness evidence that

corroborates the paper's thesis that Gore's relative performance in the recount cannot be

explained via normal changes due to fluctuations in machine readings. Nevertheless, to ensure

that the Pinellas data does not corrupt the results most of the paper's statistics are conducted with

and without its data.

Palm Beach poses another problem. The recount in that county gave Gore an additional

859 votes relative to Bush, by far Gore's best county. The only item unique to Palm Beach

County was the use of a so called "butterfly ballot."   Unfortunately, that ballot style appears to

have lead to some voter confusion causing them to vote for Buchanan rather than Gore.[9]

However, to the degree people punched out Buchanan's chad by mistake, this should not lead a

mechanical recount to either add or subtract an unusual number of votes for Gore. Nevertheless,

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[8]See, Reuters (November 9, 2000).

[9]A butterfly punch card ballot has candidates listed alternately on both sides of the ballot
with holes running down the center. In this case Gore was listed second on the left, but was the
third hole down from the top. Some voters claim this mislead them into punching the second
hole from the top which was for Buchanan who was listed first on the right.

Palm Beach County's produced so many additional votes for Gore in the recount that many of the paper's statistics are reported both with and without its data to ensure that this single observation does not drive any of the results.

3 Empirical Analysis

3.1 Estimates of the Probability Mechanical Errors Explain the Recount Results.

Numerous press reports have discussed how vote counts are likely to increase as partially dislodged chads fall off of punch card ballots. This may have lead to the impression that both candidates reaped additional votes in nearly every county from the recount. However as the next table shows this is not true.[10]

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[10]The voting data was obtained from the Tallahassee Democrat website at http://www.tdo.com/news/local/1111.nws.recount.htm. More recent updates may have a minor impact on the results reported here.

Table 1: Florida County Vote Changes as a Result of the First Unofficial Recount		
	Bush	Gore
	All Counties	
Increase	41	45
Unchanged	16	12
Decrease	10	10
	Punch Card Ballot Counties	
Increase	15	15
Unchanged	5	3
Decrease	4	6
	Non-Punch Card Ballot Counties	
Increase	26	30
Unchanged	11	9
Decrease	6	4

Overall nothing appears out of the ordinary. For both candidates the chance that the recount increased their vote total was about four times the chance it would go down in any one county. Gore did somewhat better than Bush here, but not by much. Counties that use punch card ballots are only slightly more likely to show an increase in a candidate's vote total after the recount, than those that use some other ballot type.

Now to address the paper's primary question of whether Gore's performance in the recount is likely to have come from the typical changes that arise from measurement error. Overall, Gore benefitted from the recount gaining 1225 votes throughout Florida. What are the odds this occurred by chance? If during the recount a random ballot is read differently then the change in reading should not, on average, favor either candidate since both obtained about half

the vote.[11] This means there is a 50-50 chance any revised reading of a ballot will lead to a

relative increase in Gore's vote total. Based upon the recount there were a total of 4245 reported

"revisions," defined here as any event that changes either candidate's total. If the revisions are

unbiased then the probability that 4245 revisions events will produce 1225 or more net positive

outcomes for one candidate is about zero (t-statistic of 18.8).[12] Even if you exclude both Palm

Beach and Pinellas Counties on the grounds that both are unusual for some reason there still

remains almost no chance the results were due entirely to luck (t-statistic of 6.8).[13]

At this point some people will note that the figures in the previous paragraph understate

the chance Gore would gain 1225 votes. The vote totals are not really representative of the total

number of revisions. For example, in a punch card county if one Gore chad opened up and

another closed then they would net out and the vote total would not change. Thus, the above

statistic overstates how unlikely it is that Gore would gain 1225 votes. Unfortunately, there is no

way to recover the total number of ballots that were read differently during the recount. Instead

one can ask how many such ballots would have to exist in order to provide Gore with a 1225

vote gain with some minimum probability. The following table provides the answer for various

levels of scepticism.

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[11]For this argument it does not matter what type of ballot a county used. All that is
necessary is that any change is equally likely to favor either candidate. Since this may not be true
an adjustment will be made later on.

[12]Excluding the 400 Pinellas votes for Gore whose source is in dispute has almost no
impact on the results. In this case the t-statistic goes down to 13.3, which means there is still a
almost no chance Gore's increase is due to ballot changes that equally favor either candidate.

[13]Without Palm Beach and Pinellas Counties Gore gains 344 votes out of 2908 revisions.

Table 2: Number of Revisions Needed for Various Probability Levels that Either Candidate Gains 1225 or More Votes Throughout Florida in a Recount					
Probability	1%	5%	10%	25%	40%
Number of Chad Events	355,344	390,625	554,550	1,134,688	2,101,642
Probability of a Revision	6.1%	6.7%	9.5%	19.5%	36.1%

Before going through the analysis a few disclaimers are in order. The probability of a revision is calculated based upon the total number of Gore and Bush votes reported. Thus, this number is somewhat too high since some ballots did not register for either candidate. However, given the overall fraction of the ballots cast for the two major candidates this seems unlikely to impact the revision probability line by much. Despite these caveats, the above table makes it quite clear that revisions favored Gore to a statistically unlikely degree. This conclusion is further verified in Section 3.2. There the paper discusses the parameter estimates from a statistical recount model, and concludes that the actual revision rate was about .36%.

Since not all Florida counties used punch card ballots it is possible the varying ballot styles explain the results. It may be that pro-Gore counties were more likely to use punch card ballots than pro-Bush ones. If so, than a ballot may be more likely to change state in those counties where Gore did well simply because of the type of ballot that was used.[14] The most obvious adjustment is to redo the previous exercise but exclude Pinellas and then both Pinellas and Palm Beach Counties. Removing Pinellas removes the county where most of Gore's

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[14]The general claim is that punch card ballots are more likely to register a vote change from a recount than other ballot types. It turns out this is not true and later on the paper will provide evidence on this.

additional votes came from ballots that may not have been included in the initial tally, and thus

have nothing to do with machine read errors. Removing Palm Beach eliminates a possible

outlier that may be driving the results.

Table 3: Number of Revisions Needed for Various Probability Levels that Either Candidate Gains 987 or More Votes Throughout Florida in a Recount Excluding Pinellas County					
Probability	1%	5%	10%	25%	40%
Number of Revisions	230,680	253,584	360,000	736,611	1,364,335
Probability of a Revision	4.2%	4.7%	6.6%	13.6%	25.1%
Number of Revisions Needed for Various Probability Levels that Either Candidate Gains 344 or More Votes Throughout Florida in a Recount Excluding Palm Beach and Pinellas Counties					
Number of Revisions	28,022	30,804	43,466	89,479	165,731
Probability of a Revision	0.6%	0.6%	0.9%	1.8%	3.3%

Removing only Pinellas County leaves the basic conclusion roughly unchanged. The relative

gain by Gore in the recount is statistically unlikely unless a very substantial fraction of the ballots

produce a different answer after each reading.

Eliminating both Pinellas and Palm Beach makes the numbers in Table 3 appear

somewhat more plausible. However, this plausibility is just an illusion. The number of required

revisions listed seriously underestimates the actual number that would be required to reproduce

the 344 vote gain by Gore in these areas of Florida. This is due to "data mining." The form of

data mining involved here has to do with the fact that Palm Beach and Pinellas were removed

precisely because they both gave Gore several hundred additional votes in the recount. More

accurately we should ask the following: Suppose we randomly generate data across counties. Next, *toss out* the two counties where the candidate getting the largest boost does best. Now how many events are needed to yield that candidate a relative gain of 344 votes? The answer is quite a few. Running 2280 Monte Carlo simulations of this problem, each with 500,000 chad events, produced *not one single case* in which the leading candidate received 344 or more votes. Repeating this exercise, but this time with only one county removed once again failed to produce a single result in which the leading candidate received 344 or more votes out of 4000 trials.[15] What this means is that the probability that one candidate would receive a net gain of 344 votes after tossing out his one or two best counties is about zero even if 500,000 ballots change from either counted to uncounted or visa versa.

Since Palm Beach County has generated so much controversy another approach to the question of the where the revised ballot readings originated from is to ask whether the recount numbers were likely to have come from a chance outcome. In Palm Beach, Gore received 63.8% of the vote, and gained 643 votes after the recount. Then, for this exercise, suppose that any ballot that is read differently during the recount has a 63.8% chance of increasing Gore's total vote relative to Bush's. As before, consider the initial assumption that the total vote changes correspond to the total number of ballots that changed state. In this case the probability that Gore would gain 643 or more votes out of 859 events is once again about zero (t-statistic of 14.4). Next reverse the problem and ask how many total ballot changes would be needed to produce a gain for Gore of 643 or more votes with some probability.

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[15]The simulations assumed every county is of equal size and equally likely to vote for either candidate. Clearly this is not true. However, it seems unlikely that adding these features will dramatically impact on the results.

Table 4: Number of Revisions Needed for Various Probability Levels that Gore Gains 643 or More Palm Beach County Votes in a Recount					
Probability	1%	5%	10%	25%	40%
Number of Revisions	1970	2068	2123	2219	2287
Probability of a Revision	0.47%	0.49%	0.50%	0.53%	0.54%

In this case things look a lot better since it seems plausible that just under one-half of a percent of the ballots changed state in the recount. While this seems rather high for something like a ballot (imagine if your bank incorrectly recorded one of your transactions .5% of the time) it is not out of the realm of possibility.

Table 4 brings up an interesting possibility. Suppose counties in which Gore was favored also happened to use ballots with high revision rates. Then this correlation may explain the recount results. It is possible to test the above hypothesis by simply dividing the Florida vote into those counties that did or did not use a punch card ballot. As noted earlier 27 counties use punch card ballots and the rest some other device.[16] Of the 4245 ballot changes 2277 come from one of the punch card counties, and 1962 from the others. The combined vote in the punch card counties for Gore and Bush equals 3,680,860, while in the remainder it totals 2,139,187. Thus, punch card counties reported a revised vote once in every 1614 votes, while the other counties reported a revised vote once in every 1093 votes (note, here low numbers mean more frequent revisions). Next consider the fact that the punch card counties delivered 1.11 votes to Gore for every Bush vote, while the other counties gave Gore .84 votes for every Bush vote. Then the

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[16]This list was taken from the website of the Division of Elections - Florida Department of State (2000).

implication is that if rereading the ballots randomly changed the vote totals the revisions should

have given Bush 0.745 and Gore 0.721 votes per 1000 read, on average. If anything then the

assumption that ballot changes were equally likely to favor either candidate should be replaced

with the assumption that ballot changes were likely to favor Bush by about 3%.

Perhaps then it is the brand of punch card ballot used in Palm Beach County that explains

the increase in Gore's relative vote total after the recount. However, again the evidence does not

support this view. For example, Broward cast more votes than Palm Beach, went for Gore in

similar proportions, and used a punch card system from the same manufacturer. But, after

Broward's recount Gore actually *lost* one vote relative to Bush. Another difference is that

Broward apparently has a much lower rate of ballot changes. To complete the comparison, there

are another nine Florida counties that use the same ballot company as Palm Beach. Their data is

separated out in the following table.

Table 5: Counties Using Punch Card Ballots From the Firm Supplying Palm Beach County				
County	Total Votes	Fraction of the Total Gore plus Bush Vote Going to Gore	Total Ballot Changes	Total Ballot Changes Divided by Total Votes
Broward	563,884	.69	87	0.0154%
Collier (VM)	90,351	.33	20	0.0221%
Highlands	34,373	.41	25	0.0727%
Hillsborough	350,317	.48	75	0.0214%
Lee	179,701	.41	48	0.0267%
Marion	99,806	.45	23	0.0230%
Osceola	54,393	.52	8	0.0147%
Palm Beach	422,650	.64	859	0.2032%
Pasco	138,146	.50	15	0.0109%
Pinellas	385,452	.52	478	0.1240%
Sarasota	155,953	.47	1	0.0006%

Rather strikingly Palm Beach and Pinellas stand out as having much higher ballot change rates than any other county. If individual ballot changes are due to chance then such changes should be equally probable in every county since all of the counties used the same ballot system. However, both Palm Beach and Pinellas are far more likely to have experience a vote change than any other county.[17] If ballot changes occur at some constant rate then the process that produced 87 changes out of 563,884 in Broward should also have produced 859 ballot changes

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[17]Although, as noted earlier, most of the Pinellas changes can be explained via the bundle of ballots found after the initial count. However, to the degree any of the figures are due to recovered ballots they should not, on average, favor either candidate.

out of 422,650 in Palm Beach. Using Broward as the base the probability that the Broward rate

could have produced the Palm Beach numbers is about zero (t-statistic of 12.2). Other

comparisons are possible but it is clear that whatever forces are leading to the changing votes in

the various counties the discrepancies cannot be attributed solely to the type of ballot. This

implies that the press reports attributing Gore's remarkable increase in votes relative to Bush

cannot be explained only on the inability of typical Democrat voters to properly fill out a punch

card.[18]

3.2 Statistical Model of the Recount Process

So far the analysis has shown that mechanical read errors are unlikely to explain the relative

increase in Gore's vote total from the recount. This section quantifies the degree to which the

recount was biased in his favor by analyzing an empirical model of the process.

After the initial count the ballots can be divided up into three groups, those that recorded

a vote for Bush, those that recorded a vote for Gore, and those that recorded no vote for

President. The no vote category includes ballots with two recorded votes, those that appear

unmarked, or any other ballot from which the machines could not discern a presidential

preference. Let ΔG and ΔB represent a change in the total vote recorded for Gore or Bush after a

particular ballot has been reread. Then during the recount the following table summarizes how

ΔG and ΔB change after a ballot has been read:

[18]For example, Loney (2000).

Table 6: Probability Matrix for Changes to ΔG and ΔB After a Ballot is Reread						
	ΔG			ΔB		
	-1	0	+1	-1	0	+1
Gore Vote	$1-\alpha_G$	α_G	0	0	0	0
Bush Vote	0	0	0	$1-\alpha_B$	α_B	0
No Vote	0	$e(1-\beta_G)$	$e\beta_G$	0	$e(1-\beta_B)$	$e\beta_B$

The first row of the table states that if in the initial count a ballot was read as a vote for Gore, then there exists a probability $1-\alpha_G$ that in the recount the vote will be read as a vote for neither candidate. Row two describes a similar process for a ballot read as a vote for Bush in the initial count. Finally, row three states that a vote which was read as indicating no vote in the initial count will be read as a Gore vote with probability $e\beta_G$, and a Bush vote with probability $e\beta_B$. The parameter e represents the overall probability that a ballot initially read as no vote will change upon recounting to a vote for either Gore or Bush. Given that the ballot does drop from the no vote category, the β_G and β_B parameters then determine the likelihood that the ballot will add to Gore or Bush's total. Naturally, β_G and β_B sum to one.[19]

The two questions of interest are whether or not α_G equals α_B, and if β_G and β_B are reflective of the fraction of the vote each candidate received in a particular county. The former is simple to test, but the latter requires additional structure. If q_i represents the fraction of the vote going to candidate i in the initial count, then it seems natural to assume as a null hypothesis that

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[19]A related model exploring the distribution of vote totals from a hand recount can be found in an article by Rusin (2000).

β_i should equal q_i.[20] That is, the changes from no vote to vote should break (on average) for a candidate in proportion to his initial vote in the county. As an alternative hypothesis, let t represent a tilt parameter. Then the model assumes the values of β_G and β_B depend upon both the vote in a county and t via the following formulae:

$$\left.\begin{array}{l} \beta_B = q_B(1-t) \\ \beta_G = q_G + t(1-q_G) \end{array}\right\} \text{ for } 0 \leq t \leq 1, \tag{1}$$

and

$$\left.\begin{array}{l} \beta_B = q_B - t(1-q_B) \\ \beta_G = q_G + tq_G \end{array}\right\} \text{ for } -1 \leq t < 0. \tag{2}$$

For t equal to zero β_i does indeed equal q_i. However, as t goes towards -1, β_B goes toward 1 and β_G goes towards zero. In the opposite case as t goes to +1, β_B goes to zero and β_G goes to one. Equations (1) and (2) have been designed to ensure that $\beta_B + \beta_G = 1$ for all t. The parameter t can be thought of as the fraction of the no votes accruing to a candidate above and beyond what would be expected by chance, given the fraction of votes received by the candidate in the balloting.

The above empirical model produces the following moment conditions:

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[20]To avoid notational clutter subscripts on the β_i and q_i indicating that they vary from county to county have been suppressed.

Table 7: Moment Conditions Per Ballot Recounted					
	$E(\Delta G)$	$E(\Delta B)$	$Var(\Delta G)$	$Var(\Delta B)$	$Cov(\Delta G, \Delta B)$
Gore Vote	α_G	0	$(1-\alpha_G)\alpha_G$	0	0
Bush Vote	0	α_B	0	$(1-\alpha_B)\alpha_B$	0
No Vote	$e\beta_G$	$e\beta_B$	$(1-\beta_G)\beta_G$	$(1-\beta_B)\beta_B$	$-\beta_G\beta_B$

No votes produce a negative correlation between the candidates' vote totals because when such a ballot goes to one person it has equivalently not gone to the other. That is, a gain for one candidate is a loss for the other. Using these moment conditions the model was estimated by approximating the multinomial distribution via the bivariate normal and then calculating the parameter values via maximum likelihood. Doing so yields the following results.[21]

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[21]Data for the number of uncounted votes per county come from *Sun-Sentinal.com* (2000)

Table 8: Maximum Likelihood Estimates – Vote Counting Model

α_i: Probability a ballot recorded as a vote for i will revert to no vote in the recount.
e: Probability a ballot recorded neither for Bush or Gore will revert to a vote in their favor in the recount.
t: Tilt parameter impacts the conditional probability that a candidate will receive a vote when a ballot initially recorded as no vote changes to a positive vote for either Bush or Gore. At zero, Bush and Gore receive votes with probability equal to their fraction of the county's vote in the initial count. At one, Gore receives all such changes, at minus one Bush receives all such changes. For intermediate values the formulae are:

$$\left.\begin{array}{l} \beta_B = q_B(1-t) \\ \beta_G = q_G + t(1-q_G) \end{array}\right\} \text{ for } 0 \le t \le 1, \quad \left.\begin{array}{l} \beta_B = q_B - t(1-q_B) \\ \beta_G = q_G + tq_G \end{array}\right\} \text{ for } -1 \le t < 0.$$

Where β_i equals the probability that a vote that changes from no vote will go to candidate i, and q_i equals the fraction of the county vote going to candidate i.
L: Maximized value of the log of the likelihood function.
χ^2: The χ^2 statistic from the maximum likelihood ratio test of the unrestricted model against the restricted model.
Standard Errors in parentheses.

	Unrestricted Model	Same Probability of a Lost Vote	Same Probability of a Recovered Vote	Same Probability of Both a Lost or Recovered Vote
	All Counties			
α_B	99.8393% (0.0033)	99.8349% (0.0025)	99.8175% (0.0031)	99.8309% (0.0026)
α_G	99.8291% (0.0038)		99.8463% (0.0030)	
e	7.34% (0.0919)	7.32% (0.0085)	7.42% (0.0880)	7.46% (0.0900)
t	14.85% (1.2020)	13.28 (0.9151)	0	0
L	7512.4	7514.6	7592.9	7622.6
χ^2		4.38	161	220
	All Punch Card Counties			
α_B	99.8291% (0.0067)	99.8182% (0.0041)	99.7858% (0.0059)	99.8047% (0.0053)
α_G	99.8053% (0.0080)		99.8352% (0.0060)	
e	6.52% (0.1535)	6.47% (0.1202)	6.65% (0.1466)	6.85% (0.1478)
t	19.45% (2.2382)	14.83% (1.1528)	0	0
L	1768.4	1771.5	1809.4	1859.8
χ^2		6.20	81	183

Table 8: Maximum Likelihood Estimates – Vote Counting Model

α_i: Probability a ballot recorded as a vote for i will revert to no vote in the recount.

e: Probability a ballot recorded neither for Bush or Gore will revert to a vote in their favor in the recount.

t: Tilt parameter impacts the conditional probability that a candidate will receive a vote when a ballot initially recorded as no vote changes to a positive vote for either Bush or Gore. At zero, Bush and Gore receive votes with probability equal to their fraction of the county's vote in the initial count. At one, Gore receives all such changes, at minus one Bush receives all such changes. For intermediate values the formulae are:

$$\left.\begin{array}{l}\beta_B = q_B(1-t)\\ \beta_G = q_G + t(1-q_G)\end{array}\right\} \text{ for } 0 \le t \le 1, \quad \left.\begin{array}{l}\beta_B = q_B - t(1-q_B)\\ \beta_G = q_G + tq_G\end{array}\right\} \text{ for } -1 \le t < 0.$$

Where β_i equals the probability that a vote that changes from no vote will go to candidate i, and q_i equals the fraction of the county vote going to candidate i.

L: Maximized value of the log of the likelihood function.

χ^2: The χ^2 statistic from the maximum likelihood ratio test of the unrestricted model against the restricted model. Standard Errors in parentheses.

	Unrestricted Model	Same Probability of a Lost Vote	Same Probability of a Recovered Vote	Same Probability of Both a Lost or Recovered Vote
	All Punch Card Counties Except Pinellas			
α_B	99.8235% (0.0072)	99.8203% (0.0055)	99.8008% (0.0062)	99.8140% (0.0053)
α_G	99.8164% (0.0076)		99.8344% (0.0060)	
e	6.03% (0.1576)	6.01% (0.1519)	6.06% (0.1486)	6.19% (0.1440)
t	11.25% (2.2392)	9.85% (1.2146)	0	0
L	1440.1	1440.3	1452.7	1474.8
χ^2		0.524	25.3	69.4
	All Punch Card Counties Except Pinellas and Palm Beach			
α_B	99.7711% (0.0104)	99.7967% (0.0073)	99.7862% (0.0078)	99.7943 (0.0095)
α_G	99.8182% (0.2333)		99.8052% (0.0077)	
e	7.05% (0.0092)	7.00% (0.2267)	7.02% (0.2377)	7.08% (0.3297)
t	-6.55% (2.6420)	4.40% (1.2111)	0	0
L	1349.1	1351.3	1352.1	1357.9
χ^2		4.45	6.12	17.73

Table 8: Maximum Likelihood Estimates – Vote Counting Model

α_i: Probability a ballot recorded as a vote for i will revert to no vote in the recount.

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t: Tilt parameter impacts the conditional probability that a candidate will receive a vote when a ballot initially recorded as no vote changes to a positive vote for either Bush or Gore. At zero, Bush and Gore receive votes with probability equal to their fraction of the county's vote in the initial count. At one, Gore receives all such changes, at minus one Bush receives all such changes. For intermediate values the formulae are:

$$\left.\begin{array}{l} \beta_B = q_B(1-t) \\ \beta_G = q_G + t(1-q_G) \end{array}\right\} \text{for } 0 \leq t \leq 1, \quad \left.\begin{array}{l} \beta_B = q_B - t(1-q_B) \\ \beta_G = q_G + tq_G \end{array}\right\} \text{for } -1 \leq t < 0.$$

Where β_i equals the probability that a vote that changes from no vote will go to candidate i, and q_i equals the fraction of the county vote going to candidate i.

L: Maximized value of the log of the likelihood function.

χ^2: The χ^2 statistic from the maximum likelihood ratio test of the unrestricted model against the restricted model. Standard Errors in parentheses.

	Unrestricted Model	Same Probability of a Lost Vote	Same Probability of a Recovered Vote	Same Probability of Both a Lost or Recovered Vote
	All Non-Punch Card Counties			
α_B	99.8736% (0.0040)	99.8846% (0.0029)	99.8684% (0.0041)	99.8837% (0.0029)
α_G	99.8983% (0.0041)		99.9008% (0.0040)	
e	12.65% (0.1911)	12.63% (0.1803)	12.76% (0.1812)	12.68% (0.1826)
t	6.74% (1.3675)	8.24% (1.3145)	0	0
L	4674.7	4683.7	4687.5	4703.8
χ^2		17.86	25.46	58.13

The tilt parameter indicates that the probability that a no vote converts to a Gore vote is about 15% higher than chance would indicate in the whole sample, about 19% higher in the punch card counties, and about 7% higher in the non-punch card counties. Testing this model against the alternative that the tilt parameter equals zero can be rejected in every case at any reasonable level of significance. While punch card ballots were more likely than one might expect from the initial vote to go from no vote to a Gore vote, so were the non-punch card counties. Another item to note is that one cannot reject the hypothesis that each candidate lost

votes at about the same rate (equality of α's) in both the full, and punch card samples at the 1% level. But, in the non-punch card sample the hypothesis is clearly rejected and it is the subset in which Gore does best relative to Bush on this measure. Thus, one can conclude that Gore's advantage over Bush came from the conversion of no votes in every county, and from Gore's ability to keep already counted votes in his column in the non-punch card counties.

Consistent with the earlier tables in the paper, the estimates in Table 8 were also conducted without the data from Pinellas and Palm Beach Counties. For the most part eliminating Pinellas does not change any of the conclusions one might draw from the estimated parameters. If both Pinellas and Palm Beach are dropped from the sample then one obtains the only negative tilt parameter (thereby pro-Bush) in the entire table. However, the parameter is not very stable. When the α parameters are restricted to equality the tilt parameter changes sign. Not unexpectedly then, the χ^2 statistic for whether the tilt parameter equals zero takes on a relatively low value remaining significant only at the 5% level. Furthermore, as noted earlier in the paper, eliminating Palm Beach eliminates the county where Gore picks up the greatest number of votes, and thus it should not surprise anybody that deliberately removing it from the data base will skew the results towards the hypothesis that the tilt parameter equals zero.

Table 8 also reinforces the conclusions reached earlier in the paper that the revision rates needed to explain Gore's performance in the recount are implausibly high. The α parameters provide an estimate of the probability that a ballot registered for one candidate would revise in the recount to no vote. Using the estimates from the full data set, with the α parameters for Gore and Bush set to equality, indicates that about .17% of all such ballots were read differently in the recount. From the same model the *e* parameter indicates that about 7.42% of the ballots read as

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containing no vote in the initial count were then read as a vote for one candidate or the other in the recount. To get the overall revision rate, one can now simply weight these estimates by the fraction of ballots initially read as a vote, and no vote respectively. Doing so implies that about .36% of all ballots were read differently in the recount. As tables 2, and 3 show this figure is far below the rate needed to conclude that Gore's performance in the recount can be attributed to simple fluctuations from machine read errors.

Based upon the above estimates one can also calculate the number of votes Gore received relative to Bush above and beyond what one would have expected from machine read errors. If the machine recount was unbiased then each candidate should have lost a particular vote with equal probability in the recount, and gained in proportion to his vote in the initial tabulation. Thus, the natural base case sets the values of α to equality, and the tilt parameter (t) to zero. Since punch card and non-punch card counties produce such different parameter estimates, it seems prudent to use the parameter estimates from each sample separately. Thus, for punch card counties assume that each candidate had a 1-.998047 chance of having a vote for him reread as no vote and that each candidate should have picked up $.0685q_i$ of the votes initially read as no vote. For the non-punch card counties the parameters are 1-.99837, and .1268 respectively. Applying these parameter estimates county by county the model predicts that Gore should have gained 322 votes, rather than the 1225 he actually picked up. All of which implies a net unexpected gain for Gore of 903 votes.[22]

Overall then, the parameter estimates in Table 8 make it very difficult to attribute Gore's

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[22]If instead one uses the estimates from the entire data set, and thus does not control for the ballot type, then Gore would have been expected to pick up 450 votes in the recount.

relative gain in votes over Bush to any particular feature of balloting process. Gore does

unexpectedly well in all counties, punch card counties, and non-punch card counties alike. To

the degree there is any variation it has to do with the reason Gore did so well. In the punch card

counties it was due to his ability to pick up votes among votes previously counted as no vote,

while in the non-punch card counties he gained from that as well as a relative ability to hold on to

those votes that were counted as his the first time through. Thus, the statistical model provides

yet additional evidence that human intervention played a role in the final recount tallies.

3.3 Which Count was Biased?

So far the analysis has shown that the recount figures are very unlikely to have been produced by

random chance. Under one interpretation, this may simply prove that the initial count, rather

than the recount, was biased.[23] Supposedly then, the people that conducted the initial count used

their discretion to bias it and then the media attention forced an honest recount. Intuitively, there

are two reasons to believe this is unlikely. First, the press has not indicated that the ballots were

run through more than once during the initial tally. If so, then the preferences of the counters

could not have played a role in the outcome. Second, when the first count was conducted nobody

knew just how close the election was. That means there was little incentive for those conducting

the count to influence it, and other concerns (like getting home for the night) should have

dominated. In contrast, when the recount was conducted everybody knew every vote counted

and thus partisan biases would be expected to play their strongest possible role.

Beyond intuition one can also use the data to help indicate which count appears to have

been subject to the greater degree of human influence. Suppose, for now, that the recount was in

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[23]See, for example, Miller (2000).

fact responsible for the removal of partisan biases. Then it should be the case that vote revisions

by county will be correlated in some manner with the initial votes received by each candidate.

Two possible hypotheses come to mind. First, the stronger a candidate's vote within a county the

easier it was for his representatives to influence the first count. The recount then removed this

bias. Accordingly, the recount should have given Gore an unexpectedly large fraction of the

added votes in counties where he did poorly, and visa versa. The second hypothesis is that if a

county runs strongly for one candidate then those doing the counting have no incentive to alter

the reported result. In that case the hotly contested counties will show the largest unexpected

revisions. Furthermore, the revisions should simply reflect which side happened to have the

stronger personalities on hand during the initial tabulations and thus be evenly distributed

between Bush and Gore. The following figure contains the result of that exercise.
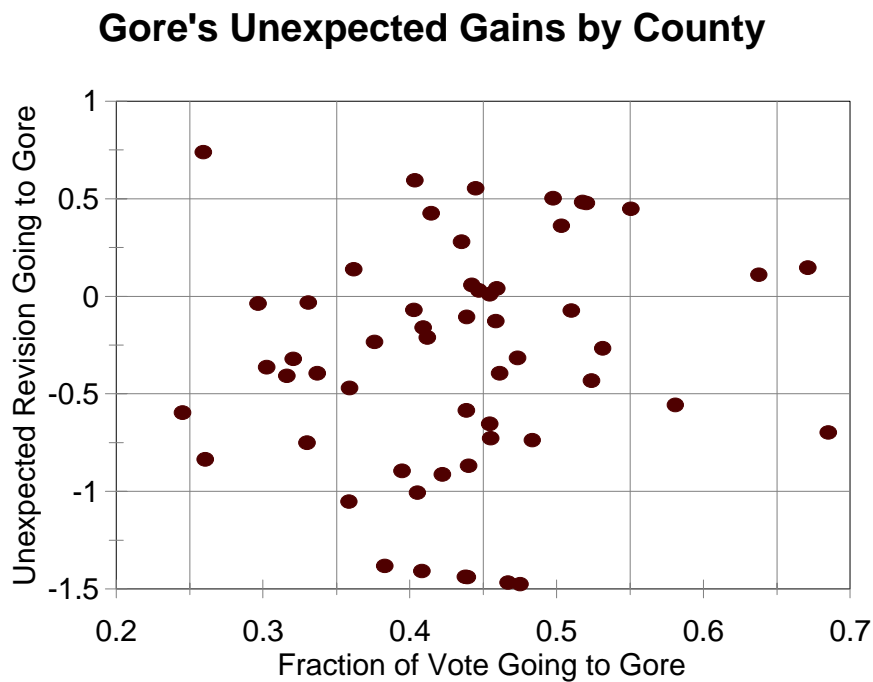
Gore's Unexpected Gains by County

Figure 1

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The unexpected revision going to Gore equals the total fraction of the of votes gained by Gore in a county divided by the total number of votes changed in that county minus Gore's faction of the county vote.[24]

The cloud of data displayed in **Figure 1** does not appear to have any discernable pattern. Instead, it looks remarkably like a textbook scatter diagram displaying two uncorrelated random variables. As such, neither of the hypothesis under which biases in the initial vote might explain Gore's unexpectedly large gains in the recount are supported. This implies that those arguing for the conclusion that the recount simply removed the impact of human judgement must also argue that partisan officers were able to alter the initial tally unimpeded by a community's support for their particular candidate. Otherwise the only other logical conclusion is that it is the recount that reflects the impact of partisan bias.

4 Conclusion

Despite the fact that ballot changes seem to favor Gore by an unlikely amount, the numbers do not prove once and for all that the recount changes were a result of skullduggery. As noted before, human judgements were used to determine how often to count the ballots and that probably helped to skew the results. It is also possible, although the author thinks unlikely, that Gore voters are more likely to ambiguously mark their ballots relative to Bush voters. However, absent some evidence that physical conditions account for the gain by Gore in the recount it

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[24]In formula form: (Number of revised votes going to Gore)/(Number of revised votes going to Gore + Number of revised votes going to Bush) - Fraction of the initial vote going to Gore. Numbers on the y-axis below minus one occur in those counties where Gore lost votes in the recount while Bush gained.

seems fairly likely that human judgement and all that it implies is responsible for the current outcome.

The debate has now moved on to whether or not to proceed with a hand recount. While those involved with recounts by machine retained some discretionary powers, their ability to influence the result was clearly circumscribed. Nevertheless, even with those restrictions it appears statistically likely that their preferences may have had an impact on the reported result. Relative to a machine count, one by hand offers far more opportunities for human biases to influence the outcome. Additional recounts may therefore produce newer numbers but not necessarily more accurate indications of who "won" the election on November 7.

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